Digital Signal Processing Introduction

Lecture-1

15-March-16

Course Assessment

- Total assessment 100%.
  - Midterm : 30%
  - Final Exam: 50%
  - Internal Evaluation: 20%

- Internal Evaluation 20%
  - Quizzes : 10%
  - Assignment: 10%

Internal Evaluation Details

- Total Quizzes 5 | Total Assignments 5

  Best of 4 Quizzes | Best of 4 Assignments

- Semester Project in form of groups will be conducted
  - Evaluation will be equal to 1 quiz and 1 assignment.

Total 5 Quizzes | Total 5 Assignments

Course Book: "Digital-Time Signal Processing", by Alan V. Oppenheim & Ronald W. Schafer & John R. Buck.

Introduction

- Digital Signal Processing (DSP) is the application of a digital computer to modify an analog or digital signal.
- The signal being processed is either temporal, spatial or both.
- Main goal is to be able to design Digital LTI filters.
- Such filters are widely used in applications such as audio entertainment systems, telecommunication and other kinds of communications, radar, video enhancement and biomedical engineering.

What is a Signal?

- A signal is defined as any physical quantity that varies with time, space or another independent variable.
- A signal is a function of independent variables such as time, distance, position, temperature and pressure.
- A signal is a real or complex valued function of one or more real variables.
- When the function depends on a single variable, the signal is said to be one-dimensional.
- When function depends on two or more variables, the signal is said to be multidimensional.

What is Signal Processing?

- A signal carries information and the objective of signal processing is to extract useful information carried by the signal.
- Signal processing is concerned with the mathematical representation of the signal and the algorithmic operation carried out on it to extract the information present.
- The signal is processed by a system.
- A system is defined as a physical device that performs an operation on a signal.

Advantages of DSP

- A digital programmable systems allows flexibility in reconfiguring the digital signal processing operations by changing the program. In analog redesign of hardware is required.
- In digital accuracy depends on word length, floating vs. fixed point arithmetic etc. In analog depends on components.
- Can be stored on disk.
- Cheaper to implement.
- Small size.
- Several filters need several boards in analog, whereas in digital same DSP processor is used for many filters.

Disadvantages of DSP

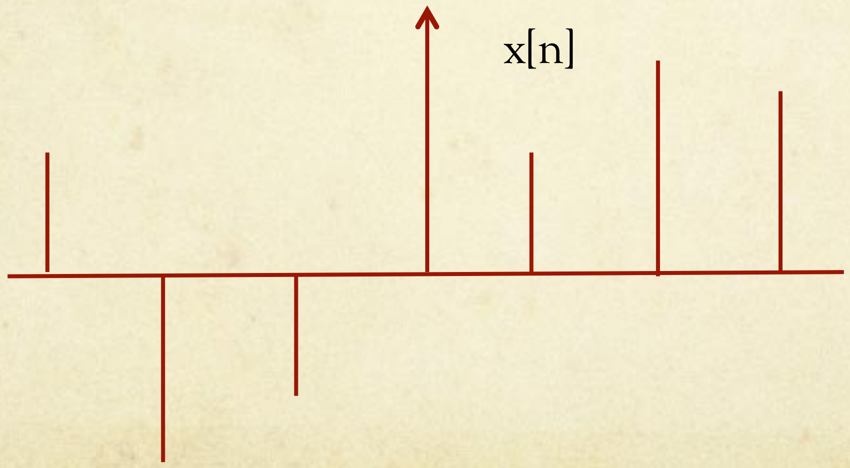
- When analog signal is changing very fast, it is difficult to convert digital form.
- Finite word length problems.
- When the signal is weak, within a few tenths of millivolts, we cannot amplify the signal after it is digitized.
- DSP hardware is more expensive than general purpose microprocessors & microcontrollers.

Applications of DSP

- Filtering.
- Speech synthesis in which white noise is filtered on a selective frequency basis in order to get an audio signal.
- Speech compression and expansion for use in radio voice communication.
- Speech recognition.
- Image processing, filtering, edge effects and enhancement.
- PCM used in telephone communication.
- Wave form generation.

Review of Discrete-Time Signals

- The number of elements in the set as well as possible values of each element is finite and countable.
- It can be represented with computer bits and stored on a digital storage medium.

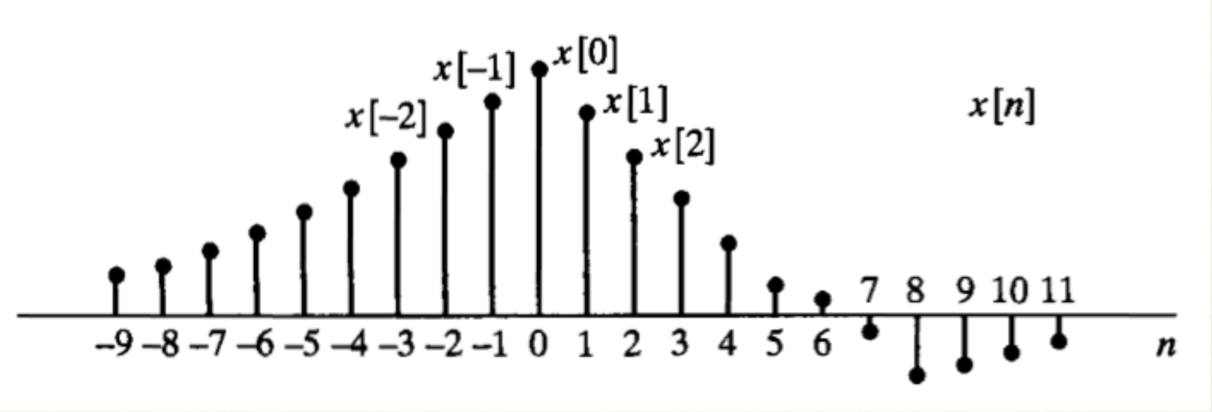


Sequences

- Discrete-time signals are represented mathematically as sequences of numbers.
- It is denoted by x[n] and formally written as: $x=\{x[n]\}, -\infty<n<\infty$
  - Where n is an integer.
- In practical setting, such sequences often arise from periodic sampling of an analog signal.
- The numeric value of the nth number in the sequence is equal to the value of the analog signal, $x_a(t)$, at time nT, i.e., $x[n] = x_a(nT), -\infty<n<\infty$
  - The quantity T is called the sampling period and its reciprocal is the sampling frequency.

Sequences (cont.)

- Graphical representation of discrete-time signal is as follows:



Basic Sequences & Sequence Operations

- The product and sum of two sequences x[n] and y[n] are defined as the sample by sample product and sum, respectively.
- Multiplication of a sequence x[n] by a number α is defined as multiplication of each sample value by α.
- A sequence y[n] is said to be a delayed or shifted version of a sequence x[n] if: $y[n] = x[n-n_0]$.
  - Where n_0 is an integer.

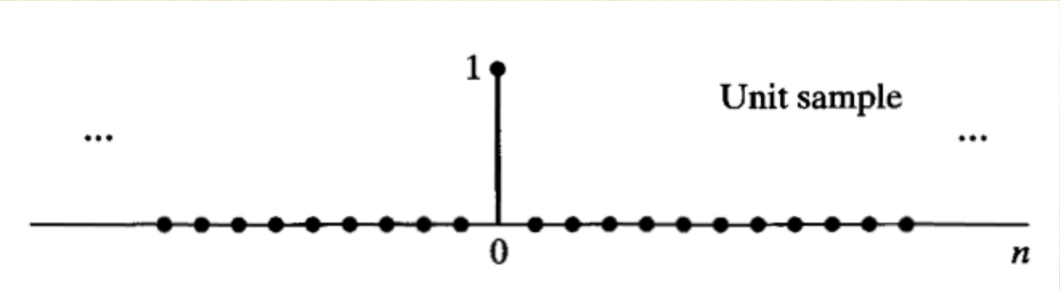
Basic Sequences & Sequence Operations (cont.)

- **Unit Sample/Impulse Sequence** :

$$\delta[n] = \begin{cases} 0, & n \neq 0, \\ 1, & n = 0. \end{cases}$$

- More generally any sequence can be expressed as:

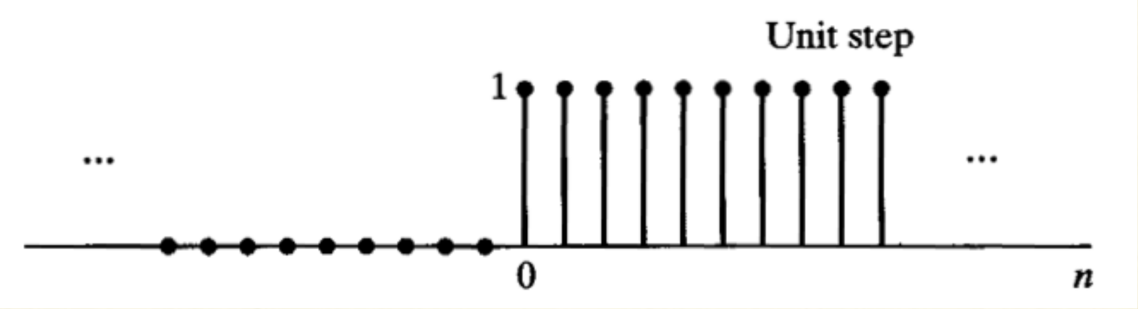
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



Basic Sequences & Sequence Operations (cont.)

- **<u>Unit Step Sequence:</u>**

$$u[n] = \begin{cases} 1, & n \geq 0, \\ 0, & n < 0. \end{cases}$$



- The unit step is related to the impulse by:

$$u[n] = \sum_{k=-\infty}^{n} \delta[k] \quad or \quad u[n] = \sum_{k=0}^{n} \delta[n-k]$$

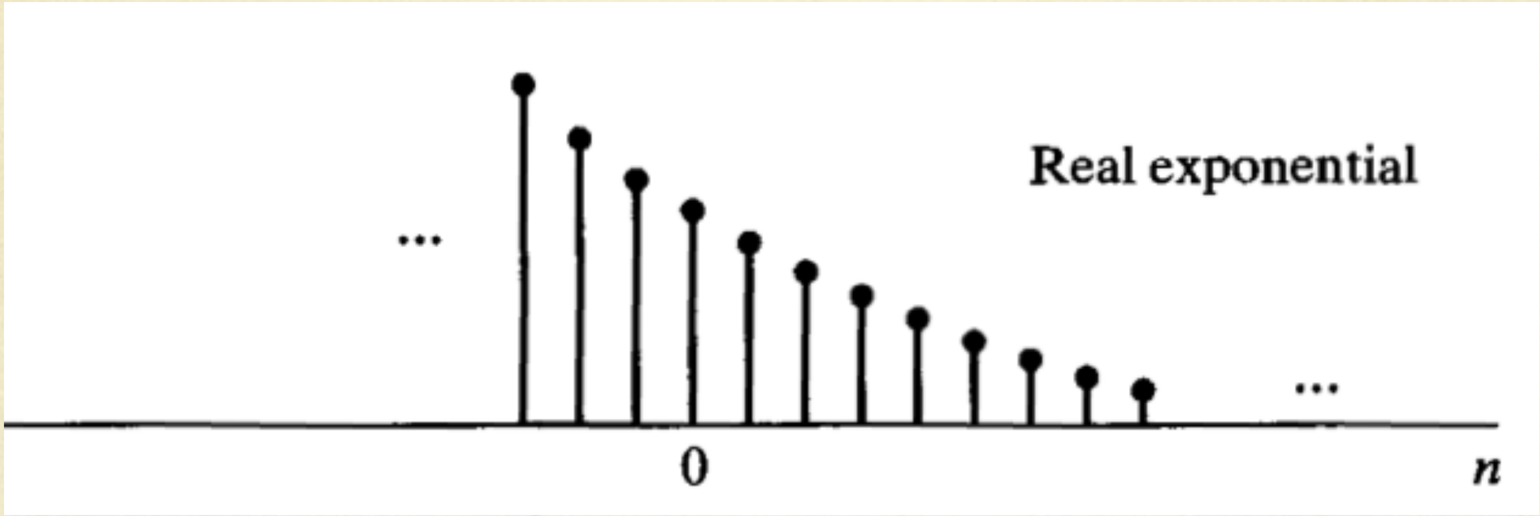
Basic Sequences & Sequence Operations (cont.)

- **Exponential Sequences:**

$$\delta[n] = u[n] - u[n-1]$$

  - The general form of an exponential sequence is: $x[n] = A\alpha^n$
  - If A and α are real numbers, then the sequence is real.
  - If $0<\alpha<1$ and A is positive, then the sequence values are positive and decrease with increasing n.
  - If $-1<\alpha<0$, the sequence values alternate in sign, but again decrease in magnitude with increasing n.
  - If $|\alpha|>1$, then the sequence grows in magnitude as n increases.

Basic Sequences & Sequence Operations (cont.)



Combination of Basic Sequences

- If we want an exponential sequence that is zero for n<0:

$$x[n] = \begin{cases} A\alpha^n, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

- A much simpler expression is:

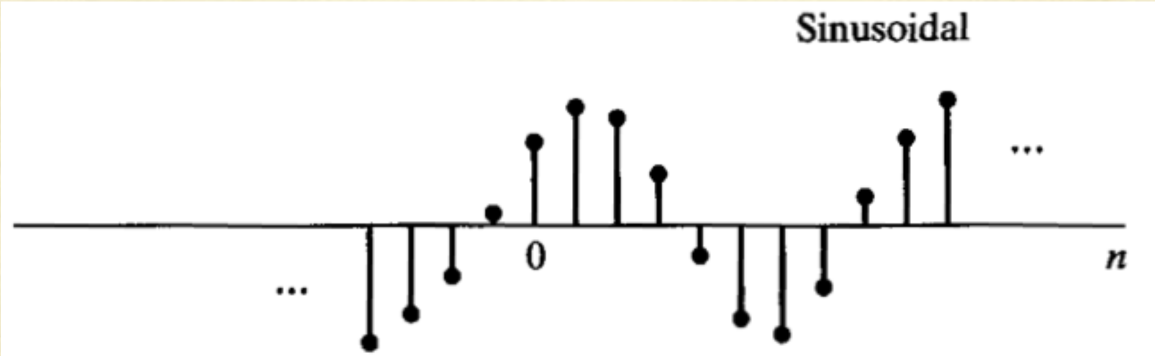
$$x[n] = A\alpha^n u[n]$$

Basic Sequences & Sequence Operations (cont.)

- **Sinusoidal Sequences:**

$$x[n] = A\cos(\omega_0 n + \phi), \quad for \quad all \quad n$$

- With A and Φ real constants.



- Euler's relation allows us to relate complex exponentials and sinusoids as:

$$e^{j\omega_0 n} = \cos\omega_0 n + j\sin\omega_0 n$$

Basic Sequences & Sequence Operations (cont.)

- And: $$A\cos(\omega_0 n+\phi)=\frac{A}{2}e^{j\phi}e^{j\omega_0 n}+\frac{A}{2}e^{-j\phi}e^{-j\omega_0 n}$$
- The general discrete time complex exponential can be written in terms of real exponential and sinusoidal signals.
- If we write C and α in polar form $C=|C|e^{j\theta}$ and $\alpha=|\alpha|e^{j\omega_0}$ then,

$$C\alpha^n=|C||\alpha|^n\cos(\omega_0 n+\phi)+j|C||\alpha|^n\sin(\omega_0 n+\phi)$$

- Thus for $|\alpha|=1$, the real and imaginary part of a complex exponential sequence are sinusoidal.
- If $|\alpha|<1$, they correspond to sinusoid sequence multiplied by a decaying exponential.
- If $|\alpha|>1$, they correspond to sinusoid sequence multiplied by a growing exponential.

Basic Sequences & Sequence Operations (cont.)

- **Periodic & Aperiodic:**
  - A periodic sequence is a sequence for which: x[n] = x[n+N], for all n.
    - Where the period N is necessarily an integer.
  - If this condition for periodicity is tested for the discrete-time sinusoid, then:

$$A\cos(\omega_0 n + \phi) = A\cos(\omega_0 n + \omega_0 N + \phi)$$

  - Which requires that:

$$\omega_0 N = 2\pi k$$

  - Where k is an integer.
  - A similar statement holds for the complex exponential sequence $Ce^{j\omega_0 n}$ i.e., periodicity with period N requires that:

$$e^{j\omega_0 (n+N)} = e^{j\omega_0 n}$$

Basic Sequences & Sequence Operations (cont.)

- Which is true only for $\omega_0 N = 2\pi k$.
- Consequently, complex exponential and sinusoidal sequences are not necessarily periodic in n with period $(2\pi/\omega_0)$ and depending on the value of ω_0, may not be periodic at all.

Example #1

- Determine whether or not each of the following sequences is periodic. If your answer is yes, determine the period.
  1. $x[n] = A \cos [(3\pi/7)n - (\pi/8)]$
  2. $x[n] = e^{j[(n/8)-\pi]}$

- Solution:

1. $x[n] = A \cos [(3\pi/7)n - (\pi/8)]$
   - $x[n]$ is periodic if $x[n] = x[n+N]$ for some integer value of N. So,
   - $x[n+N] = A \cos [(3\pi/7)n +(3\pi/7)N - (\pi/8)$
   - $x[n+N] =x[n]$ if $(3\pi/7)N$ is an integer of multiple 2π.
   - The smallest value of N for which this is true is N=14.
   - Therefore the sequence is periodic with period 14.

Example #1 (cont.)

- Solution:

1. $x[n] = e^{j[(n/8) - \pi]}$
   - $x[n+N] = e^{j[(n/8) + (N/8) - \pi]}$

     $= e^{j[(n/8) - \pi]} e^{j(N/8)} = x[n] e^{j(N/8)}$
   - The factor $e^{j(N/8)}$ is unity for $(N/8)$ an integer multiple of 2π.
   - This requires that: $N/8 = 2\pi R$.
   - Where N and R are both integers. This is not possible since π is an irrational number. Therefore this sequence is not periodic.