Waq

# ISRA UNIVERSITY

Islamabad Campus

**Department of Electrical Engineering**
**Program: B.E. (Electrical)**
**Semester – Spring 2016**

**EL-322 Digital Signal Processing**

**Assignment – 1 Solution** **Due Date: 30/03/2016**
**Marks: 10** **Handout Date: 23/03/2016**

Question # 1:

For each of the following systems, determine whether the system is Stable, Causal, Linear, Time Invariant and Memory less:

1. $T(x[n]) = ax[n] + b$
2. $T(x[n]) = x[-n]$

Solution:

1. $T(x[n]) = ax[n] + b$
   - Stable:
     $|T(x[n])| = |ax[n] + b| \leq a|M| + |b|$, Which is stable for finite a and b.
   - Causal:
     This doesn't use future values of x [n] so it is causal.
   - Linear:
     $$y_3(n) = T\{a_1x_1(n) + a_2x_2(n)\}$$
     $$= a[a_1x_1(n) + a_2x_2(n)] + b$$
     Here above equation is written from given system equation with $x(n) = a_1x_1(n) + a_2x_2(n)$. The response of the system to two inputs $x_1(n)$ and $x_2(n)$ when applied separately is given as:
     $$y_1(n) = T\{x_1(n)\} = ax_1(n) + b \text{ and}$$
     $$y_2(n) = T\{x_2(n)\} = ax_2(n) + b$$
     The linear combination of two outputs given by above equation will be,
     $$y'_3(n) = a_1y_1(n) + a_2y_2(n)$$
     $$= a_1[ax_1(n) + b] + a_2[ax_2(n) + b]$$
     Hence, $y_3(n) \neq y'_3(n)$. Hence the system is non linear.

   - Time Invariant or Variant:
     The response of the system to the input delayed b 'k' samples will be:
     $$y(n, k) = T\{x(n - k)\}$$
     $$= ax(n - k) + b$$

And the delayed output by 'k' samples will be obtained by replacing 'n' by (n-k) in given system equation i.e.,

$$y(n-k) = ax(n-k) + b$$

Hence, $y(n,k) = y(n-k)$. Hence the system is Shift/Time Invariant.

- Memory less:

  y[n] depends on the $n^{th}$ value of x[n] only, so it is memory less.

2. $T(x[n]) = x[-n]$

- Stable:

  $|T(x[n])| \leq |x[-n]| \leq M$, so it is stable.

- Causal:

  For n<0 , it depends on the future value of x[n], so it is not causal.

- Linear:

$$y_3(n) = T\{ax_1(n) + bx_2(n)\}$$
$$= ax_1[-n] + bx_2[-n]$$

  Here above equation is written from given system equation with $x(n) = a_1x_1(n) + a_2x_2(n)$. The response of the system to two inputs $x_1(n)$ and $x_2(n)$ when applied separately is given as:

$$y_1(n) = T\{x_1(n)\} = x_1(-n) \text{ and}$$
$$y_2(n) = T\{x_2(n)\} = x_2(-n)$$

  The linear combination of two outputs given by above equation will be,

$$y'_3(n) = ay_1(n) + by_2(n)$$
$$= ax_1(-n) + bx_2(-n)$$

  Hence, $y_3(n) = y'_3(n)$. Hence the system is linear.

- Time Invariant or Variant:

  The response of the system to the input delayed b 'k' samples will be:

$$y(n,k) = T\{x(n-k)\}$$
$$= x(-n-k)$$

  And the delayed output by 'k' samples will be obtained by replacing 'n' by (n-k) in given system equation i.e.,

$$y(n-k) = x(-[n-k]) = x(-n+k)$$

  Hence, $y(n,k) \neq y(n-k)$. Hence the system is Shift/Time Variant.

- Memory less:

  For $n \neq 0$, it depends on a value of x other than the $n^{th}$ value, so it is not memory less.

Question # 2:

Find the frequency response $H(e^{j\omega})$ of the linear time-invariant system whose input and output satisfy the difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

Solution:

The difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

Taking the Fourier transform of both sides,

$$Y(e^{j\omega})\left[1 - \frac{1}{2}e^{-i\omega}\right] = X(e^{j\omega})[1 + 2e^{-j\omega} + e^{-j2\omega}]$$

Hence, the frequency response is:

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

$$H(e^{j\omega}) = \frac{1 + 2e^{-j\omega} + e^{-j2\omega}}{1 - \frac{1}{2}e^{-i\omega}}$$

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**Good Luck**