ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Spring 2016

EL-322 Digital Signal Processing

Assignment – 2 Solution **Due Date: 13/04/2016**
Marks: 10 **Handout Date: 06/04/2016**

Question # 1:

Find the z-transform of the following sequences:

1. $x[n] = \cos(n\omega_0)u(n)$
2. $x[n] = \left(\frac{1}{2}\right)^n u(n+2) + (3)^n u(-n-1)$

Solution:

1. $x[n] = \cos(n\omega_0)u(n)$

   We can write:

$$x[n] = \cos(n\omega_0)u(n) = \frac{1}{2}\left[e^{jn\omega_0} + e^{-jn\omega_0}\right]u(n)$$

   The z-transform for this is:

$$X(z) = \frac{1}{2}\left[\frac{1}{1 - e^{j\omega_0}z^{-1}}\right] + \frac{1}{2}\left[\frac{1}{1 - e^{-j\omega_0}z^{-1}}\right]$$

   With a ROC $|z| > 1$, combining the two terms we get:

$$X(z) = \frac{1 - (\cos\omega_0)z^{-1}}{1 - 2(\cos\omega_0)z^{-1} + z^{-2}} \quad ,|z| > 1$$

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2. $x[n] = \left(\frac{1}{2}\right)^n u(n+2) + (3)^n u(-n-1)$

   The x (n) is the sum of two sequences. We will first find the transforms separately and then will combine them.

$$\left(\frac{1}{2}\right)^n u(n+2) \ or \ \left(\frac{1}{2}\right)^{n+2} u(n+2) \leftrightarrow \frac{z^2}{1 - \frac{1}{2}z^{-1}}$$

And second term is left sided exponential sequence then:

$$(3)^n u(-n-1) \leftrightarrow -\frac{1}{1-3z^{-1}}$$

Finally, for the z-transform of x (n) we have:

$$X(z) = \frac{z^2}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

ROC is:

$$\frac{1}{2} < |z| < 3$$

Question # 2:

The z-transform of a sequence x(n) is:

$$X(z) = \frac{z + 2z^{-2} + z^{-3}}{1 - 3z^{-4} + z^{-5}}$$

If the ROC includes the unit circle, find the DTFT of x (n) at $\omega = \pi$.

Solution:

If X (z) is the z-transform of x (n), and the unit circle is within the ROC, the DTFT of x (n) may be found by evaluating X(z) around the unit circle.

$$X(e^{j\omega}) = X(z)|_{z=-1}$$

Therefore, the DTFT at $\omega = \pi$ is:

$$X(e^{j\omega})\big|_{\omega=\pi} = X(z)|_{z=e^{j\pi}} = X(z)|_{z=-1}$$

And we have:

$$X(e^{j\omega})\big|_{\omega=\pi} = \frac{z + 2z^{-2} + z^{-3}}{1 - 3z^{-4} + z^{-5}}\bigg|_{z=e^{j\pi}} = \frac{-1+2\pm 1}{1-3-1} = 0$$

Good Luck