ISRA UNIVERSITY
Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Spring 2016

EL-322 Digital Signal Processing

Quiz – 1 Solution
Marks: 10 **Handout Date: 30/03/2016**

Question # 1:

Determine whether the specified input-output relationship is linear and/or shift-invariant:

$$y[n] = 2x[n] + 3$$

Solution:

- Linear or Non-Linear:

$$T\{x_1(n)\} = 2x_1(n) + 3$$
$$T\{x_2(n)\} = 2x_2(n) + 3$$

$$y_3(n) = T\{ax_1(n) + bx_2(n)\}$$
$$= 2[ax_1(n) + bx_2(n)] + 3$$

Here above equation is written from given system equation with $x(n) = ax_1(n) + bx_2(n)$. The response of the system to two inputs x_1(n) and x_2(n) when applied separately is given as:

$$y_1(n) = T\{x_1(n)\} = 2x_1(n) + 3 \text{ and}$$
$$y_2(n) = T\{x_2(n)\} = 2x_2(n) + 3$$

The linear combination of two outputs given by above equation will be,

$$y'_3(n) = ay_1(n) + by_2(n)$$
$$= a[2x_1(n) + 3] + b[2x_2(n) + 3]$$

Hence, $y_3(n) \neq y'_3(n)$. Hence the system is non linear.

- Time Invariant or Variant:

The response of the system to the input delayed b 'k' samples will be:

$$y(n,k) = T\{x(n-k)\}$$
$$= 2x(n-k) + 3$$

And the delayed output by 'k' samples will be obtained by replacing 'n' by (n-k) in given system equation i.e.,

$$y(n-k) = 2x(n-k) + 3$$

Hence, $y(n,k) = y(n-k)$. Hence the system is Shift/Time Invariant.

Question # 2:

Find the frequency response $H(e^{j\omega})$ of the linear time-invariant system whose input and output satisfy the difference equation:

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1]$$

Solution:

The difference equation:

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1]$$

Taking Fourier transform of both sides:

$$Y\left[e^{j\omega}\right]-\frac{3}{4}Y\left[e^{j\omega}\right]e^{-j\omega}+\frac{1}{8}Y\left[e^{j\omega}\right]e^{-2j\omega}=2.X\left[e^{j\omega}\right]e^{-j\omega}$$

$$Y\left[e^{j\omega}\right]\left[1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-2j\omega}\right]=2.X\left[e^{j\omega}\right]e^{-j\omega}$$

The system function is:

$$H\left[e^{j\omega}\right]=\frac{Y\left[e^{j\omega}\right]}{X\left[e^{j\omega}\right]}$$

$$H\left[e^{j\omega}\right]=\frac{2.e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-2j\omega}}$$
