ISRA UNIVERSITY
Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Spring 2016

EL-322 Digital Signal Processing

Quiz – 2 Solution
Marks: 10 **Handout Date: 06/04/2016**

Question # 1:

Find the DTFT of the following sequence:

$$cos(\omega_0 n)u(n)$$

Solution:

$$x(n) = cos(\omega_0 n)u(n)$$

$$X(\omega) = F\{x(n)\} = \sum_{n=-\infty}^{\infty} \{cos(\omega_0 n)u(n)\}e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2}\right] e^{-j\omega n}$$

$$= \frac{1}{2}\left\{\sum_{n=0}^{\infty}\left[e^{j(\omega_0-\omega)}\right]^n + \sum_{n=0}^{\infty}\left[e^{-j(\omega_0+\omega)}\right]^n\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{1-e^{j(\omega_0-\omega)}} + \frac{1}{1-e^{-j(\omega_0+\omega)}}\right\}$$

$$= \frac{1}{2}\left\{\frac{1-e^{-j(\omega_0+\omega)} + 1-e^{j(\omega_0-\omega)}}{1+e^{-2j\omega} - e^{-j\omega}(e^{j\omega_0} + e^{j-\omega_0})}\right\}$$

$$X(\omega) = \frac{1 - e^{-j\omega}\cos\omega_0}{1 + e^{-2j\omega} - 2e^{-j\omega}\cos\omega_0}$$

Question # 2:

Solve the following LCCDE for y(n) assuming zero initial conditions:

$$y[n] - 0.25y[n-1] = x[n] - x[n-2]$$

For $x[n] = \delta[n]$.

Solution:

$$y[n] - 0.25y[n-1] = x[n] - x[n-2]$$

Taking DTFT of each term in the difference equation we get,

$$Y\left[e^{j\omega}\right] - 0.25e^{-j\omega}Y\left[e^{j\omega}\right] = X\left[e^{j\omega}\right] - e^{-2j\omega}X\left[e^{j\omega}\right]$$

Because the DTFT of x (n) is $X\left[e^{j\omega}\right] = 1$,

$$Y\left[e^{j\omega}\right] = \frac{1 - e^{-2j\omega}}{1 - 0.25e^{-j\omega}} = \frac{1}{1 - 0.25e^{-j\omega}} - \frac{e^{-2j\omega}}{1 - 0.25e^{-j\omega}}$$

Using the DTFT pair,

$$(0.25)^n u(n) \leftrightarrow \frac{1}{1 - 0.25e^{-j\omega}}$$

The inverse of DTFT of $Y\left[e^{j\omega}\right]$ may easily be found using the linearity and shift properties:

$$y[n] = (0.25)^n u(n) - (0.25)^{n-2} u(n-2)$$
