-Digital Signal Processing- Basic Structures of IIR Systems

Lecture-12
04-May-16

Basic Structures of IIR Systems

- We obtained block diagram representations of the direct form I and direct form II or canonic direct form, structures for a LTI system whose input and output satisfy a difference equation of the form:

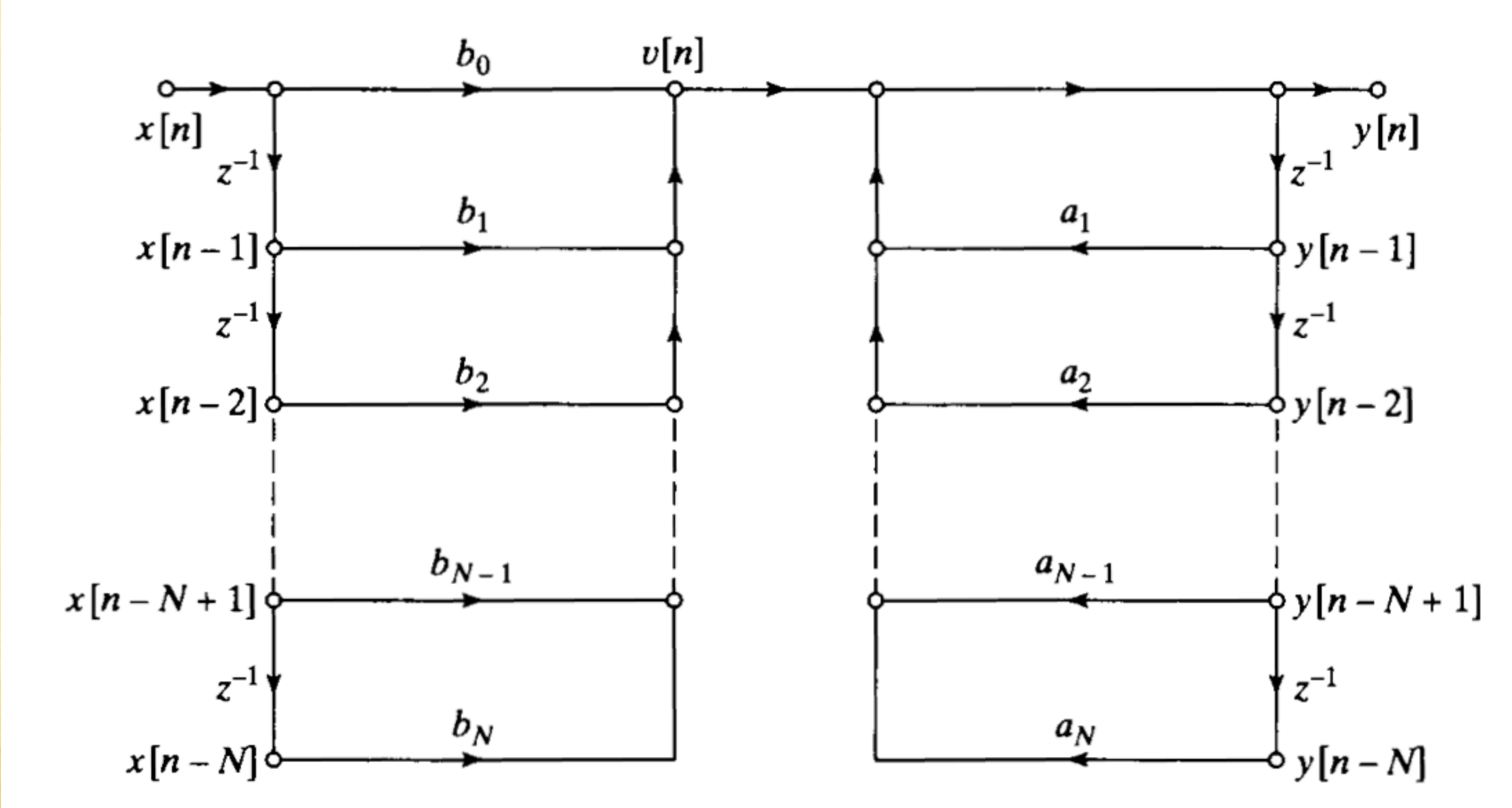
$$y[n]-\sum_{k=1}^{N}a_k y[n-k]=\sum_{k=0}^{M}b_k y[n-k]$$

- With corresponding rational system function:

$$H[z]=\frac{\sum_{k=0}^{M}b_k z^{-k}}{1-\sum_{k=1}^{N}a_k z^{-k}}$$

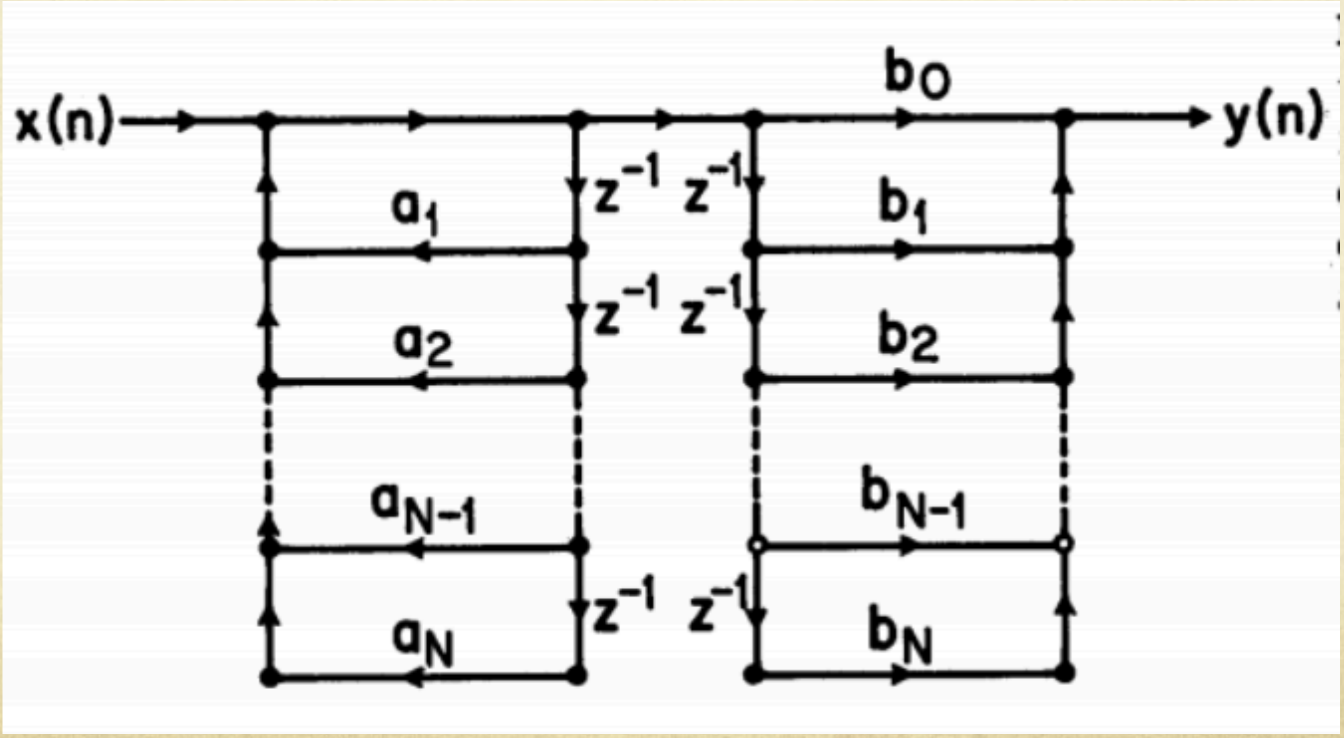
Basic Structures of IIR Systems (cont.)

- The figure below is the direct form I structure using signal flow graph conventions:



Basic Structures of IIR Systems (cont.)

- Flow graph representation of a general difference equation based on interchanging the order in which the poles and zeros are cascaded:



Basic Structures of IIR Systems (cont.)

- Z-transform factorization and difference equation corresponding to the above network:

$$y(n)=\sum_{k=0}^{M}b_kx(n-k)+\sum_{k=1}^{N}a_ky(n-k)$$

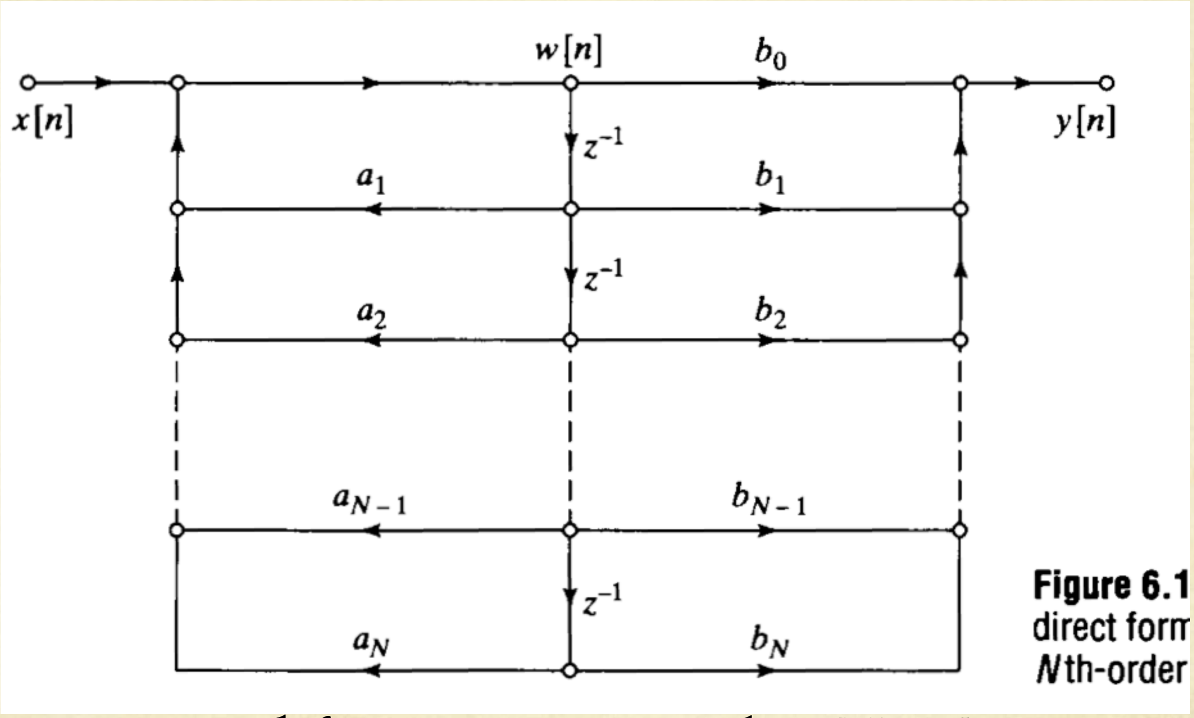
$$H(z)=\left[\frac{1}{1-\sum_{k=1}^{N}a_kz^{-k}}\right]\left[\sum_{k=0}^{M}b_kz^{-k}\right]$$

$$y_1(n)=x(n)+\sum_{k=1}^{N}a_ky_1(n-k)$$

$$y(n)=\sum_{k=0}^{M}b_ky_1(n-k)$$

Basic Structures of IIR Systems (cont.)

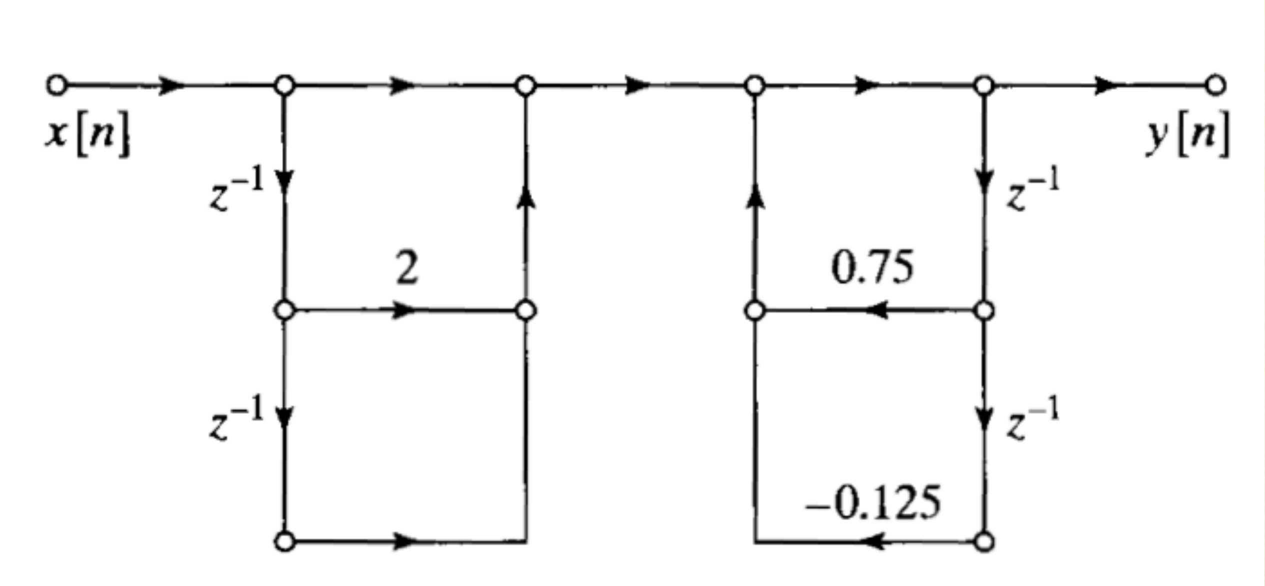
- The figure below shows the signal flow graph representation$ of the direct form II structure.



- Again we assumed for convenience that N=M.

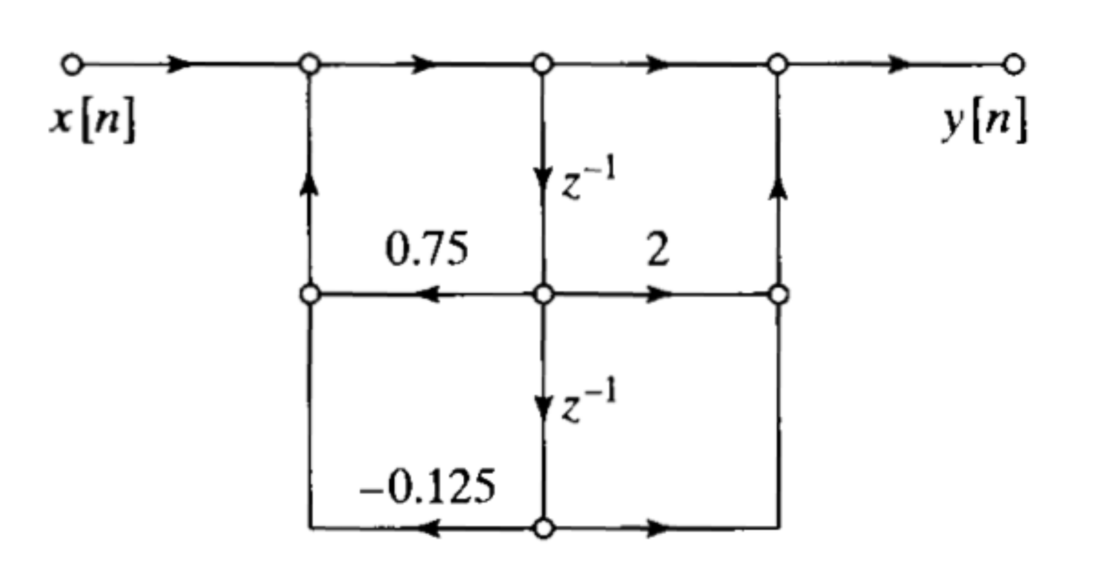
Example #1

- Illustration of Direct form I and Direct for II structures.
- Consider the system: $H[z] = \dfrac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$
  - Direct Form I:



Example #1 (cont.)

- Direct Form II:



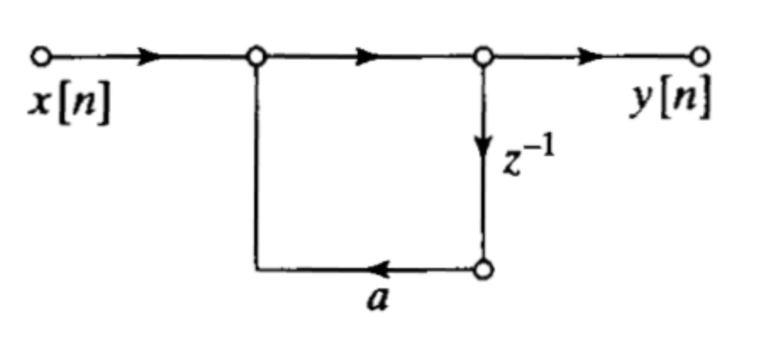
Transposed Forms

- Using signal flow graphs, we can transform a given system into a different network structure while maintaining the same system function.
- One of these procedures, called flow graph reversal or transposition.
- Transposition of a flow graph is accomplished by:
  - reversing the directions of all branches in the network while keeping the branch transmittances as they were.
  - Reversing the roles of the input and output so that source nodes become sink nodes and vice versa.
- For single input, single output systems, the resulting flow graph has the same system function as the original graph if the input and output nodes are interchanged.
- Transfer function remains the same.

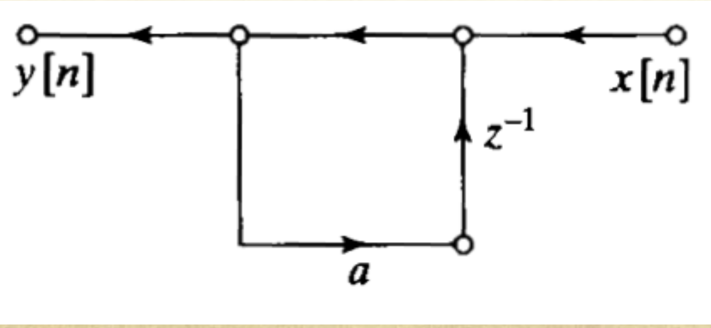
Example #2

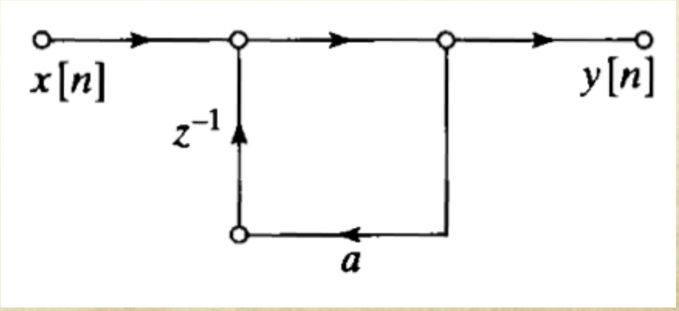
- Consider the system function of a first order system flow graph shown below:

$$H(z) = \frac{1}{1 - az^{-1}}$$

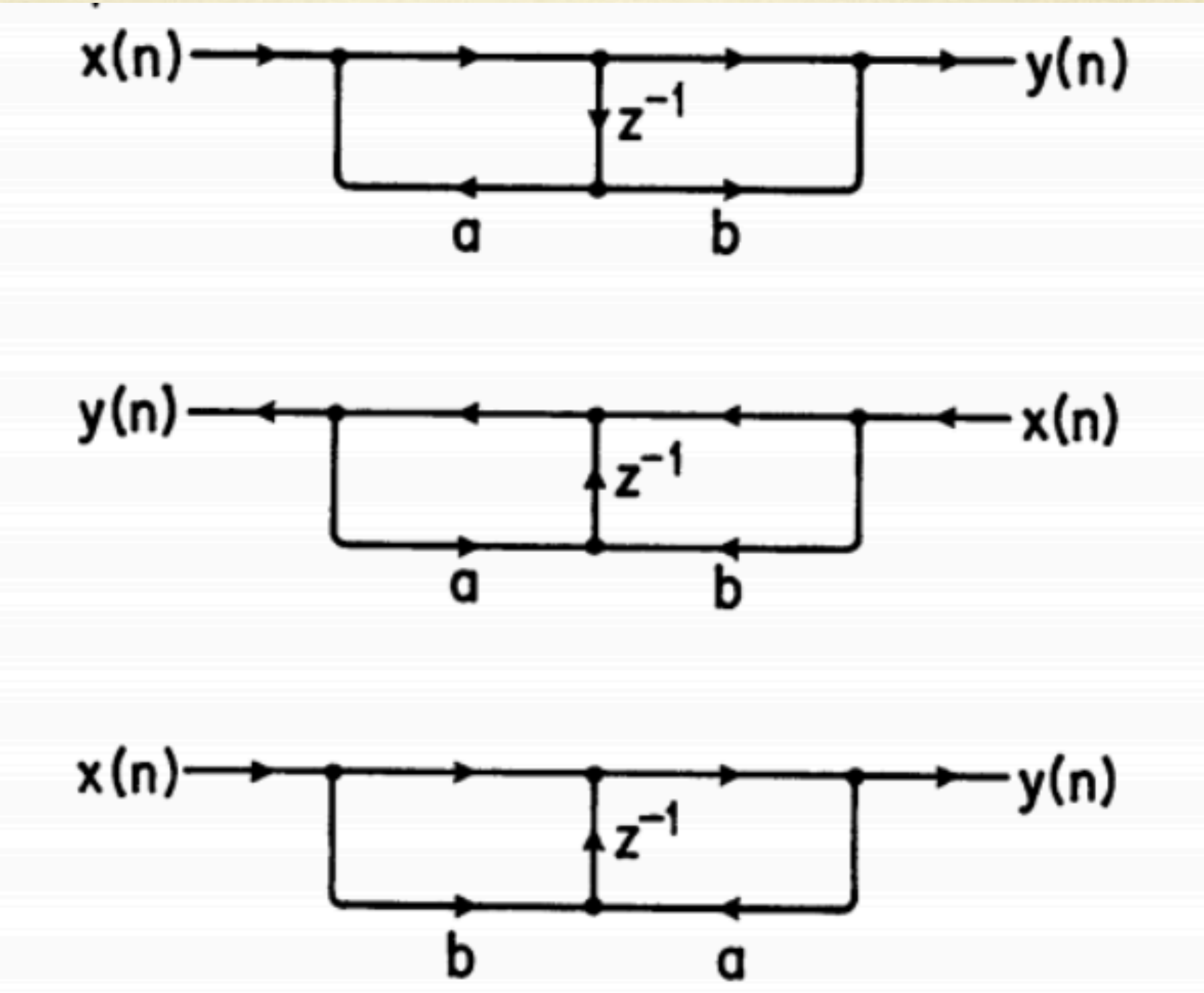


- Solution:

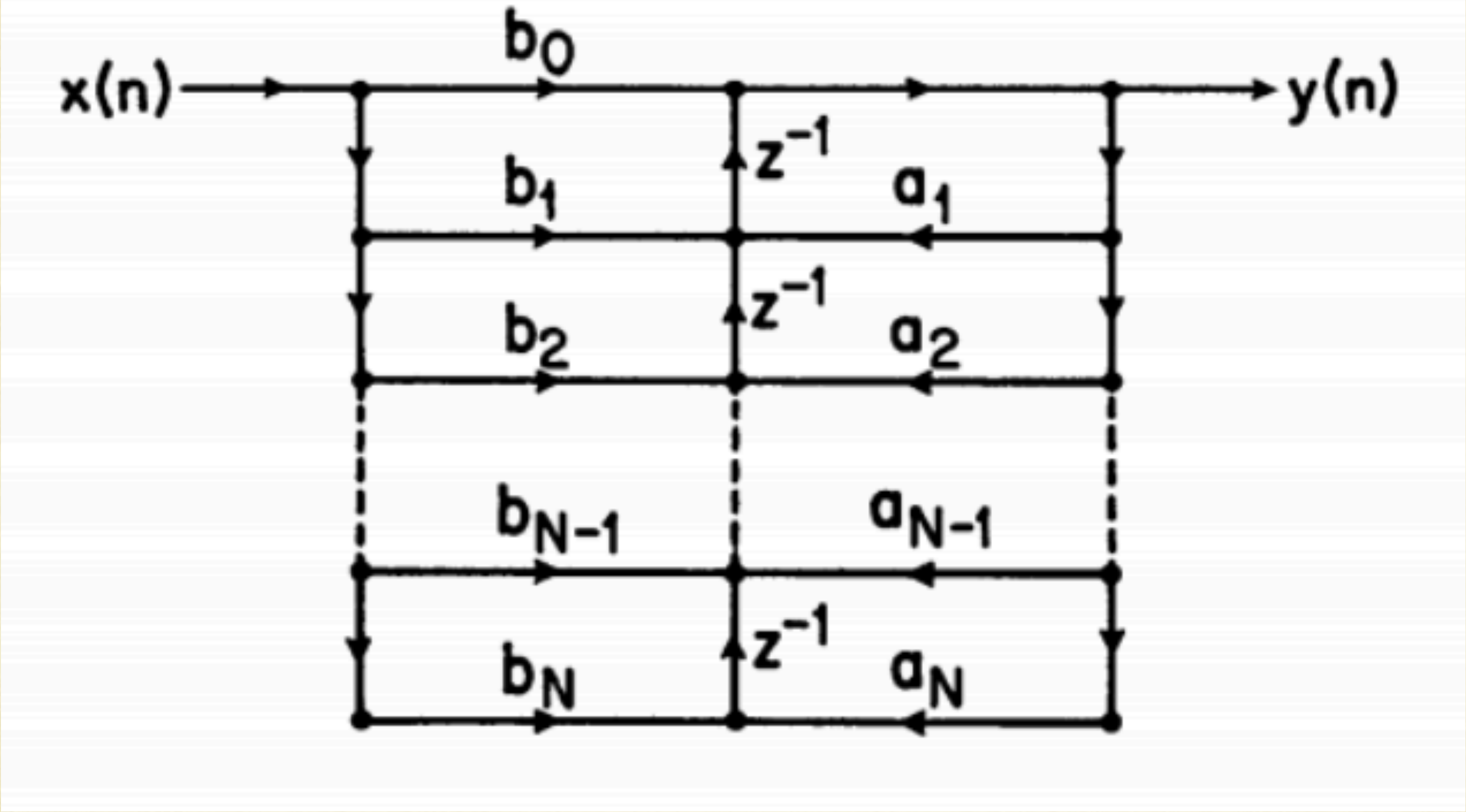




Example #3



Example #4



Cascade Form

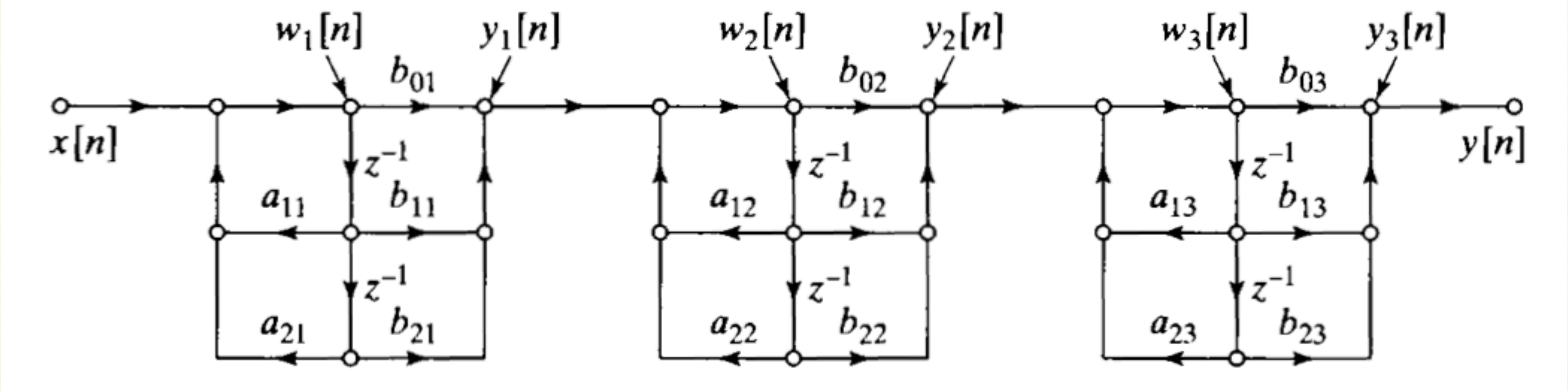
- Factorization of the *z*-transform for the cascade structure:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

$$= A \prod \frac{1 + \beta_{1k} z^{-1} + \beta_{2k} z^{-2}}{1 - \alpha_{1k} z^{-1} - \alpha_{2k} z^{-2}}$$

Cascade Form (cont.)

- Due to finite word length effects, each such cascade realization behaves differently from others.
- Cascade structure with a direct form II realization of each second-order subsystem.



Cascade Form (cont.)

- The difference equations represented by a general cascade of direct form II second-order sections are of the form:

$$y_0[n] = x[n]$$

$$w_k[n] = a_{1k} w_k[n-1] + a_{2k} w_k[n-2] + y_{k-1}[n-1], \quad k = 1, 2, \ldots, N_s$$

$$y_k[n] = b_{0k} w_k[n] + b_{1k} w_k[n-1] + b_{2k} w_k[n-2], \quad k = 1, 2, \ldots, N_s$$

$$y[n] = y_{N_s}[n]$$

Example #5

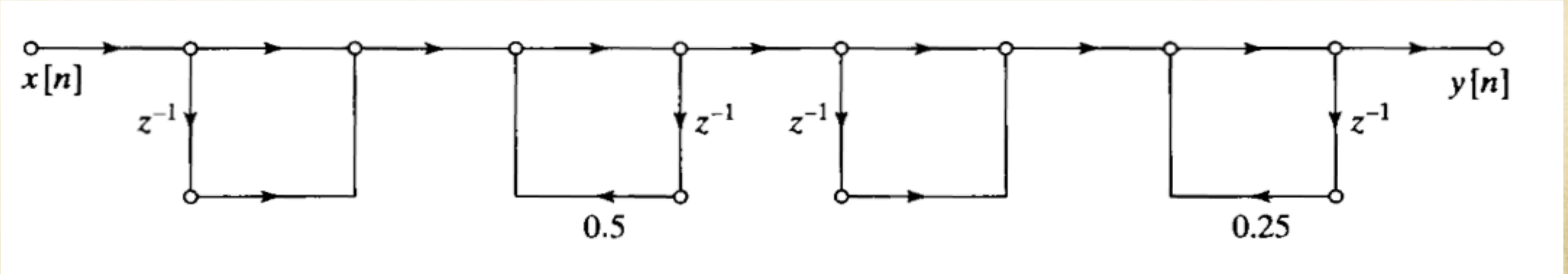
- Lets consider the system :

$$H(z)=\frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}=\frac{\left(1+z^{-1}\right)\left(1+z^{-1}\right)}{\left(1-0.5z^{-1}\right)\left(1-0.25z^{-1}\right)}$$

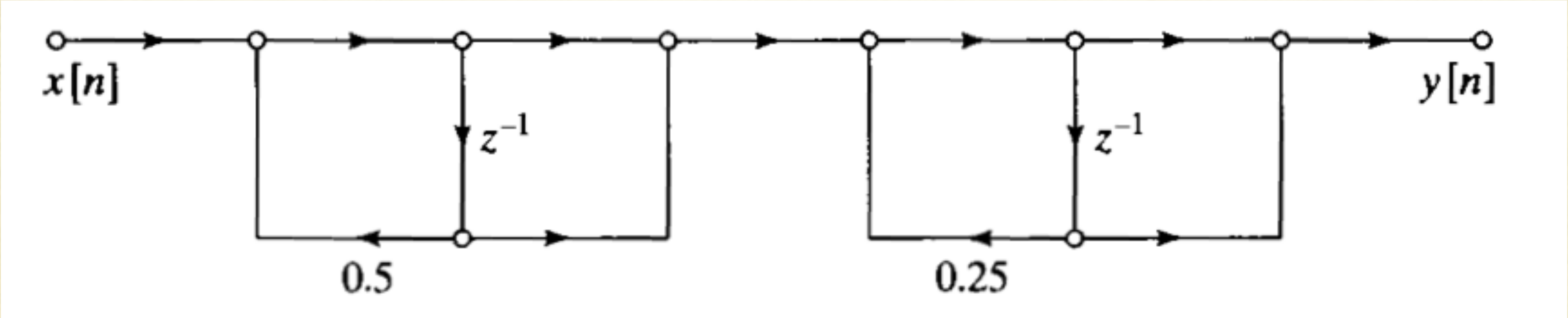
- Since all of the poles and zeros are real, a cascade structure with first-order sections has real coefficients.

- If the poles and/or zeros were complex, only a second-order section would have real coefficients.

- The two equivalent cascade structures are shown below:



Example #5 (cont.)



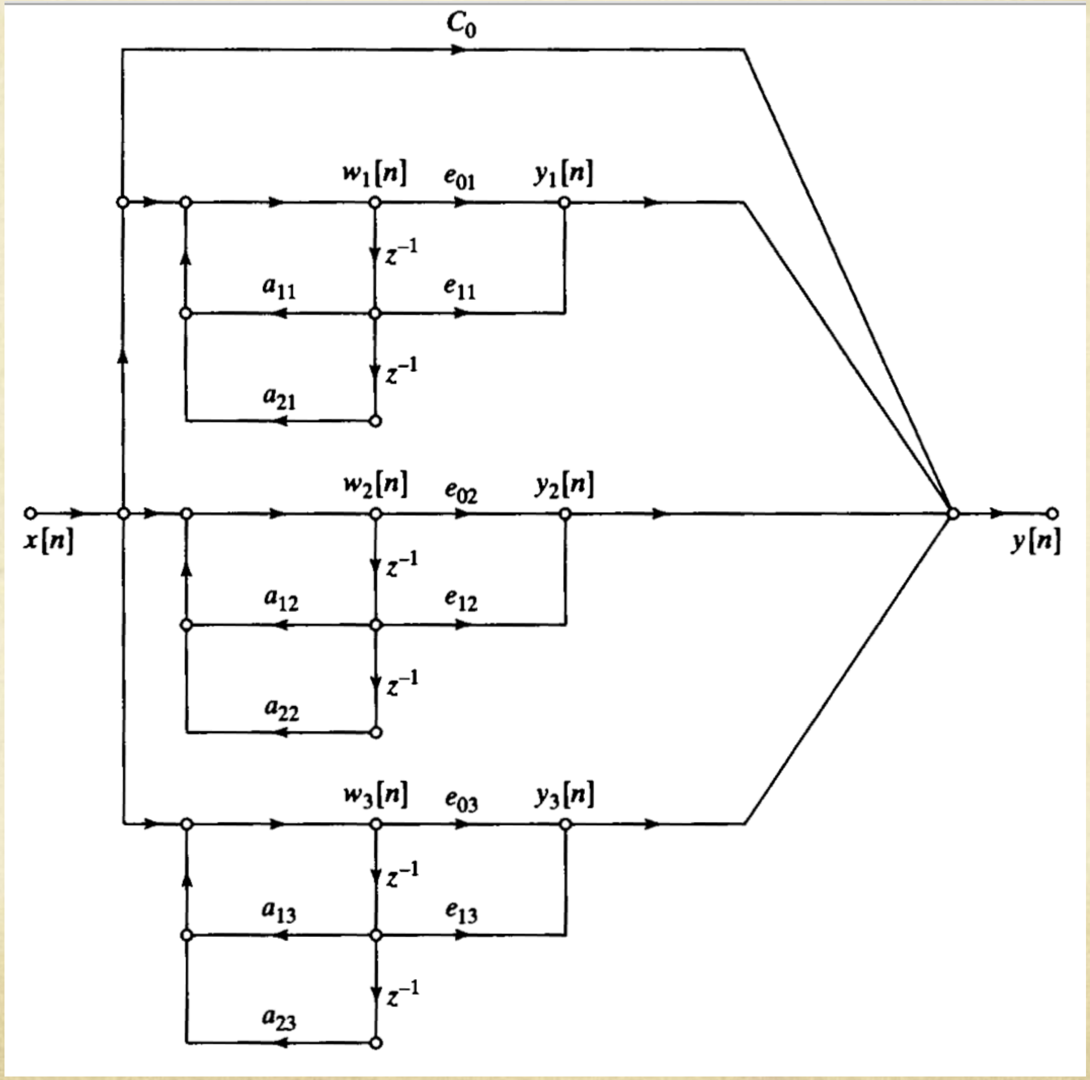
Parallel Form

- Equivalently, expressing the transfer function as a sum using partial fraction expansion gives a parallel structure:

$$H(z) = \sum_{k=0}^{N_p} C_k z^{-k} + \sum_{k=1}^{N_p} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

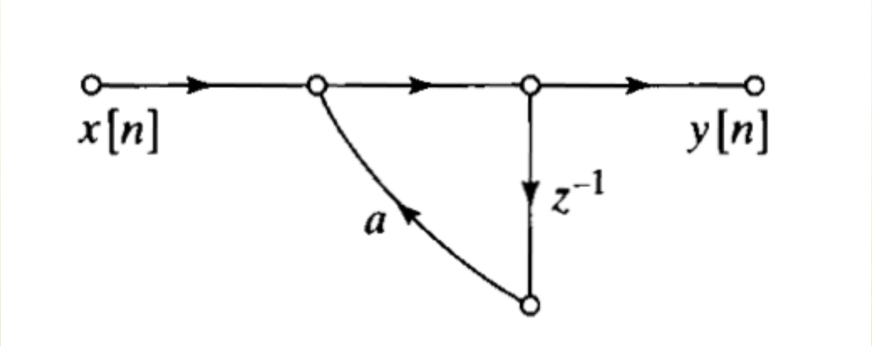
- A partial fraction expansion of the transfer function in z^{-1} leads to the parallel form I structure.
- A direct partial fraction expansion of the transfer function in z leads to the parallel form II structure.
- Parallel form structure for sixth order system with the real and complex poles is shown below:

Parallel Form (cont.)



Feedback in IIR Systems

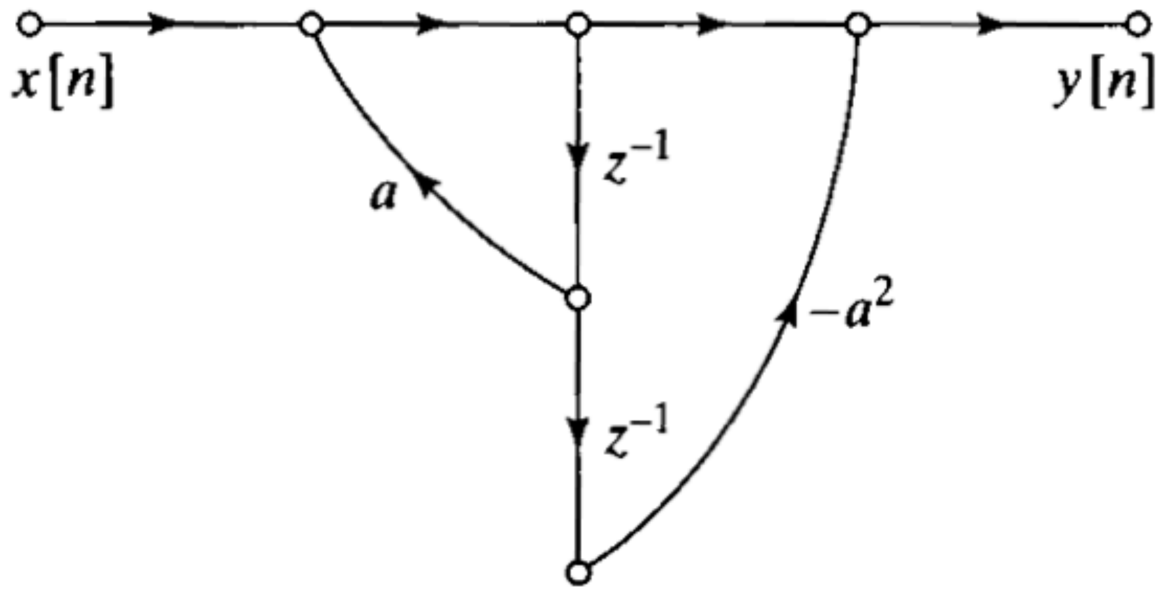
- All the flow graphs of this section have feedback loops, i.e., they have closed paths that begin at a node and return to that node by traversing branches only in the direction of their arrowheads.
- Such a structure in the flow graph implies that a node variable in a loop depends directly or indirectly on itself.
- A simple example is shown below which represents the difference equation: y[n] = ay[n-1] + x[n].



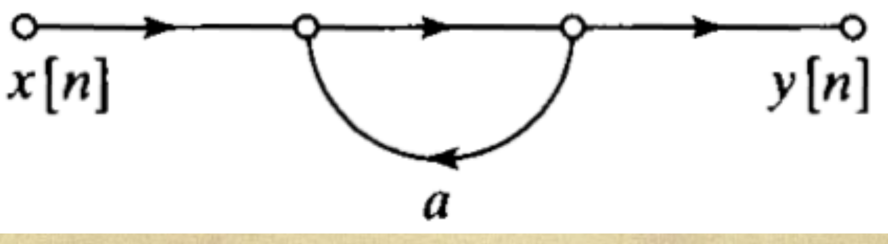
Feedback in IIR Systems (cont.)

- If a system has poles, a corresponding block diagram or signal flow graph will have feedback loops. (But neither poles in the system function nor loops in the network are sufficient for the impulse response to be infinitely long.)
- A delay element is necessary in the feedback loop otherwise it is non-computable. (the structure should be modified to eliminate the non- computable loops.)

Feedback in IIR Systems (cont.)



(b)



Example #6

- Consider the discrete time linear causal system defined by the difference equation:

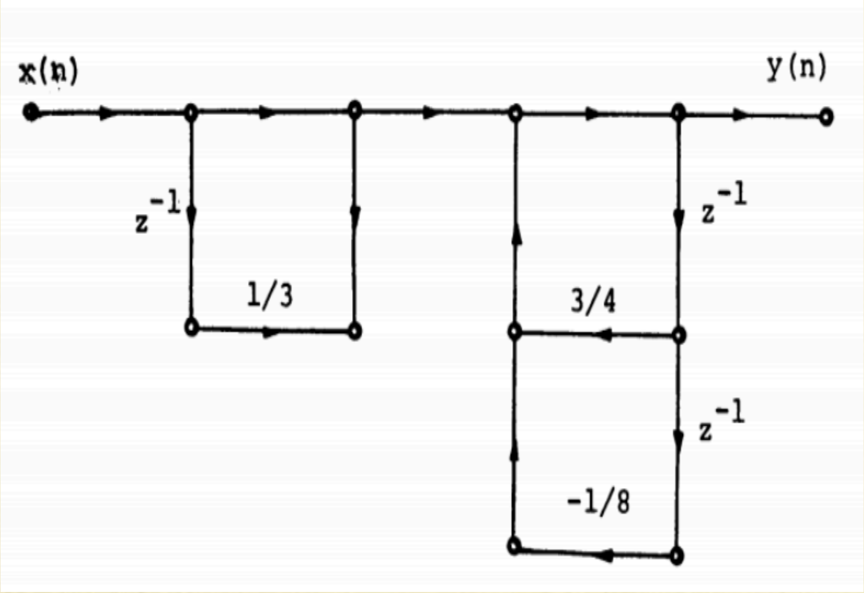
$$y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)+\frac{1}{3}x(n-1)$$

- Draw a signal flow graph to implement this system in each of the following forms:
  - Direct form I.
  - Direct form II.
  - Cascade.
  - Parallel.

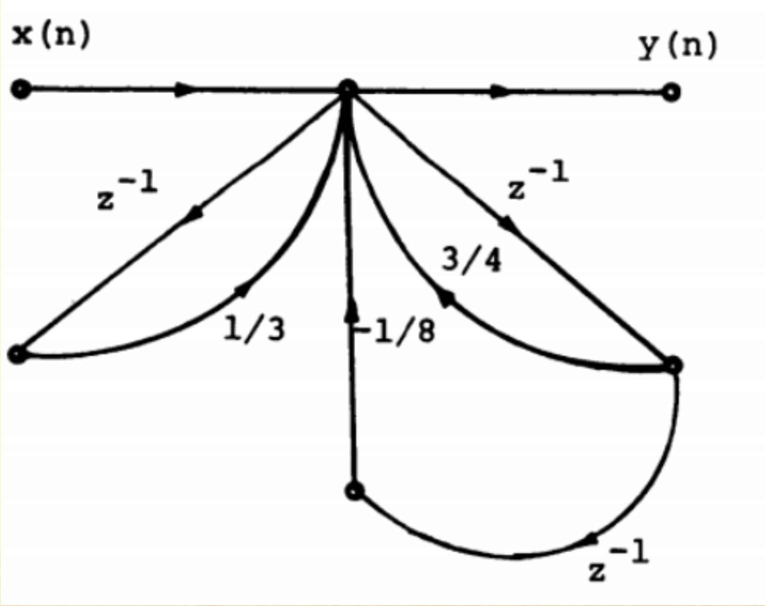
- For the cascade and parallel forms use only first-order sections.

Example #6 Solution

- (a): Direct form I corresponds to first implementing the right-hand side of the difference equation (i.e., the zeros) followed by te left-hand side (i.e. the poles). Thus the direct form I for this difference equation is:

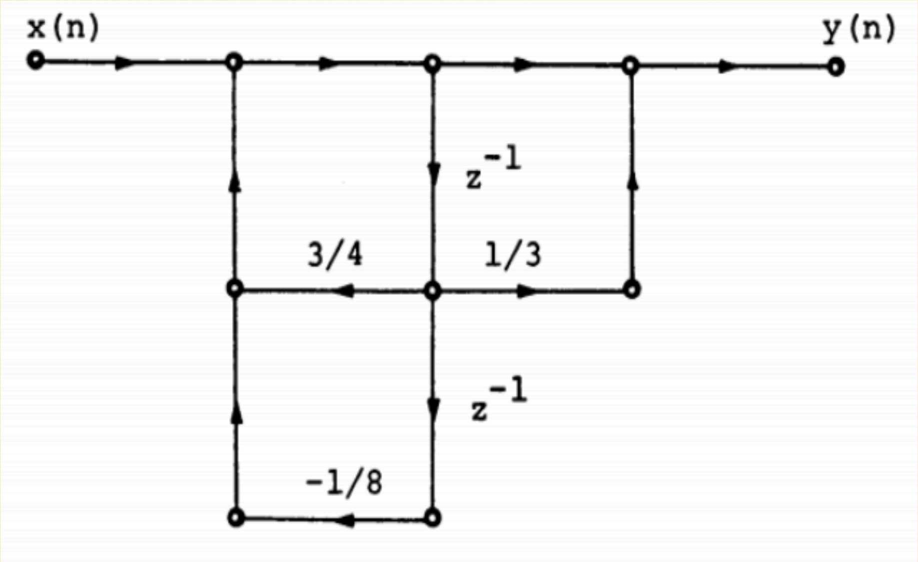


OR

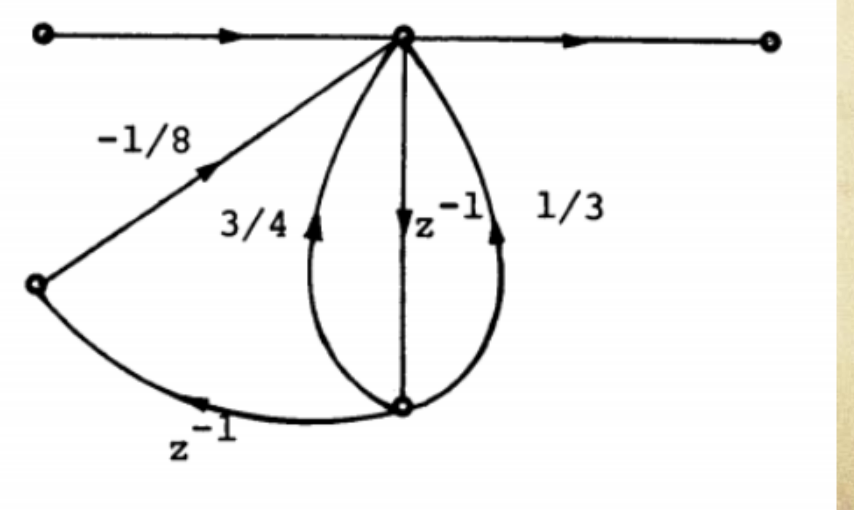


Example #6 Solution (cont.)

- (b) The direct form II corresponds to implementing the poles first, followed by the zeros:



or alternatively,



Example #6 Solution (cont.)

- (c): In the cascade form using firs-order sections, we must first factor the system function into a cascade of two first-order systems.
- Applying the *z*-transform to both sides of the difference equation:

$$Y(z)\left[1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right]=X(z)\left[1+\frac{1}{3}z^{-1}\right]$$

or

$$H(z)=\frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}=\frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)}$$

Example #6 Solution (cont.)

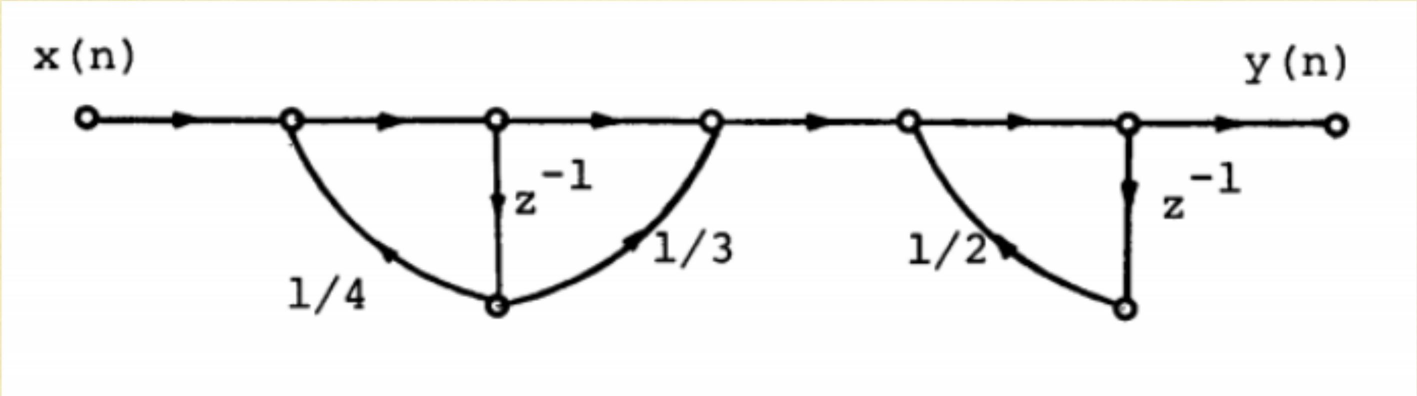
- In developing the cascade form, we can include the zero with either pole and arrange the cascade in either order.

- For example writing H(z) as:

$$H(z) = \left[\frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{4} z^{-1}} \right] \left[\frac{1}{1 - \frac{1}{2} z^{-1}} \right]$$

- And using the direct form II for the first subsection leads to the cascade form shown below:

Example #6 Solution (cont.)



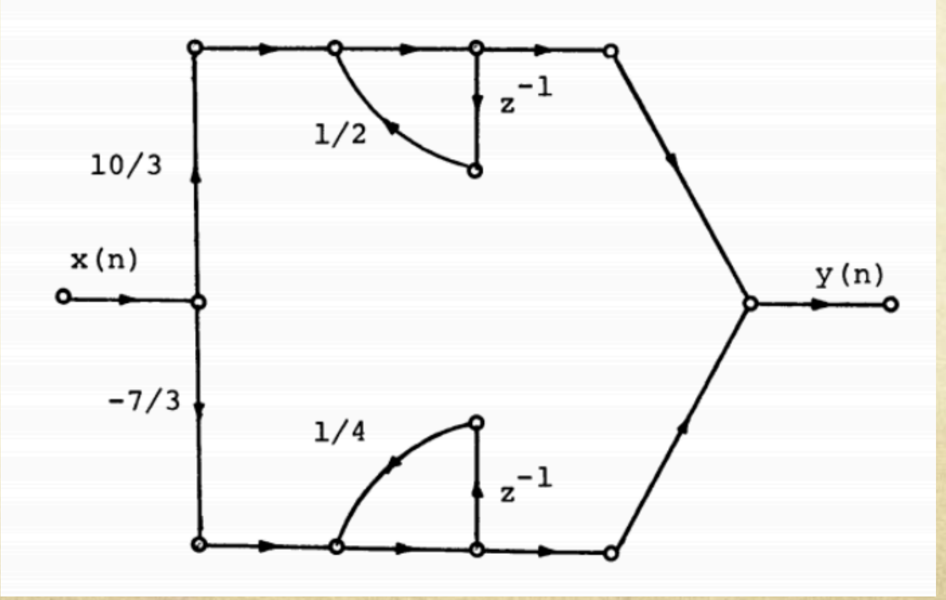
- This flow graph can also be collapsed somewhat as we have done with those in (a) and (b).

Example #6 Solution (cont.)

- (d): The parallel form corresponds to expanding H(z) in a partial fraction expansion. Thus,

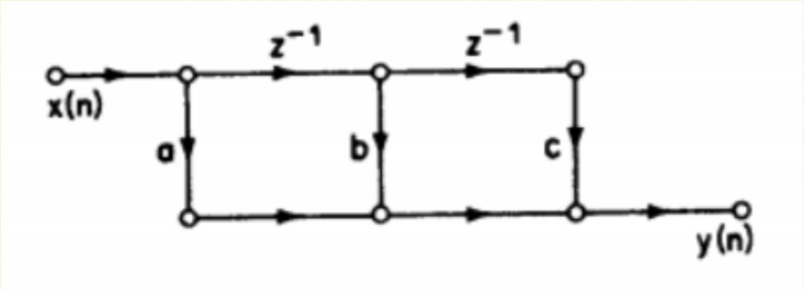
$$H(z) = \frac{10/3}{1 - \frac{1}{2}z^{-1}} + \frac{-7/3}{1 - \frac{1}{4}z^{-1}}$$

- Leading us to the flow graph shown below:



Example #7

- Determine the transpose of the following network and verify that in the original case and transpose networks have the same transfer function:



- Solution:
  - By inspection of the network we can write that:

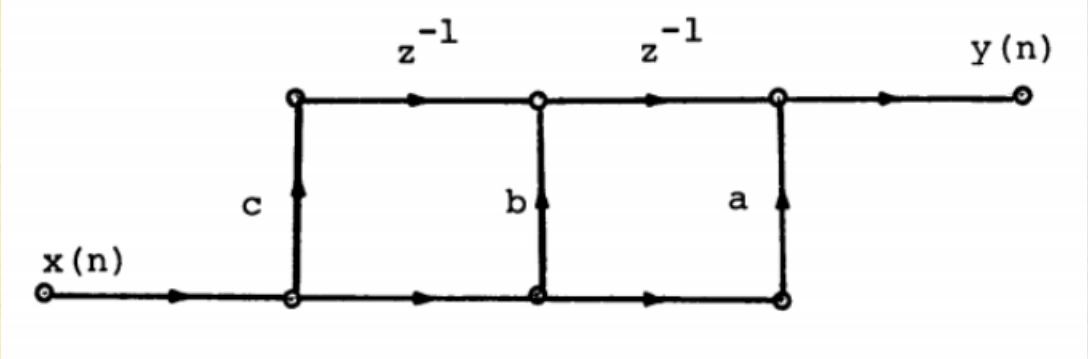
$$Y(z) = aX(z) + bz^{-1}X(z) + cz^{-2}X(z)$$

or

$$H(z) = a + bz^{-1} + cz^{-2}$$

Example #7 (cont.)

- The transpose network is:



- By inspection of this network we see that:

$$Y(z) = cz^{-2}X(z) + bz^{-1}X(z) + aX(z)$$

or

$$H(z) = a + bz^{-1} + cz^{-2}$$

- As before.