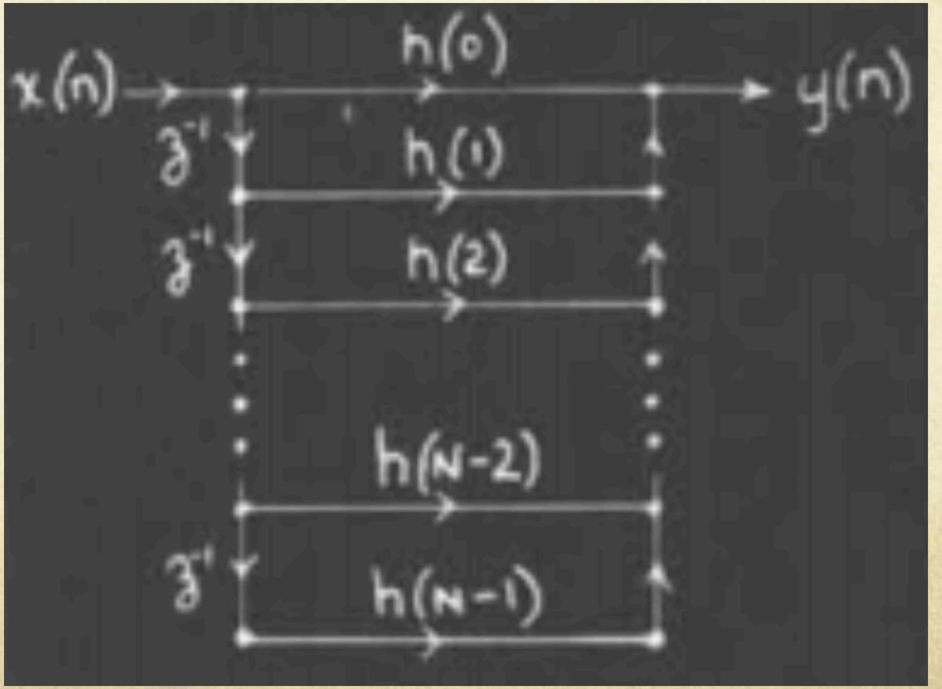
-Digital Signal Processing-
FIR Network Structures

Lecture-13
10-May-16

FIR Systems

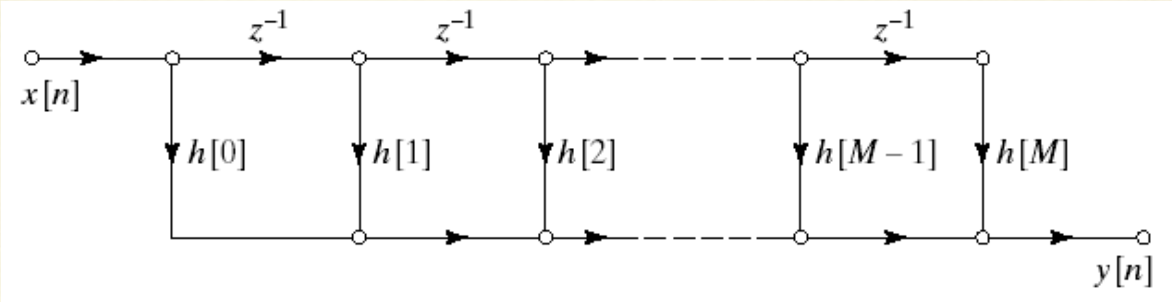
$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$y(n) = \sum_{k=0}^{N-1} h(k) x(n-k)$$

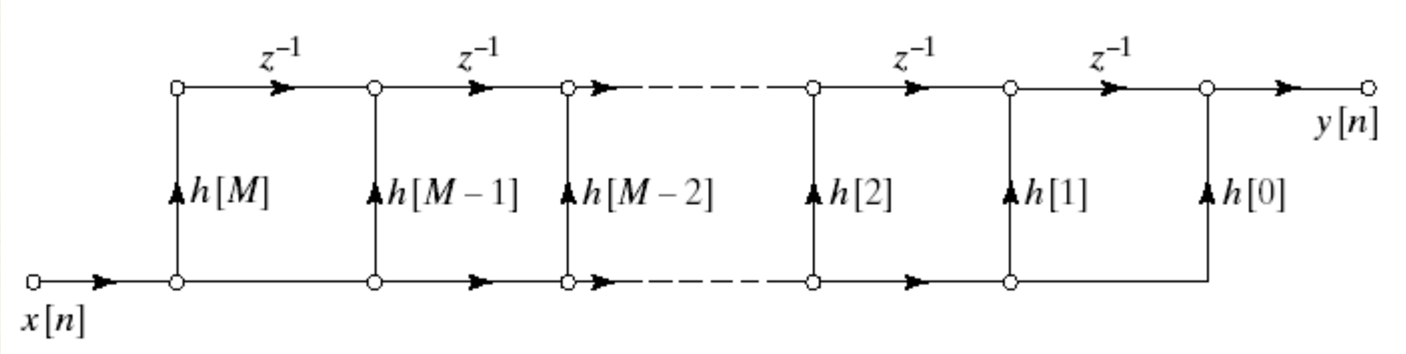


Direct Form FIR Structures

- The direct and cascade forms for IIR systems include FIR systems as a special case:



- Transpose of direct form I gives direct form II:

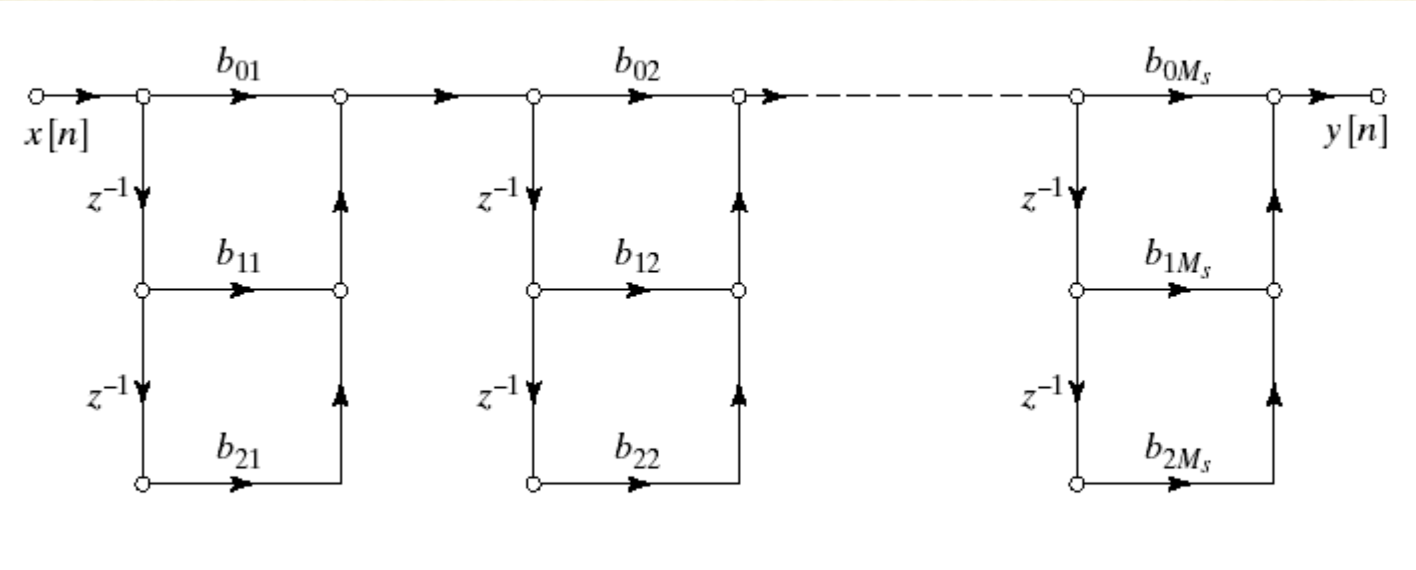


- Because of the chain of delay elements across the top of the diagram, this structure is also referred to as a tapped delay line structure or transversal filter structure.

Cascade Form

- The cascade form for FIR systems is obtained by factoring the polynomial system function. That is:

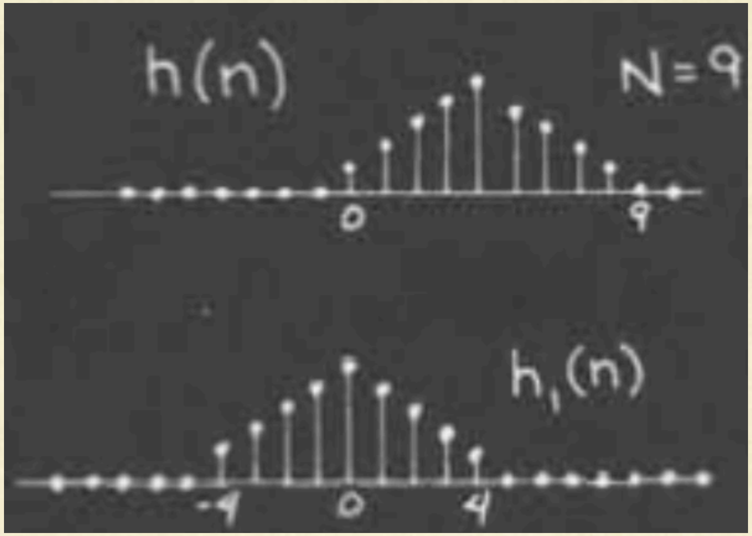
$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} = \prod_{k=1}^{M_S} \left(b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right)$$



Linear Phase FIR Systems

- Casual FIR system with generalized linear phase are symmetric.

$$h(n) = h(N-1-n)$$



$$h(n) = h_1\left(n - \frac{N-1}{2}\right)$$

$$H\left(e^{j\omega}\right) = e^{-j\omega\left(\frac{N-1}{2}\right)} H_1\left(e^{j\omega}\right)$$

$$h_1(n) \quad even \quad \Rightarrow \quad H_1\left(e^{j\omega}\right) \quad real$$

Linear Phase FIR Systems (cont.)

$$H(z)=\sum_{n=0}^{N-1}h(n)z^{-n}$$

- Assume N is even:

$$=\sum_{n=0}^{\frac{N}{2}-1}h(n)z^{-n}+\sum_{n=\frac{N}{2}}^{N-1}h(n)z^{-n}$$

$$r=(N-1)-n$$

$$n=N-1-r$$

$$=\sum_{r=0}^{\frac{N}{2}-1}h(N-1-r)z^{-(N-1-r)}$$

$$=\sum_{n=0}^{\frac{N}{2}-1}h(n)z^{-n}+\sum_{n=0}^{\frac{N}{2}-1}h(n)z^{-(N-1-n)}$$

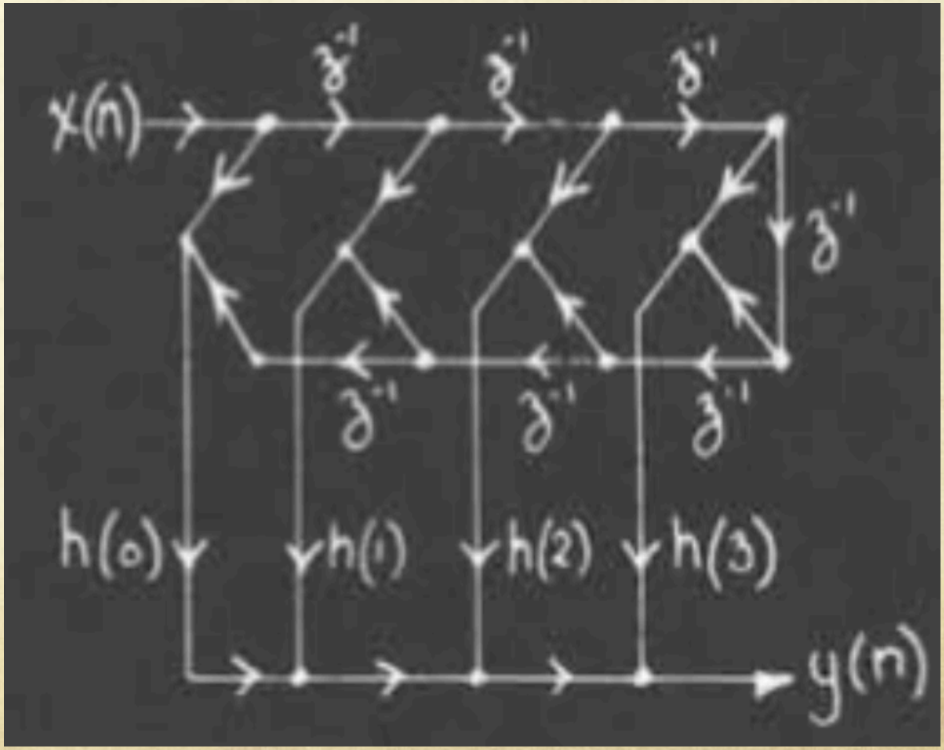
$$=\sum_{n=0}^{\frac{N}{2}-1}h(n)\left[z^{-n}+z^{-(N-1-n)}\right]$$

Linear Phase FIR Systems (cont.)

$$H(z) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\left[z^{-n} + z^{-(N-1-n)}\right]$$

$$N = 8$$

$$H(z) = \sum_{n=0}^{3} h(n)\left[z^{-n} + z^{-(-7-n)}\right]$$



Example # 1

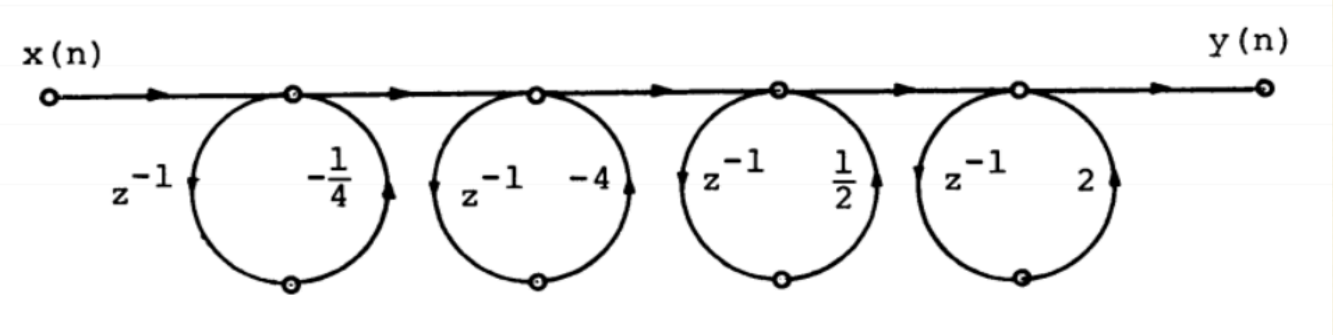
- H(z) represents the system function for an FIR linear system and is given by:

$$H(z) = \left(1 + \frac{1}{2}z^{-1}\right)\left(1 + 2z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - 4z^{-1}\right)$$

- Draw a flow graph implementation of the system in each of the following forms:
  - (a): Cascade Form
  - (b): Direct Form
  - (c): Linear phase Form

Example # 1 Solution

- (a): Since H(z) has only real zeros we will use only first-order sections in the cascade form. Figure below represents one possible ordering for these sections:

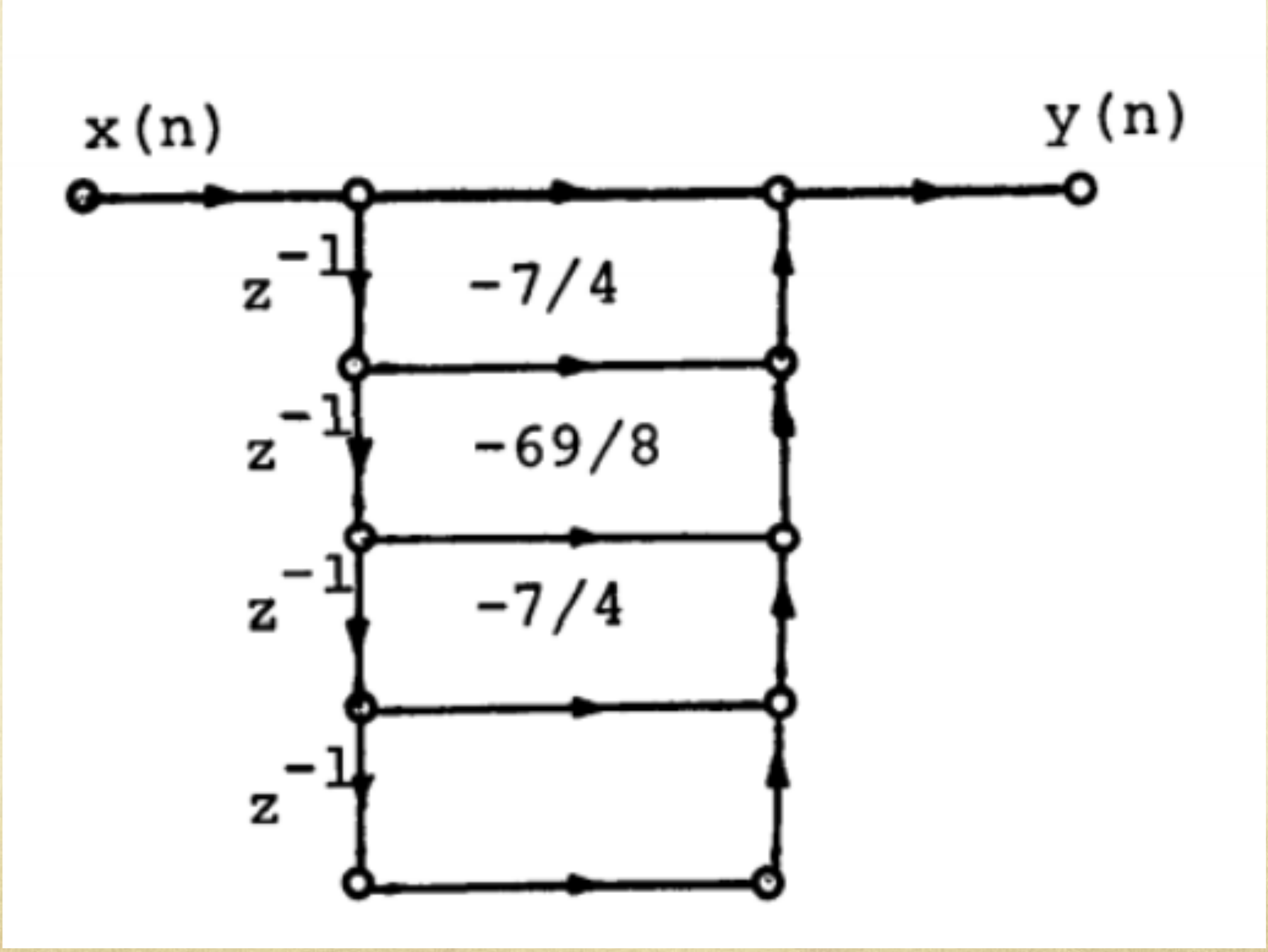


- (b): For the direct form we first express H(z) as:

$$H(z) = 1 - \frac{7}{4}z^{-1} - \frac{69}{8}z^{-2} - \frac{7}{4}z^{-3} + z^{-4}$$

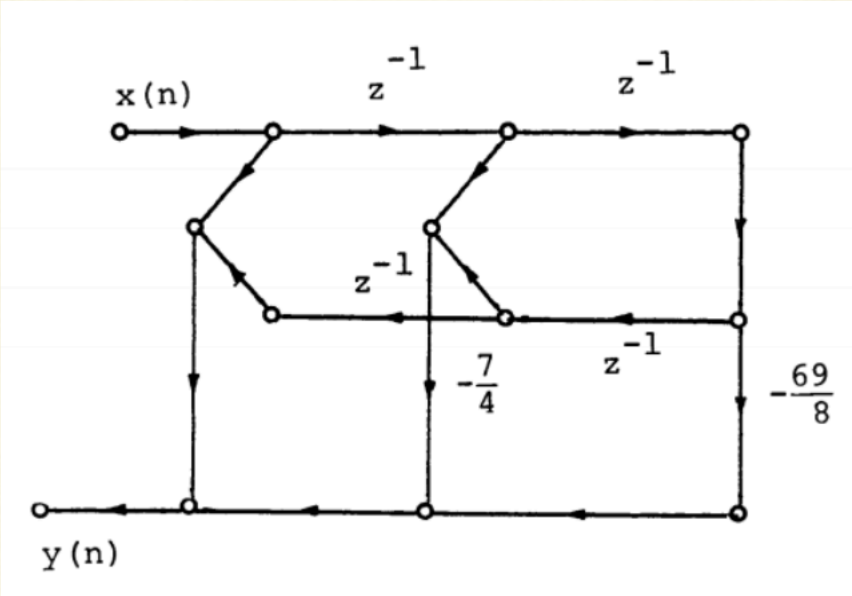
  - The direct form structure is then shown below:

Example # 1 Solution (cont.)



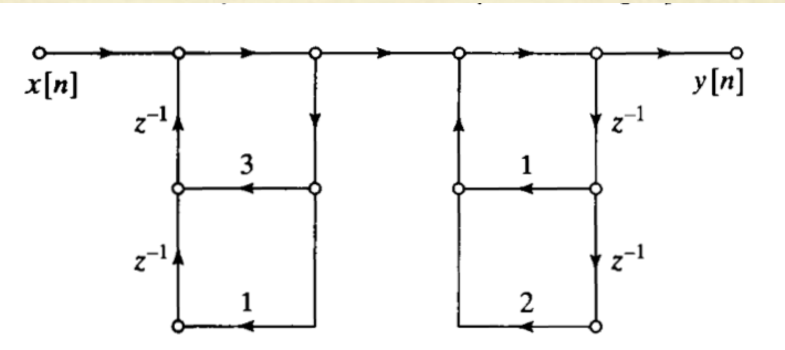
Example # 1 Solution (cont.)

- (c): Since the unit sample response if symmetrical the filter is in fact linear phase. The Linear phase form is shown below:



Example #2

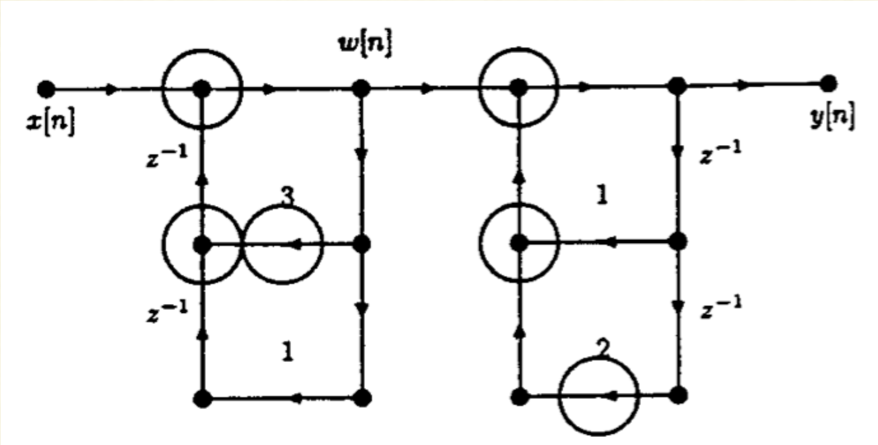
- A linear time-invariant system is realized by the flow graph shown below:



- (a): write the difference equation relating x(n) and y(n) for this flow graph.
- (b): What is the system function of the system?
- (c): In the realization of above graph, how many real multiplications and real additions are required to compute each sample of the output? (Assume that x[n] is real and assume that multiplication by 1 does not count in the total.)
- (d): The realization of above graph requires four storage registers (delay elements). Is it possible t reduce the number of storage registers by using a different structure? If so, draw the flow graph if not explain why the number of storage registers cannot be reduced.

Example #2 Solution

- The flow graph for this system is drawn below:



- (a):

$$w(n) = x(n) + 3w(n-1) + w(n-2)$$
$$y(n) = w(n) + y(n-1) + 2y(n-2)$$

Example #2 Solution

- (b):

$$W(z) = X(z) + 3z^{-1}W(z) + z^{-2}W(z)$$

$$Y(z) = W(z) + z^{-1}Y(z) + 2z^{-2}Y(z)$$

$$\frac{Y(z)}{X(z)} = H(z)$$

$$= \frac{1}{\left(1 - z^{-1} - 2z^{-2}\right)\left(1 - 3z^{-1} - z^{-2}\right)}$$

$$= \frac{1}{1 - 4z^{-1} + 7z^{-3} + 2z^{-4}}$$

- (c): Adds and multiplies are circled above: 4 real adds and 2 real multiplies per output point.

- (d): It is not possible to reduce the number of storage registers. Note that implementing H(z) above in the canonical direct form II (minimum storage registers) also requires 4 registers.