ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Spring 2016

EL-322 Digital Signal Processing

Assignment – 0 Solution | **Due Date: 08/06/2016**
Marks: 20 | **Handout Date: 30/05/2016**

Question # 1:

Find the Z-transform of the following sequences:

1. $x(n) = 2^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$
2. $x(n) = \left(\frac{1}{3}\right)^n \cos(n\omega_0)\, u(n)$

Solution:

1. $x(n) = 2^n u(n) + 3\left(\frac{1}{2}\right)^n u(n)$

(a) Because $x(n)$ is a sum of two sequences of the form $\alpha^n u(n)$, using the linearity property of the z-transform, and the z-transform pair

$$\alpha^n u(n) \overset{z}{\longleftrightarrow} \frac{1}{1-\alpha z^{-1}} \qquad |z| > |\alpha|$$

we have

$$X(z) = \frac{1}{1-2z^{-1}} + \frac{3}{1-\frac{1}{2}z^{-1}} = \frac{4-\frac{13}{2}z^{-1}}{(1-2z^{-1})\left(1-\frac{1}{2}z^{-1}\right)} \qquad |z| > 2$$

2. $x(n) = \left(\frac{1}{3}\right)^n \cos(n\omega_0)\, u(n)$

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(c) As we saw in Problem 4.3(b), the z-transform of $\cos(n\omega_0)u(n)$ is

$$\cos(n\omega_0)u(n) \stackrel{z}{\longleftrightarrow} \frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}} \qquad |z|>1$$

Therefore, using the exponentiation property,

$$\alpha^n x(n) \stackrel{z}{\longleftrightarrow} X(\alpha^{-1}z)$$

we have $$\left(\frac{1}{3}\right)^n \cos(n\omega_0)u(n) \stackrel{z}{\longleftrightarrow} \frac{1-\frac{1}{3}(\cos\omega_0)z^{-1}}{1-\frac{2}{3}(\cos\omega_0)z^{-1}+\frac{1}{9}z^{-2}}$$

with a region of convergence $|z|>\frac{1}{3}$.

Question # 2:

Evaluate the convolution of the two sequences: (Using Z-transform Property)

$$h(n) = (0.5)^n u(n), \qquad and \qquad x(n) = 3^n u(-n)$$

Solution:

$$h(n) = (0.5)^n u(n) \qquad \text{and} \qquad x(n) = 3^n u(-n)$$

To evaluate this convolution, we will use the convolution property of the z-transform. The z-transform of $h(n)$ is

$$H(z) = \frac{1}{1 - \frac{1}{2} z^{-1}} \qquad |z| > \frac{1}{2}$$

and the z-transform of $x(n)$ may be found from the time-reversal and shift properties, or directly as follows:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{0} 3^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3} z\right)^n = \frac{1}{1 - \frac{1}{3} z} = -\frac{3 z^{-1}}{1 - 3 z^{-1}} \qquad |z| < 3$$

Therefore, the z-transform of the convolution, $y(n) = x(n) * h(n)$, is

$$Y(z) = -\frac{1}{1 - \frac{1}{2} z^{-1}} \cdot \frac{3 z^{-1}}{1 - 3 z^{-1}}$$

The region of convergence is the intersection of the regions $|z| > \frac{1}{2}$ and $|z| < 3$, which is $\frac{1}{2} < |z| < 3$. To find the inverse z-transform, we perform a partial fraction expansion of $Y(z)$,

$$Y(z) = \frac{A}{1 - \frac{1}{2} z^{-1}} + \frac{B}{1 - 3 z^{-1}}$$

where
$$A = \left[\left(1 - \frac{1}{2} z^{-1}\right) Y(z)\right]_{z = \frac{1}{2}} = \frac{6}{5}$$

and
$$B = [(1 - 3z^{-1}) Y(z)]_{z=3} = -\frac{6}{5}$$

Therefore, it follows that

$$y(n) = \left(\frac{6}{5}\right)\left(\frac{1}{2}\right)^n u(n) + \left(\frac{6}{5}\right) 3^n u(-n-1)$$

Question # 3:

Find the Inverse Z-transform of the second-order system:

$$X(z) = \frac{1 + \frac{1}{4}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)^2}, \quad |z| > 2$$

Solution:

Here we have a second-order pole at $z = \frac{1}{2}$. The partial fraction expansion for $X(z)$ is

$$X(z) = \frac{A_1}{1 - \frac{1}{2}z^{-1}} + \frac{A_2}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

The constant A_1 is

$$A_1 = \frac{1}{2}\left[\frac{d}{dz}\left(1 - \frac{1}{2}z^{-1}\right)^2 X(z)\right]_{z=1/2} = \frac{1}{2}\left[-\frac{1}{4}z^{-2}\right]_{z=1/2} = -\frac{1}{2}$$

and the constant A_2 is

$$A_2 = \left[\left(1 - \frac{1}{2}z^{-1}\right)^2 X(z)\right]_{z=1/2} = \frac{3}{2}$$

Therefore,

$$X(z) = -\frac{\frac{1}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{\frac{3}{2}}{\left(1 - \frac{1}{2}z^{-1}\right)^2}$$

and

$$x(n) = -\left(\frac{1}{2}\right)^{n+1} u(n) + 3(n+1)\left(\frac{1}{2}\right)^{n+1} u(n)$$

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Good Luck