ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Spring 2016

EL-322 Digital Signal Processing

Assignment – 4 & 5 Solution **Due Date: 01/05/2016**
Marks: 20 **Handout Date: 25/05/2016**

Question # 1:

a) We wish to design a discrete time low pass filter using the bilinear transformation on a continuous-time ideal low pass filter. Assume that the continuous time prototype filter has cutoff frequency $\Omega_c = 2\pi(2000) rad/s$ and we choose the bilinear transformation parameter T=0.4ms. What is the cutoff frequency ω_c for the resulting discrete-time filter?

b) For the analog transfer function

$$H_a(s) = \frac{2}{(s+1)(s+3)}$$

Determine H (z) if T=1s, using Impulse Invariance method.

Solution:

a) Using the bilinear transform frequency mapping equation:

$$\omega_c = 2\tan^{-1}\left(\frac{\Omega_c T}{2}\right)$$

$$= 2\tan^{-1}\left(\frac{2\pi(2000)(0.4\times10^{-3})}{2}\right) = 0.7589\pi\, rad$$

b) Given, $H_a(s) = \frac{2}{(s+1)(s+3)}$

Using partial fractions, $H_a(s)$ can be expressed as:

$$H_a(s) = \frac{A}{s+1} + \frac{B}{s+3}$$

$$A = (s+1)H_a(s)|_{s=-1} = \left.\frac{2}{s+3}\right|_{s=-1} = 1$$

$$B = (s+3)H_a(s)|_{s=-3} = \left.\frac{2}{s+1}\right|_{s=-3} = -1$$

$$\therefore \qquad H_a(s) = \frac{1}{s+1} - \frac{1}{s+3} = \frac{1}{s-(-1)} - \frac{1}{s-(-3)}$$

By Impulse invariant transformation we know that:

$$\frac{A}{s-p_i} \to \frac{A}{1-e^{p_i T}z^{-1}}$$

Here $H_a(s)$ has two poles and $p_1 = -1 \ and \ p_2 = -3$.
Therefore, the system function of the digital filter is:

$$H(z) = \frac{1}{1-e^{p_1 T}z^{-1}} - \frac{1}{1-e^{p_2 T}z^{-1}}$$
$$= \frac{1}{1-e^{-T}z^{-1}} - \frac{1}{1-e^{-3T}z^{-1}}$$

Question # 2:

We wish to design an FIR low pass filter satisfying the specifications:

$$0.98 < H(e^{j\omega}) < 1.02, \qquad 0 \le |\omega| \le 0.63\pi$$
$$-0.15 < H(e^{j\omega}) < 0.15, \qquad 0.65 \le |\omega| \le \pi$$

By applying a Kaiser window to the impulse response h_d (n) for the ideal discrete time low pass filter with cutoff $\omega_c = 0.64\pi$. Find the values of β and M required to satisfy this specification. ($\delta = 0.02$).

Solution:

Since,

$$\delta = 0.02$$
$$A = -20\log_{10}(0.02) = 33.9794$$
$$\beta = 0.5842(33.9794 - 21)^{0.4} + 0.07886(33.9794 - 21) = 2.65$$
$$M = \frac{A-8}{2.285\Delta\omega} = \frac{33.9794 - 8}{2.285(0.65\pi - 0.63\pi)} = 180.95 \to 181$$

Good Luck