Solutions to the Examples in Lecture#19:

Solution of Example #3:

Let us perform the four-point circular convolution of the two sequences $h(n)$ and $x(n)$ shown in Fig. 6.4.

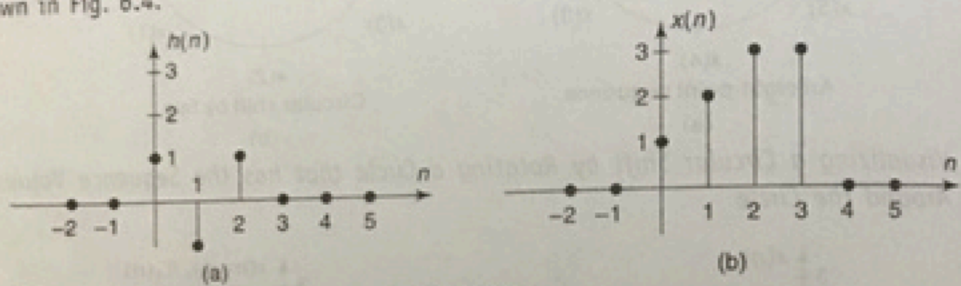


Fig. 6.4

The four-point circular convolution is

$$y(n) = \left[\sum_{k=0}^{3} \tilde{h}(n-k)\tilde{x}(k)\right]\mathcal{R}_4(n)$$

which may be performed graphically, as follows. The value of $y(n)$ at $n = 0$ is

$$y(0) = \sum_{k=0}^{3} \tilde{h}(-k)\tilde{x}(k)$$

Shown in Fig. 6.4(c) is a plot of the sequence $\tilde{h}(-k)\mathcal{R}_4(k)$. To evaluate $y(0)$, we multiply this sequence by $x(k)$ and sum the product from $k = 0$ to $k = 3$. The result is $y(0) = 1$. Next to find the value of $y(1)$, we evaluate the sum

$$y(1) = \sum_{k=0}^{3} \tilde{h}(1-k)\tilde{x}(k)$$

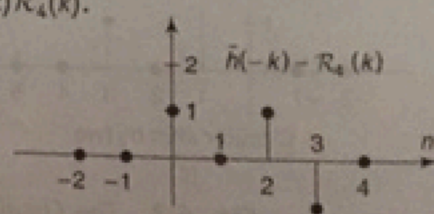


Fig. 6.4(c)

Shown in Fig. 6.4(d) is a plot of $\tilde{h}(1-k)\mathcal{R}_4(n)$. Multiplying by $\tilde{x}(k)$ and summing from $k = 0$ to $k = 3$, we find that $y(1) = 4$. Repeating for $n = 2$ and $n = 3$, we have

$$y(2) = \sum_{k=0}^{3} \tilde{h}(2-k)\tilde{x}(k) = 2$$

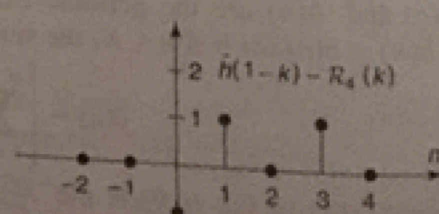


Fig. 6.4(d)

$$y(3) = \sum_{k=0}^{3} \tilde{h}(3-k)\,\tilde{x}(k) = 2$$

Therefore, $$y(n) = h(n) \circledast_4 x(n) = \delta(n) + 4\delta(n-1) + 2\delta(n-2) + 2\delta(n-3)$$

By comparison, the linear convolution of $h(n)$ with $x(n)$ is the following six-length sequence:

$$h(n) * x(n) = \delta(n) + \delta(n-1) + 2\delta(n-2) + 2\delta(n-3) + 3\delta(n-5)$$

Solution of Example #4:

−1 0 1 2 3 4 5 6 7

8.12. (a)

$$x[n] = \cos(\frac{\pi n}{2}), \quad 0 \le n \le 3$$

transforms to

$$X[k] = \sum_{n=0}^{3} \cos(\frac{\pi n}{2}) W_4^{kn}, \quad 0 \le k \le 3$$

The cosine term contributes only two non-zero values to the summation, giving:

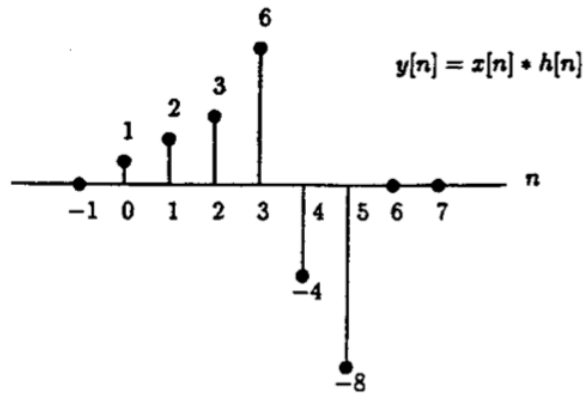
$$\begin{aligned} X[k] &= 1 - e^{-j\pi k}, \quad 0 \le k \le 3 \\ &= 1 - W_4^{2k} \end{aligned}$$

(b)

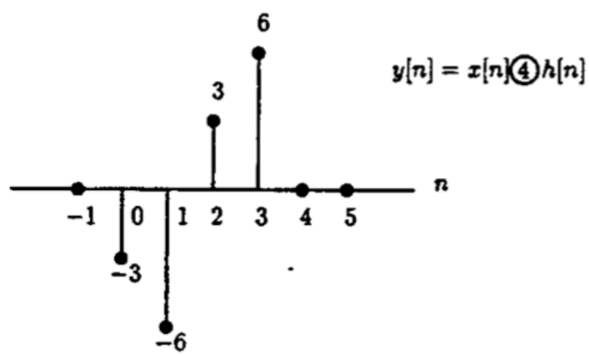
$$h[n] = 2^n, \quad 0 \le n \le 3$$

$$\begin{aligned} H[k] &= \sum_{n=0}^{3} 2^n W_4^{kn}, \quad 0 \le k \le 3 \\ &= 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k} \end{aligned}$$

(c) Remember, circular convolution equals linear convolution plus aliasing. We need $N \geq 3+4-1=6$ to avoid aliasing. Since $N=4$,we expect to get aliasing here. First, find $y[n]=x[n]*h[n]$:



For this problem, aliasing means the last three points ($n=4,5,6$) will wrap-around on top of the first three points, giving $y[n]=x[n]④h[n]$:



(d) Using the DFT values we calculated in parts (a) and (b):

$$\begin{aligned} Y[k] &= X[k]H[k] \\ &= 1 + 2W_4^k + 4W_4^{2k} + 8W_4^{3k} - W_4^{2k} - 2W_4^{3k} - 4W_4^{4k} - 8W_4^{5k} \end{aligned}$$

Since $W_4^{4k} = W_4^{0k}$ and $W_4^{5k} = W_4^k$

$$Y[k] = -3 - 6W_4^k + 3W_4^{2k} + 6W_4^{3k}, \quad 0 \le k \le 3$$

Taking the inverse DFT:

$$y[n] = -3\delta[n] - 6\delta[n-1] + 3\delta[n-2] + 6\delta[n-3], \quad 0 \le n \le 3$$