ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

EL-322 Digital Signal Processing

Assignment – 1 Solution | **Due Date: 28/07/2016**
Marks: 20 | **Handout Date: 21/07/2016**

Question # 1:

Solve the linear system by Gauss-Jordan elimination:

$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x \qquad\qquad - 3w = -3$$

Solution:

The augmented matrix is:

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

-2R1+R2, 1R1+R3, -3R1+R4

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

1/3 R2, then -1 new R2+R3, -3 new R2+R4

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R2+R1

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The corresponding system of equation is:

$$x - w = -1 \text{ or } x = w - 1$$
$$y - 2z = 0 \text{ or } y = 2z$$

Let z=s and w=t. The solution is x= t-1, y = 2s, z= s, w = t.

Question # 2:

Use the given information to find A:

$$(I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$$

Solution:

$$((I + 2A)^{-1})^{-1} = \frac{1}{-1.5 - 2.4} \begin{bmatrix} 5 & -2 \\ -4 & -1 \end{bmatrix}$$

$$= -\frac{1}{13} \begin{bmatrix} 5 & -2 \\ -4 & -1 \end{bmatrix}$$

$$I + 2A = \begin{bmatrix} -\frac{5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{bmatrix}$$

Thus:

$$2A = \begin{bmatrix} -\frac{5}{13} & \frac{2}{13} \\ \frac{4}{13} & \frac{1}{13} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{18}{13} & \frac{2}{13} \\ \frac{4}{13} & -\frac{12}{13} \end{bmatrix}$$

Hence:

$$A = \begin{bmatrix} -\frac{9}{13} & \frac{1}{13} \\ \frac{2}{13} & -\frac{6}{13} \end{bmatrix}$$

Question # 3:

Use the inversion algorithm to find the inverse of the given matrix, if the inverse exists:

$$\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$$

Solution:

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

-1R1:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

-2R1+R2 and 4R1+R3:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 2 & 1 & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

1/10 R2:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

10R2+R3:

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -\frac{7}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{array}\right]$$

Since there is a row of zeros on the left side, $\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$ is not invertible.

Question # 4:

Find all values of the unknown constant (s) in order for A to be symmetric:

1. $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$
2. $A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$

Solution:

1. $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$

   For A to be symmetric,

   $$a_{12} = a_{21}\ or - 3 = a + 5, so\ a = -8.$$

2. $A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$

   For A to be symmetric,

   $$a_{12} = a_{21}\ or\ a_{13} = a_{31}\ then, x^2 = 0, so\ x = 0.$$

Good Luck