ISRA UNIVERSITY
Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

MS-121 Linear Algebra

Assignment – 2 Solution **Due Date: 03/08/2016**
Marks: 20 **Handout Date: 30/07/2016**

Question # 1:

Evaluate the determinant of the given matrix by cofactor expansion:

$$\begin{bmatrix} 3 & 6 & 0 & 1 \\ -2 & 3 & 1 & 4 \\ 1 & 0 & -1 & 1 \\ -9 & 2 & -2 & 2 \end{bmatrix}$$

Solution:

Expand the determinant with first row:

$$det(A) = \begin{vmatrix} 3 & 6 & 0 & 1 \\ -2 & 3 & 1 & 4 \\ 1 & 0 & -1 & 1 \\ -9 & 2 & -2 & 2 \end{vmatrix}$$

$$det(A) = 3\underbrace{\begin{vmatrix} 3 & 1 & 4 \\ 0 & -1 & 1 \\ 2 & -2 & 2 \end{vmatrix}}_{a} - 6\underbrace{\begin{vmatrix} -2 & 1 & 4 \\ 1 & -1 & 1 \\ -9 & -2 & 2 \end{vmatrix}}_{b} + 0\underbrace{\begin{vmatrix} -2 & 3 & 4 \\ 1 & 0 & 1 \\ -9 & 2 & 2 \end{vmatrix}}_{c} - 1\underbrace{\begin{vmatrix} -2 & 3 & 1 \\ 1 & 0 & -1 \\ -9 & 2 & -2 \end{vmatrix}}_{d} \rightarrow 1$$

Lets solve first for a, b, c & d:

$$det(a) = \begin{vmatrix} 3 & 1 & 4 \\ 0 & -1 & 1 \\ 2 & -2 & 2 \end{vmatrix} ; Expand\ with\ first\ column$$

$$det(a) = 3\begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} - 0\begin{vmatrix} 1 & 4 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix}$$

$$det(a) = 3[(-1\times 2) - (1\times -2)] - 0[(1\times 2) - (4\times -2)] + 2[(1\times 1) - (4\times -1)]$$

$$det(a) = 3[0] - 0[10] + 2[5] \Rightarrow 10$$

$$det(b) = \begin{vmatrix} -2 & 1 & 4 \\ 1 & -1 & 1 \\ -9 & -2 & 2 \end{vmatrix} ; Expand\ with\ second\ column$$

$$det(b) = -1\begin{vmatrix} 1 & 1 \\ -9 & 2 \end{vmatrix} - 1\begin{vmatrix} -2 & 4 \\ -9 & 2 \end{vmatrix} - 2\begin{vmatrix} -2 & 4 \\ 1 & 1 \end{vmatrix}$$

$$det(b) = -1[(1\times 2) - (1\times -9)] - 1[(-2\times 2) - (4\times -9)] - 2[(-2\times 1) - (4\times 1)]$$
$$det(b) = -1[11] - 1[32] - 2[-6] \Rightarrow -31$$

There is no need to compute for (c) as it will eventually turn out to be zero.

$$det(d) = \begin{vmatrix} -2 & 3 & 1 \\ 1 & 0 & -1 \\ -9 & 2 & -2 \end{vmatrix}; Expand\ with\ second\ column$$
$$det(d) = -3\begin{vmatrix} 1 & -1 \\ -9 & -2 \end{vmatrix} + 0\begin{vmatrix} -2 & 1 \\ -9 & -2 \end{vmatrix} - 2\begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix}$$
$$det(d) = -3[(1\times -2) - (-1\times -9)] + 0[(-2\times -2) - (1\times -9)] - 2[(-2\times -1) - (1\times 1)]$$
$$det(d) = -3[-11] + 0[13] - 2[1] \Rightarrow 31$$

Put values of a, b, c & d in equation (1):

$$det(A) = 3(10) - 6(-31) + 0 - 1(31) \Rightarrow 185\ Ans\ check$$

Question # 2:

Solve by Cramer's rule:

$$\begin{aligned} 4x + 5y \quad &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

Solution:

The augmented matrix is:

$$\left(\begin{array}{ccc|c} 4 & 5 & 0 & 2 \\ 11 & 1 & 2 & 3 \\ 1 & 5 & 2 & 1 \end{array}\right)$$

First find out the determinant of main matrix, which is:

$$A = \begin{bmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$
$$det(A) = \begin{vmatrix} 4 & 5 & 0 \\ 11 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}; expand\ with\ first\ row$$
$$det(A) = 4\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 5\begin{vmatrix} 11 & 2 \\ 1 & 2 \end{vmatrix} + 0 \Rightarrow -132$$

Now replace the first column of main matrix with the solution vector and find its determinant:

$$D_1 = \begin{vmatrix} 2 & 5 & 0 \\ 3 & 1 & 2 \\ 1 & 5 & 2 \end{vmatrix}; expand\ with\ first\ row$$
$$D_1 = 2\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} - 5\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} + 0 \Rightarrow -36$$

Now replace the second column of main matrix with the solution vector and find its determinant:

$$D_2 = \begin{vmatrix} 4 & 2 & 0 \\ 11 & 3 & 2 \\ 1 & 1 & 2 \end{vmatrix}; expand\ with\ third\ column$$

$$D_2 = 0\begin{vmatrix} 11 & 3 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 4 & 2 \\ 11 & 3 \end{vmatrix} \Rightarrow -24$$

Now replace the third column of main matrix with the solution vector and find its determinant:

$$D_3 = \begin{vmatrix} 4 & 5 & 2 \\ 11 & 1 & 3 \\ 1 & 5 & 1 \end{vmatrix}; expand\ with\ first\ row$$

$$D_3 = 4\begin{vmatrix} 1 & 3 \\ 5 & 1 \end{vmatrix} - 5\begin{vmatrix} 11 & 3 \\ 1 & 1 \end{vmatrix} + 2\begin{vmatrix} 11 & 1 \\ 1 & 5 \end{vmatrix} \Rightarrow 12$$

Now:

$$x_1 = \frac{D_1}{D} = \frac{-36}{-132} \Rightarrow \frac{3}{11}$$

$$x_2 = \frac{D_2}{D} = \frac{-24}{-132} \Rightarrow \frac{2}{11}$$

$$x_3 = \frac{D_3}{D} = \frac{12}{-132} \Rightarrow -\frac{1}{11}$$

Good Luck