



ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering

Program: B.E. (Electrical)

Semester – Summer 2016

MS-121 Linear Algebra

Assignment – 3 **Solution**

Marks: 15

Due Date: 18/08/2016

Handout Date: 10/08/2016

Question # 1:

Find the Euclidean distance between \mathbf{u} and \mathbf{v} :

- i. $\mathbf{u} = (0, -2, -1, 1), \mathbf{v} = (-3, 2, 4, 4)$
- ii. $\mathbf{u} = (2, -1), \mathbf{v} = (-2, 2)$
- iii. $\mathbf{u} = (1, 2, -3, 0), \mathbf{v} = (5, 1, 2, -2)$

Solution:

- i. $\mathbf{u} = (0, -2, -1, 1), \mathbf{v} = (-3, 2, 4, 4)$

Euclidean distance is as follows:

$$\begin{aligned}d(\mathbf{u}, \mathbf{v}) &= \|\mathbf{u} - \mathbf{v}\| \\&= \sqrt{(0 - (-3))^2 + ((-2) - 2)^2 + (-1 - 4)^2 + (1 - 4)^2} \\&= \sqrt{3^2 + (-4)^2 + (-5)^2 + (-3)^2} \Rightarrow \sqrt{59} \text{ Ans}\end{aligned}$$

- ii. $\mathbf{u} = (2, -1), \mathbf{v} = (-2, 2)$

Euclidean distance is as follows:

$$\begin{aligned}d(\mathbf{u}, \mathbf{v}) &= \|\mathbf{u} - \mathbf{v}\| \\&= \sqrt{(2 - (-2))^2 + (-1 - 2)^2} \\&= \sqrt{4^2 + (-3)^2} = \sqrt{25} \Rightarrow 5 \text{ Ans}\end{aligned}$$

- iii. $\mathbf{u} = (1, 2, -3, 0), \mathbf{v} = (5, 1, 2, -2)$

Euclidean distance is as follows:

$$\begin{aligned}d(\mathbf{u}, \mathbf{v}) &= \|\mathbf{u} - \mathbf{v}\| \\&= \sqrt{(1 - 5)^2 + (2 - 1)^2 + (-3 - 2)^2 + (0 - (-2))^2} \\&= \sqrt{(-4)^2 + 1^2 + (-5)^2 + 2^2} \Rightarrow \sqrt{46} \text{ Ans}\end{aligned}$$

Question # 2:

Find the cosine of the angle θ between \mathbf{u} and \mathbf{w} :

- i. $\mathbf{u} = (2,3), \mathbf{w} = (5, -7)$
- ii. $\mathbf{u} = (1, -5,4), \mathbf{w} = (3,3,3)$

Solution:

- i. $\mathbf{u} = (2,3), \mathbf{w} = (5, -7)$
Cosine of the angle θ is:

$$\begin{aligned}\cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \\ &= \frac{(2)(5) + (3)(-7)}{\sqrt{2^2 + 3^2} \sqrt{5^2 + (-7)^2}} \\ &= \frac{-11}{\sqrt{13}\sqrt{74}} \Rightarrow -\frac{11}{\sqrt{962}} \text{ Ans}\end{aligned}$$

- ii. $\mathbf{u} = (1, -5,4), \mathbf{w} = (3,3,3)$
Cosine of the angle θ is:

$$\begin{aligned}\cos \theta &= \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|} \\ &= \frac{(1)(3) + (-5)(3) + (4)(3)}{\sqrt{1^2 + (-5)^2 + 4^2} \sqrt{3^2 + 3^2 + 3^2}} \\ &= \frac{0}{\sqrt{42}\sqrt{27}} \Rightarrow 0 \text{ Ans}\end{aligned}$$

Question # 3:

Find the vector component of \mathbf{x} along \mathbf{a} :

- i. $\mathbf{x} = (6,5,4), \mathbf{a} = (1, -1,3)$
- ii. $\mathbf{x} = (3, -2,6), \mathbf{a} = (1,2, -7)$

Solution:

- i. $\mathbf{x} = (6,5,4), \mathbf{a} = (1, -1,3)$
The vector component of \mathbf{x} along \mathbf{a} is:

$$\begin{aligned}\text{proj}_{\mathbf{a}} \mathbf{x} &= \frac{\mathbf{x} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \mathbf{a} = \frac{(6)(1) + (5)(-1) + (4)(3)}{(\sqrt{1^2 + (-1)^2 + 3^2})^2} (1, -1,3) \\ &= \frac{13}{11} (1, -1,3) \Rightarrow \left(\frac{13}{11}, -\frac{13}{11}, \frac{39}{11}\right) \text{ Ans}\end{aligned}$$

- ii. $\mathbf{x} = (3, -2,6), \mathbf{a} = (1,2, -7)$
The vector component of \mathbf{x} along \mathbf{a} is:

$$\text{proj}_{\mathbf{a}} \mathbf{x} = \frac{\mathbf{x} \cdot \mathbf{a}}{\|\mathbf{a}\|^2} \mathbf{a} = \frac{(3)(1) + (-2)(2) + (6)(-7)}{(\sqrt{1^2 + 2^2 + (-7)^2})^2} (1,2, -7)$$

$$= -\frac{43}{54}(1, 2, -7) \Rightarrow \left(-\frac{43}{54}, -\frac{43}{27}, \frac{301}{54}\right) \text{ Ans}$$

Good Luck