ISRA UNIVERSITY
Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

MS-121 Linear Algebra

Assignment – 4 Solution **Due Date: 25/08/2016**
Marks: 20 **Handout Date: 19/08/2016**

Question # 1:

Perform the following operations on the given matrices:

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix},\ B = \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix},\ C = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$$

- **i.** $A + B$
- **ii.** $(A + B)^T$
- **iii.** $C + B$
- **iv.** $3(A\times B)$
- **v.** $2tr(C)$

Solution:

i. $A + B$

$$A + B = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 4 & 5 & 4 \\ 1 & 2 & 2 & 3 \\ 7 & 0 & 3 & 5 \end{bmatrix}\ Ans$$

ii. $(A + B)^T$

$$(A + B)^T = \begin{bmatrix} -2 & 4 & 5 & 4 \\ 1 & 2 & 2 & 3 \\ 7 & 0 & 3 & 5 \end{bmatrix}^T \Rightarrow \begin{bmatrix} -2 & 1 & 7 \\ 4 & 2 & 0 \\ 5 & 2 & 3 \\ 4 & 3 & 5 \end{bmatrix}\ Ans$$

iii. $C + B$

$$C + B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}$$

C+B is undefined as both the matrices are of unequal lengths.

iv. $3(A\times B)$

$$3(A\times B) = 3\left(\begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix} \times \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}\right)$$

This product is undefined, as we cannot multiply 3 x 4 and a 3 x 4 matrix together. For matrix multiplication the number of columns of the 1st matrix must equal the number of rows of the 2nd matrix.

v. $2tr(C)$

$$2tr(C) = 2tr\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} = 2(2+5) = 2(7) \Longrightarrow 14\ Ans$$

Question # 2:

Solve the following system of equations with Gauss Jordan Elimination (Reduced Row Echelon Form):

$$\begin{cases} x + y + 2z = 8 \\ -x - 2y + 3z = 1 \\ 3x - 7y + 4z = 10 \end{cases}$$

Solution:

The augmented matrix form of the above equations is:

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

Applying row operations:

R1+R2, -3R1+R3

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

-1R2

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

R1-R2

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

10R2+R3

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right]$$

-1/52 R3

$$\left[\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

-7R3+R1, 5R3+R2

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Hence the solution is: $x_1 \Longrightarrow 3, x_2 \Longrightarrow 1, x_3 \Longrightarrow 2.$

Question # 3:

Compute the inverse of following 2 x 2 matrices:

i. $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

ii. $B = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$

Solution:

i. $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$

$$B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}^{-1} = \frac{1}{(2)(4)-(-3)(4)} \begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix}$$

$$= \frac{1}{8+12} \begin{bmatrix} 4 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{20} & \frac{3}{20} \\ -\frac{4}{20} & \frac{2}{20} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix} Ans$$

ii. $B = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$

$$B = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}^{-1} = \frac{1}{(-3)(-2)-(7)(1)} \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix}$$

$$= \frac{1}{6-7} \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix} = -1 \begin{bmatrix} -2 & -7 \\ -1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix} Ans$$

Good Luck