ISRA UNIVERSITY
Islamabad Campus

**Department of Electrical Engineering**
**Program: B.E. (Electrical)**
**Semester – Summer 2016**

**MS-121 Linear Algebra**

**Quiz – 1**
**Marks: 20** **Handout Date: 2/08/2016**

**Question # 1:**

Solve the following system by using the Gauss-Jordan elimination method:

$$\begin{cases} x + y + z = 5 \\ 2x + 3y + 5z = 8 \\ 4x + 5z = 2 \end{cases}$$

**Solution:**

The augmented matrix of the system is as following:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{array}\right]$$

Perform row operations until a reduced row echelon form is obtained:

$$\xrightarrow{-2R1+R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 4 & 0 & 5 & 2 \end{array}\right]$$

$$\xrightarrow{-4R1+R3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -4 & 1 & -18 \end{array}\right]$$

$$\xrightarrow{4R2+R3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{array}\right]$$

$$\xrightarrow{\frac{1}{13}R3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{-3R3+R2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{-1R3+R1} \left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{-1R2+R1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Hence the solution is:

$$x = 3, y = 4, z = -2$$

**Question # 2:**

Find the determinant of the matrix with row reduction method or cofactor expansion:

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 3 & 0 & 0 & 5 \\ 2 & 1 & 4 & -3 \\ 1 & 0 & 5 & 0 \end{bmatrix}$$

**Solution:**

Row Reduction Method:

$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 3 & 0 & 0 & 5 \\ 2 & 1 & 4 & -3 \\ 1 & 0 & 5 & 0 \end{bmatrix}$$

$$-3R1 + R2 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & -6 & 8 \\ 2 & 1 & 4 & -3 \\ 1 & 0 & 5 & 0 \end{bmatrix}$$

$$-2R1 + R3 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & -6 & 8 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 5 & 0 \end{bmatrix}$$

$$-1R1 + R4 \rightarrow \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 0 & -6 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

$$R2 \leftrightarrow R3 \rightarrow -\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -6 & 8 \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

$$-\frac{1}{6}R3 \rightarrow -\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 3 & 1 \end{bmatrix}$$

$$-3R3 + R4 \rightarrow -\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$Restore\ the\ R3\ to\ original \rightarrow -\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -6 & 8 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Multiply the main diagonal elements:

$$-(1 \times 1 \times (-6) \times 5) \Longrightarrow 30\ Ans$$

Cofactor Expansion:

$$\det(A) = \begin{vmatrix} 1 & 0 & 2 & -1 \\ 3 & 0 & 0 & 5 \\ 2 & 1 & 4 & -3 \\ 1 & 0 & 5 & 0 \end{vmatrix} ; \textit{expand with second column}$$

$$\det(A) = -0\begin{vmatrix} 3 & 0 & 5 \\ 2 & 4 & -3 \\ 1 & 5 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & 5 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ 1 & 5 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ 2 & 4 & -3 \end{vmatrix}$$

$$\det(A) = -0 + 0 - 1\underbrace{\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ 1 & 5 & 0 \end{vmatrix}}_{(a)} + 0$$

$$\det(a) = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ 1 & 5 & 0 \end{vmatrix} ; \textit{expand with second column}$$

$$\det(a) = -2\begin{vmatrix} 3 & 5 \\ 1 & 0 \end{vmatrix} + 0\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} - 5\begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix}$$

$$\det(a) = -2(0-5) + 0 - 5(5+3)$$

$$\Longrightarrow -30; \textit{putting it back in main determinant}$$

$$\det(A) = -0 + 0 - 1\underset{(a)}{(-30)} + 0 \Longrightarrow 30\ Ans$$

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