ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

MS-121 Linear Algebra

Quiz – 2 Solution
Marks: 20 **Handout Date: 11/08/2016**

Question # 1:

Use Cramer's rule to solve the following system of equations:

$$\begin{cases} 2x + y + z = 3 \\ x - y - z = 0 \\ x + 2y + z = 0 \end{cases}$$

Solution:

Coefficient of matrix's determinant:

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = [(-2) + (-1) + (2)] - [(-1) + (-4) + (1)] \Rightarrow 3$$

$$D_x = \begin{vmatrix} 3 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 2 & 1 \end{vmatrix} = [(-3) + (0) + (0)] - [(0) + (-6) + (0)] = -3 + 6 \Rightarrow 3$$

$$D_y = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{vmatrix} = [(0) + (-3) + (0)] - [(0) + (0) + (3)] = -3 - 3 \Rightarrow -6$$

$$D_z = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \\ 1 & 2 & 0 \end{vmatrix} = [(0) + (0) + (6)] - [(-3) + (0) + (0)] = 6 + 3 \Rightarrow 9$$

Using Cramer's rule:

$$x = \frac{D_x}{D} = \frac{3}{3} = 1, \qquad y = \frac{D_y}{D} = \frac{-6}{3} = -2, \qquad z = \frac{D_z}{D} = \frac{9}{3} = 3$$

Question # 2:

Check whether the following 3 x 3 matrix is invertible or not:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & 4 & 6 \end{bmatrix}$$

Solution:

$$det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 2 & 4 & 6 \end{vmatrix} ; expand\ with\ second\ column$$

$$= -1\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} + 0\begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}$$

$$= -1(0) + 0(0) - 1(0) \Rightarrow 0$$

Hence, the determinant is 0 therefore the inverse of a given matrix doesn't exist.

Question # 3:

Verify that **det (AB) = det (A) det (B):**

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 \\ 5 & 8 \end{bmatrix},$$

Solution:

$$AB = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & 3 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 3\times(-1) + 1\times5 & 3\times3 + 1\times8 \\ 2\times(-1) + 1\times5 & 2\times3 + 1\times8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 17 \\ 3 & 14 \end{bmatrix}$$

$$det(AB) = \begin{bmatrix} 2 & 17 \\ 3 & 14 \end{bmatrix} = (2)(14) - (17)(3) = 28 - 51 \Rightarrow -23$$

$$det(A) = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = (3)(1) - (1)(2) \Rightarrow 1$$

$$det(B) = \begin{bmatrix} -1 & 3 \\ 5 & 8 \end{bmatrix} = (-1)(8) - (3)(5) \Rightarrow -23$$

$$det(A)det(B) = 1\times(-23) \Rightarrow -23$$

Hence, **det (AB) = det (A) det (B)** so proved.
