ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

MS-121 Linear Algebra

Quiz – 3 Solution
Marks: 20 **Handout Date: 18/08/2016**

Question # 1:

Find the cosine of the angle $\boldsymbol{\theta}$ between **u** and **w**:

1. $\mathbf{u} = (1, -5, 4),\ \boldsymbol{w} = (3,3,3)$
2. $\mathbf{u} = (2,5),\ \boldsymbol{w} = (4, -3)$

Solution:

1. $\mathbf{u} = (1, -5, 4),\ \boldsymbol{w} = (3,3,3)$

   Cosine of the angle $\boldsymbol{\theta}$ is:

$$\cos\theta = \frac{\boldsymbol{u}.\boldsymbol{v}}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}$$

$$= \frac{(1)(3) + (-5)(3) + (4)(3)}{\sqrt{1^2 + (-5)^2 + 4^2}\sqrt{3^2 + 3^2 + 3^2}}$$

$$= \frac{\mathbf{0}}{\sqrt{42}\sqrt{27}} \Longrightarrow 0\ Ans$$

2. $\mathbf{u} = (2,5),\ \boldsymbol{w} = (4, -3)$

   Cosine of the angle $\boldsymbol{\theta}$ is:

$$\cos\theta = \frac{\boldsymbol{u}.\boldsymbol{v}}{\|\boldsymbol{u}\|\|\boldsymbol{v}\|}$$

$$= \frac{(2)(4) + (5)(-3)}{\sqrt{2^2 + 5^2}\sqrt{4^2 + (-3)^2}}$$

$$= \frac{\mathbf{-7}}{\sqrt{29}\sqrt{25}} \Longrightarrow \frac{\mathbf{-7}}{\sqrt{725}}\ Ans$$

Question # 2:

Whether the following set of vector in R^4 is linearly dependent?

$$(3,8,7,-3),(1,5,3,-1),(2,-1,2,6),(1,4,0,3)$$

Solution:

The set S= {$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$} of vectors in R^4 is linearly dependent if the only solution of $c_1\boldsymbol{v}_1 + c_2\boldsymbol{v}_2 + c_3\boldsymbol{v}_3 + c_4\boldsymbol{v}_4 = 0$.

The equation:
$c_1(3,8,7,-3) + c_2(1,5,3,-1) + c_3(2,-1,2,6) + c_4(1,4,0,3) = 0$ Generates the following equations:

$$3c_1 + c_2 + 2c_3 + c_4 = 0$$
$$8c_1 + 5c_2 - c_3 + 4c_4 = 0$$
$$7c_1 + 3c_2 + 2c_3 = 0$$
$$-3c_1 - c_2 + 6c_3 + 3c_4 = 0$$

The matrix form of the above equations is:

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 8 & 5 & -1 & 4 \\ 7 & 3 & 2 & 0 \\ -3 & -1 & 6 & 3 \end{bmatrix}$$

Finding the determinant of above matrix:

$$det(A) = \begin{vmatrix} 3 & 1 & 2 & 1 \\ 8 & 5 & -1 & 4 \\ 7 & 3 & 2 & 0 \\ -3 & -1 & 6 & 3 \end{vmatrix}; expand\ with\ third\ column$$

$$det(A) = -1\begin{vmatrix} 8 & 5 & -1 \\ 7 & 3 & 2 \\ -3 & -1 & 6 \end{vmatrix} + 4\begin{vmatrix} 3 & 1 & 2 \\ 7 & 3 & 2 \\ -3 & -1 & 6 \end{vmatrix} - 0\begin{vmatrix} 3 & 1 & 2 \\ 8 & 5 & -1 \\ -3 & -1 & 6 \end{vmatrix} + 3\begin{vmatrix} 3 & 1 & 2 \\ 8 & 5 & -1 \\ 7 & 3 & 2 \end{vmatrix}, \rightarrow equ(1)$$

Find the determinant of matrices inside the det (A):

$$\begin{vmatrix} 8 & 5 & -1 \\ 7 & 3 & 2 \\ -3 & -1 & 6 \end{vmatrix} = 8\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} - 5\begin{vmatrix} 7 & 2 \\ -3 & 6 \end{vmatrix} - 1\begin{vmatrix} 7 & 3 \\ -3 & -1 \end{vmatrix}$$
$$= 8(20) - 5(48) - 1(2) \Longrightarrow -82$$

$$\begin{vmatrix} 3 & 1 & 2 \\ 7 & 3 & 2 \\ -3 & -1 & 6 \end{vmatrix} = 3\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} - 1\begin{vmatrix} 7 & 2 \\ -3 & 6 \end{vmatrix} + 2\begin{vmatrix} 7 & 3 \\ -3 & -1 \end{vmatrix}$$
$$= 3(20) - 1(48) + 2(2) \Longrightarrow 16$$

$$\begin{vmatrix} 3 & 1 & 2 \\ 8 & 5 & -1 \\ 7 & 3 & 2 \end{vmatrix} = 3\begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix} - 1\begin{vmatrix} 8 & -1 \\ 7 & 2 \end{vmatrix} + 2\begin{vmatrix} 8 & 5 \\ 7 & 3 \end{vmatrix}$$
$$= 3(13) - 1(23) + 2(-11) \Longrightarrow -6$$

Now insert the above-calculated determinants in equ. (1):

$$det(A) = -1(-82) + 4(16) - 0 + 3(-6) \Rightarrow 128 \neq 0$$

The system has only the trivial solution and the vectors are **Linearly Independent**.
