ISRA UNIVERSITY
Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester – Summer 2016

MS-121 Linear Algebra

Quiz – 4 Solution
Marks: 20 **Handout Date: 25/08/2016**

Question # 1:

Find the characteristic equation and Eigenvalues of the following matrix:

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

To find the characteristic equation of the matrix A, lets consider the following statement:

$$\det(\lambda I - A) = 0$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}\right) = \det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}\right)$$

$$\det\left(\begin{bmatrix} \lambda - 1 & 3 & -3 \\ -3 & \lambda + 5 & -3 \\ -6 & 6 & \lambda - 4 \end{bmatrix}\right); expand\ with\ first\ row$$

$$\det\left(\begin{bmatrix} \lambda - 1 & 3 & -3 \\ -3 & \lambda + 5 & -3 \\ -6 & 6 & \lambda - 4 \end{bmatrix}\right)$$

$$= \lambda - 1 \begin{vmatrix} \lambda + 5 & -3 \\ 6 & \lambda - 4 \end{vmatrix} - 3 \begin{vmatrix} -3 & -3 \\ -6 & \lambda - 4 \end{vmatrix} + (-3) \begin{vmatrix} -3 & \lambda + 5 \\ -6 & 6 \end{vmatrix}$$

$$= \lambda - 1[(\lambda + 5)(\lambda - 4) - (-3)(6)] - 3[(-3)(\lambda - 4) - (-3)(-6)] + (-3)[(-3)(6) - (\lambda + 5)(-6)]$$

$$= \lambda - 1[(\lambda^2 - 4\lambda + 5\lambda - 20) + 18] - 3[(-3\lambda + 12) - 18] + (-3)[(-18) - (-6\lambda - 30)]$$

$$= \lambda - 1[\lambda^2 + \lambda - 20 + 18] - 3[-3\lambda + 12 - 18] + (-3)[-18 + 6\lambda + 30]$$

$$= \lambda - 1[\lambda^2 + \lambda - 2] - 3[-3\lambda - 6] + (-3)[6\lambda + 12]$$

$$= \lambda^3 + \lambda^2 - 2\lambda - \lambda^2 - \lambda + 2 + 9\lambda + 18 - 18\lambda - 36$$

$$= \lambda^3 - 12\lambda - 16$$

$$\det(\lambda I - A) = \lambda^3 - 12\lambda - 16 = 0$$
$$\lambda^3 - 12\lambda - 16 = 0$$
$$(\lambda - 4)(\lambda^2 + 4\lambda + 4) = 0$$
$$(\lambda - 4) = 0, \lambda^2 + 4\lambda + 4 = 0$$
$$\lambda = 4$$

Use quadratic formula for the other term:

$$\lambda = \frac{-4 \pm \sqrt{4^2 - 4.1.4}}{2} = \frac{-4 \pm 0}{2} \Rightarrow -2$$

Therefore, the eigenvalues of A are $\lambda = 4, -2$

<u>Question # 2:</u>

Find the determinant of the given matrix by (a) Cofactor expansion & (b) Arrow Technique:

$$A = \begin{bmatrix} 3 & -4 & a \\ a^2 & 1 & 2 \\ 2 & a-1 & 4 \end{bmatrix}$$

<u>Solution:</u>

a) Cofactor Expansion:

$$\det(A) = \begin{vmatrix} 3 & -4 & a \\ a^2 & 1 & 2 \\ 2 & a-1 & 4 \end{vmatrix} ; \textit{expand with 1st row}$$
$$= 3\begin{vmatrix} 1 & 2 \\ a-1 & 4 \end{vmatrix} - (-4)\begin{vmatrix} a^2 & 2 \\ 2 & 4 \end{vmatrix} + a\begin{vmatrix} a^2 & 1 \\ 2 & a-1 \end{vmatrix}$$
$$= 3[(1)(4) - (2)(a-1)] - (-4)[(a^2)(4) - (2)(2)]$$
$$+ a[(a^2)(a-1) - (1)(2)]$$
$$= 3[4 - 2a + 2] - (-4)[4a^2 - 4] + a[a^3 - a^2 - 2]$$
$$= 3[-2a + 6] + 4[4a^2 - 4] + a[a^3 - a^2 - 2]$$
$$= -6a + 18 + 16a^2 - 16 + a^4 - a^3 - 2a$$
$$\det(A) = a^4 - a^3 + 16a^2 - 8a + 2 \; Ans$$

b) Arrow Technique:

$$\det(A) = \begin{vmatrix} 3 & -4 & a \\ a^2 & 1 & 2 \\ 2 & a-1 & 4 \end{vmatrix} \begin{matrix} 3 & -4 \\ a^2 & 1 \\ 2 & a-1 \end{matrix}$$

$$\det(A) = \left[(3\times1\times4) + (-4\times2\times2) + \left(a\times a^2(a-1)\right)\right]$$
$$- \left[(-4\times a^2\times4) + \left(3\times2(a-1)\right) + (a\times1\times2)\right]$$
$$= \left[(12) + (-16) + \left(a^3(a-1)\right)\right] - \left[(-16a^2) + \left(6(a-1)\right) + (2a)\right]$$
$$= [12 - 16 + (a^4 - a^3)] - [-16a^2 + (6a - 6) + 2a]$$
$$= [-4 + a^4 - a^3] - [-16a^2 + 8a - 6]$$
$$= -4 + a^4 - a^3 + 16a^2 - 8a + 6 \Rightarrow a^4 - a^3 + 16a^2 - 8a + 2 \; Ans$$
