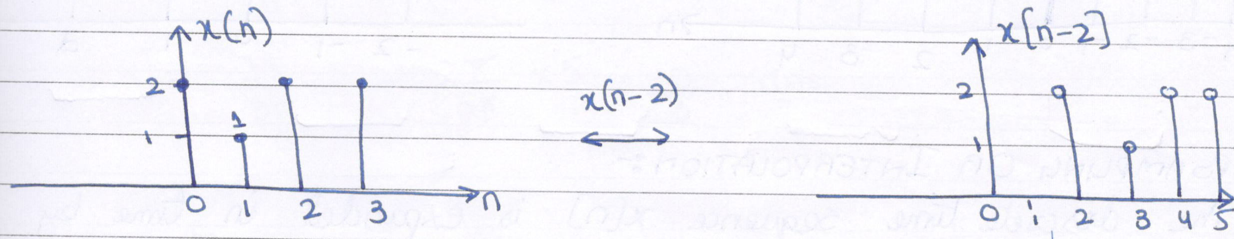
Lecture Notes
17th October 2016

Date: MONDAY, 17 OCT, 2016

TIME SHIFT:-

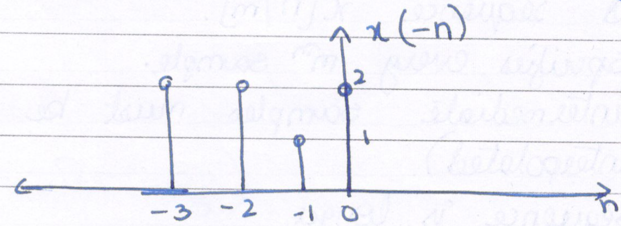
⇒ Two signals are identical in shape but that are displaced or shifted relative to each other.

$$x[n] \Leftrightarrow x[n-n_0]$$
$$x(t) \leftrightarrow x(t-t_0)$$



TIME REVERSAL:-

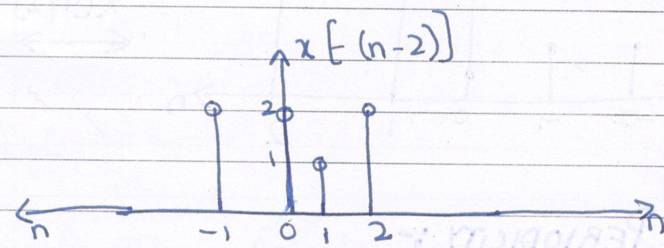
$x[n] \xleftrightarrow{x[n]}$



TIME SHIFT & REVERSAL:-

$$x[-(n-2)]$$

$x[n-2] \xleftrightarrow{x[-(n-2)]}$



TIME SCALING:-

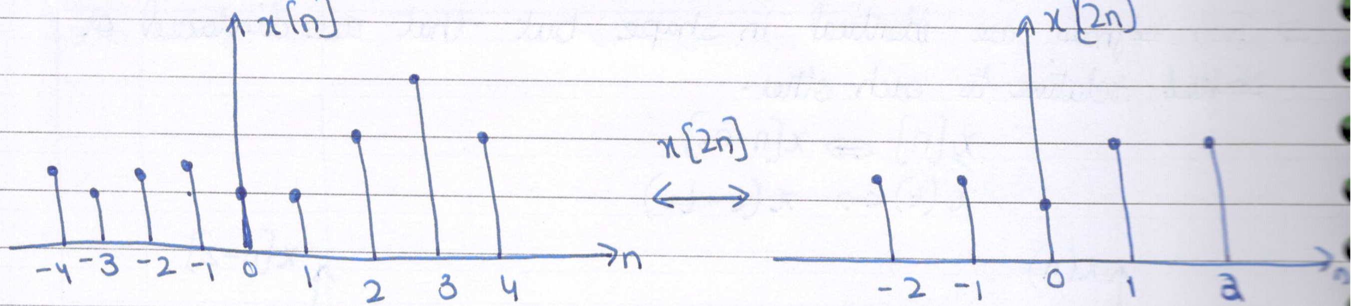
⇒ The discrete time sequence $x[n]$ is compressed in time by multiplying the index n by an integer k. to produce the time scaled sequence $x[nk]$.

⇒ This extracts every k^{th} sample of $x[n]$.

⇒ Intermediate samples are lost.

⇒ The sequence is shorter.

⇒ Also known as Downsampling or decimation.



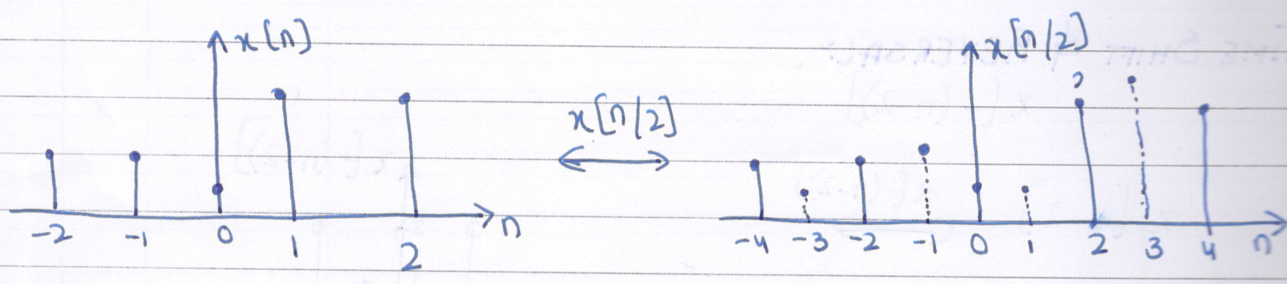
Upsampling or Interpolation :-

⇒ The discrete time sequence $x[n]$ is expanded in time by dividing the index n by an integer m. to produce the time scaled sequence $x[n/m]$.

⇒ This specifies every m^{th} sample.

⇒ The intermediate samples must be synthesized (set to zero, or interpolated).

⇒ The sequence is longer.



Periodicity :-

Example #2:

(a) $e^{j3\pi t/5}$

Sol:-

If $x(t)$ is a periodic signal, then there exists $T>0$ such that $x(t) = x(t+T)$.

$T = ?$

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$$\omega_0 = \frac{2\pi}{T_0}$$

$$T = \frac{2\pi}{\omega_0}$$

$$\omega_0 = \frac{3\pi}{5}$$

$$T = \frac{2\pi}{\frac{3\pi}{5}} = \frac{2\pi}{3\pi} \times 5 \Rightarrow \frac{10}{3} \text{ . ans}$$

b) $e^{j3\pi n/5}$

$N = ?$

$$\Omega_0 = \frac{2\pi}{N_0}$$

$$N_0 = \frac{2\pi k}{\Omega_0} \qquad \left(\because \Omega_0 = \frac{3\pi}{5}\right)$$

$$\frac{N_0}{K} = \frac{2\pi}{\frac{3\pi}{5}} = \frac{2 \times 5}{3} \Rightarrow \frac{10}{3}$$

$N_0 \Rightarrow 10$ ans

PROBLEM #1:-

let $x(t) = \cos\left(\overset{\omega_x}{\frac{3\pi}{4}}(t + T_x) + \theta_x\right)$

$\omega_x = \frac{3\pi}{4}$, $T_x = \frac{1}{2}$, $\theta_x = \pi/4$. Calculate period of $x(t)$?

Sol:-

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\frac{3\pi}{4}} = \frac{2\pi}{3\pi} \times 4 \Rightarrow \frac{8}{3} \text{ s.}$$

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PROBLEM #2:-

Let $x[n] = \cos(\Omega_x (n + P_x) + \theta_x)$

Determine the period of $x[n]$ when $\Omega_x = \frac{3}{4}$, $P_x = 1$, $\theta_x = 2/4$

SOL:-

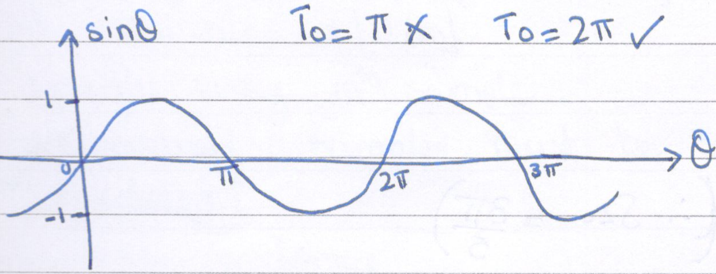
$\Omega N = 2\pi k$ $k > 0$ some integer

$$\frac{N}{k} = \frac{2\pi}{\Omega}$$

$$\frac{N}{k} = \frac{2\pi}{3/4} = \frac{2\pi}{3} \times 4 \Rightarrow \frac{8\pi}{3}$$

$N = 8\pi$ which is irrational hence, it is not periodic.

$T_0 = \pi$ ✗ $T_0 = 2\pi$ ✓



$\sin\theta = \sin(\theta \pm n\,2\pi)$

$x(t) = x(t \pm nT_0)$

Fundamental frequency $= f_0 = \frac{1}{T_0}$ Hz

Fundamental angular freq $= \omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T_0}$ rad/sec

T_0 may or may not be an integer.

for discrete time signal $x[n] = x[n \pm mN]$

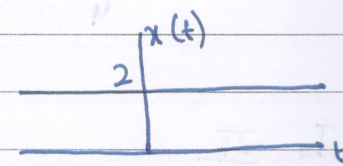
$m =$ integer, N fundamental period (must be an integer)

Fundamental frequency $= \frac{1}{N}$ Hz.

Q: dc value is P (T/F) ⇒ True its periodic.

$x(t) = 2 \;\forall t$.

$x(t) = x(t \pm nT_0)$



T_0 is undefined incase of dc value

$f_0 = 0$ because cycles $= 0$