Signal & Systems

Continuous & Discrete Signals

3rd October 16

Introduction

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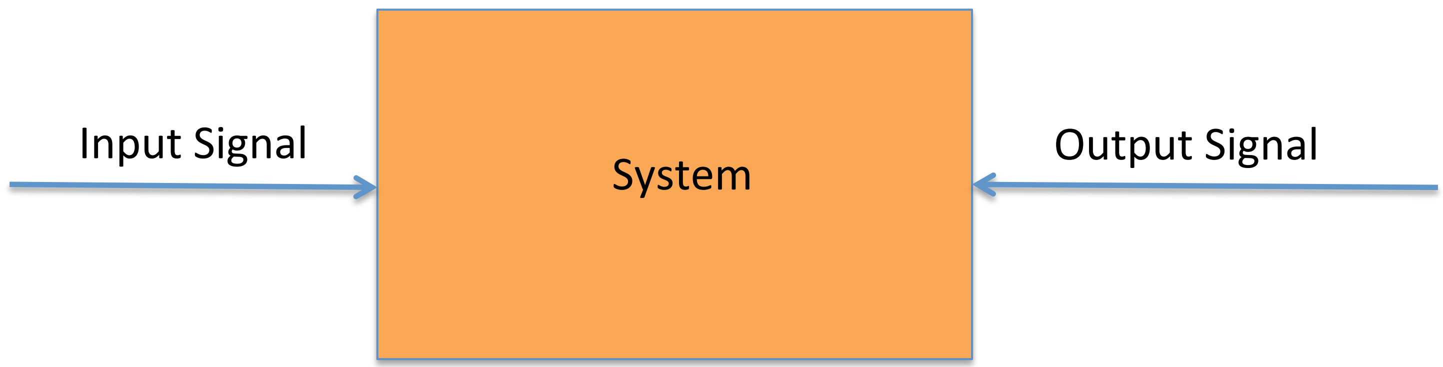
What is a Signal?



- If a function represents a physical quantity or variable containing the information about the behavior and nature of the phenomenon.
- A quantitative description of a physical phenomenon, event or process.
- Signals are function of time or a sequence in time.
- A function is mathematically represented as a function of independent variable and denoted by x(t).

What is a Signal?

- Some common examples of signals are:
  - Electrical current or voltage in a circuit.
  - Audio signal: continous time in its original form and discrete time when shared on a CD.
  - Current through the resistor.
  - Voltage across the indicator.

What is a System?

- Systems are operator that accept a given signal (the input) and produces a new signal (the output).
- A device or a set of rules defining the functional relation between the input and output.
- They are simply functions that have domain and range that are sets of functions of time.

Classification of Signals

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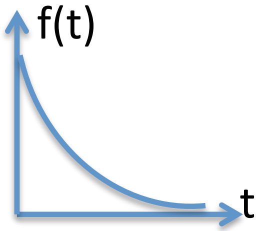
Classification

- Two main broad classification of signals are:
  - Continuous time signal
  - Discrete time signal

Continuous Time Signals

- It is an infinite and uncountable set of numbers.
- There are infinite possible values from time t and instantaneous amplitude x(t) between start and end point.
- If a signal at all values of t is a continuous variable:

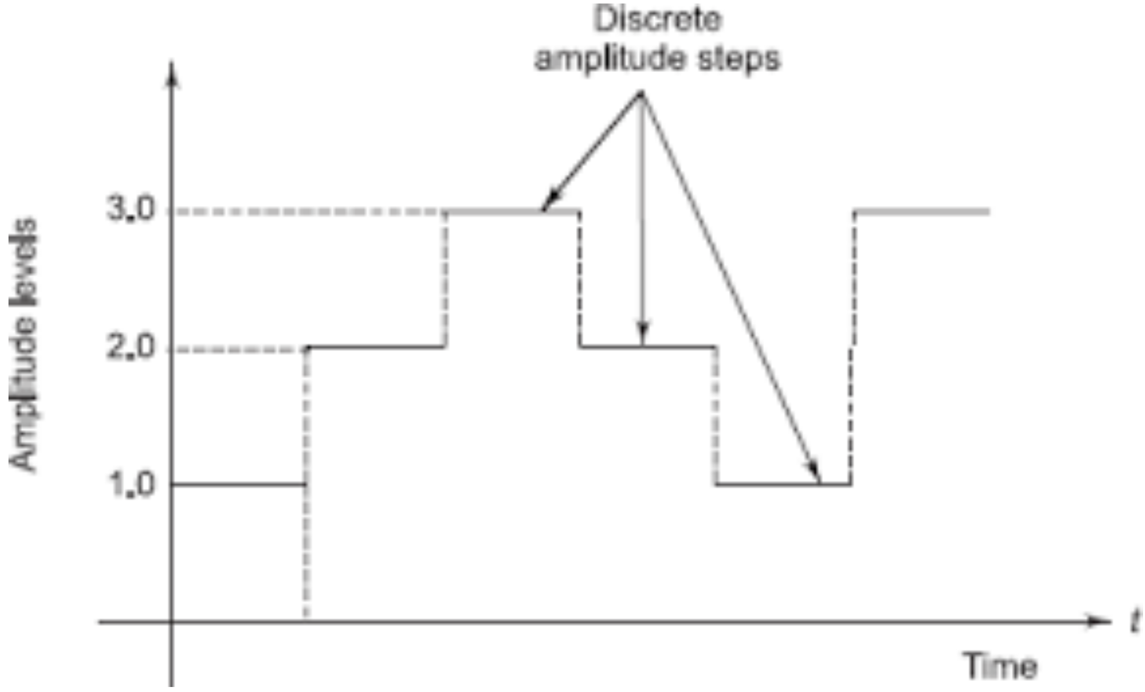


Exponential Function

- This signal is continuous in time as well as in amplitude.
  - Another example is Sinusoidal Signal.

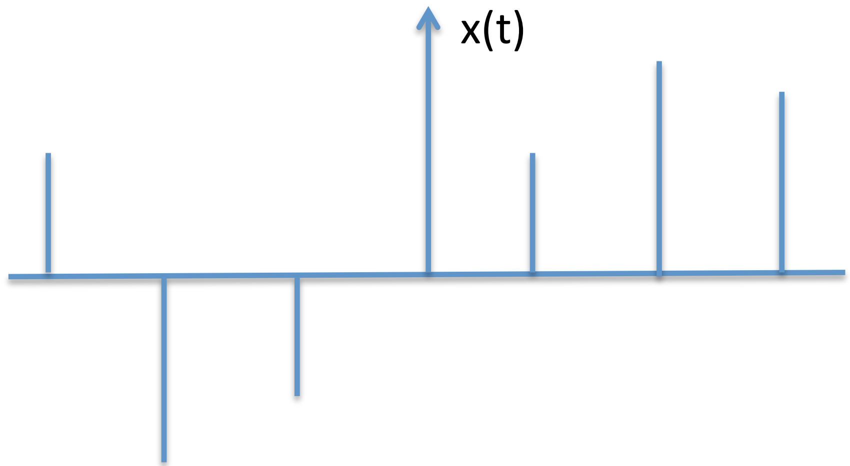
Continuous Time Signals

❖ This signal is continuous in time but discrete in amplitude.



Discrete Time Signals

- The number of elements in the set as well as possible values of each element is finite and countable.
- It can be represented with computer bits and stored on a digital storage medium.



Basic Operations

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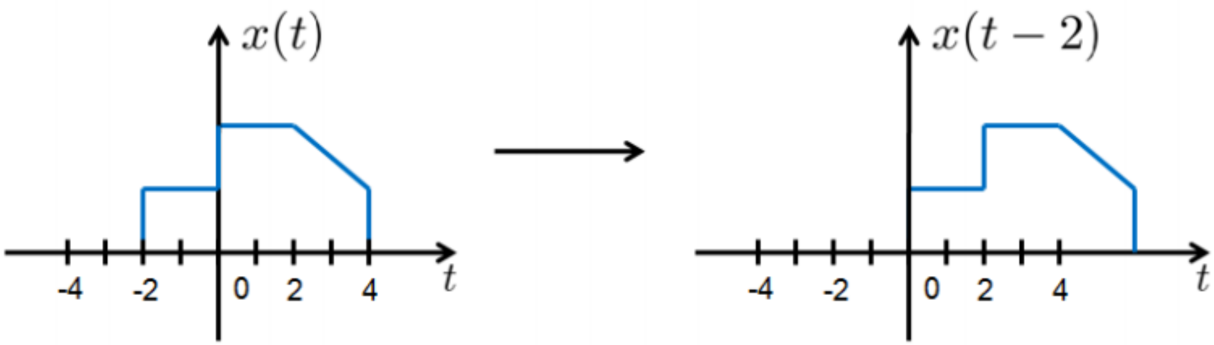
Time Shift

- For any $t_0 \in R$ and $n_0 \in Z$, time shift is an operation defined as:

$$x(t) \rightarrow x(t - t_0)$$

$$x[n] \rightarrow x[n - n_0]$$

- If $t_0 > 0$, the time shift is known as “delay”.
- If $t_0 < 0$, the time shift is known as “advance”.
- For example:

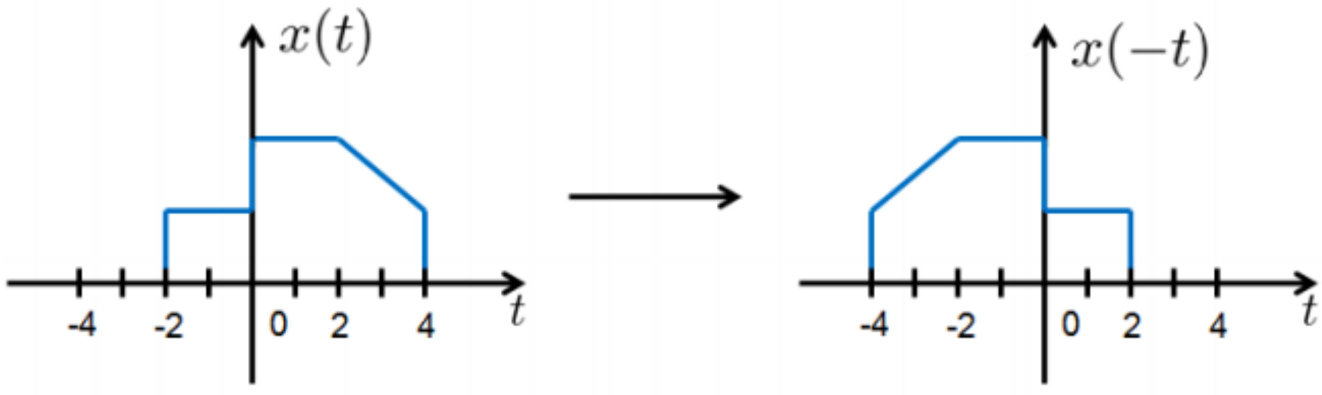


Time Reversal

- Time reversal if defined as: $x(t) \rightarrow x(-t)$

$$x[n] \rightarrow x[-n]$$

- Which can be interpreted as the "flip over the y-axis".
- For example:

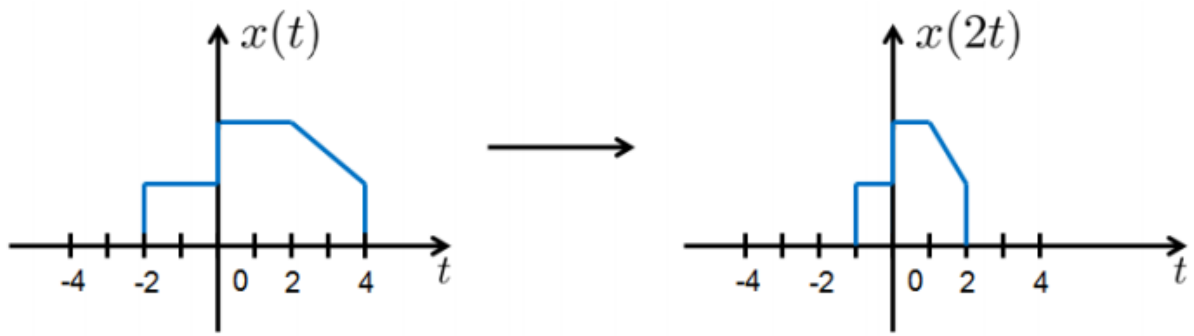


Time Scaling

- Time scaling is the operation where the time variable t is multiplied by a constant a:

$$x(t) \rightarrow x(at), \quad a > 0$$

- If a > 1, the time scale of the resultant signal is "decimated" (speed up).
- If 0 < a < 1, the time scale of the resultant signal is "expanded" (slowed down).
- For example:



Combination of Operations

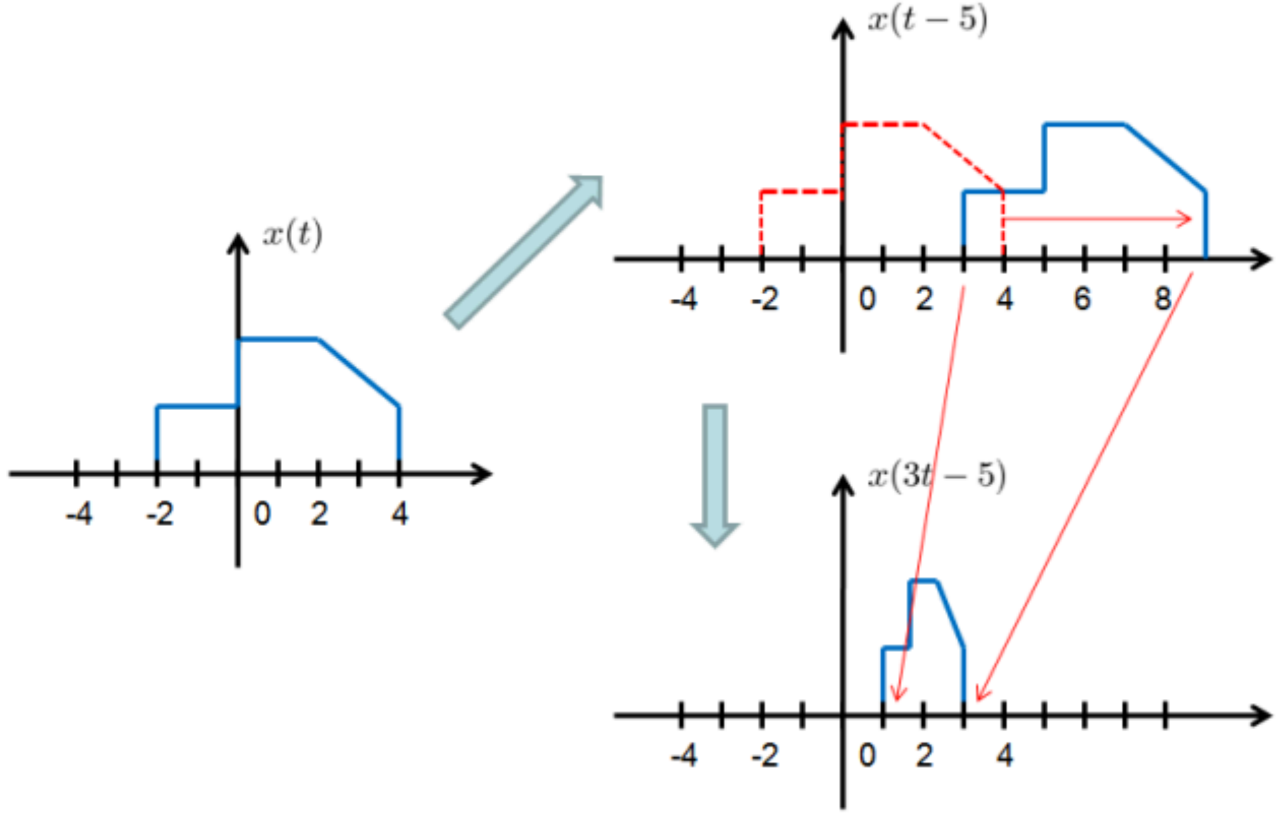
- Linear operation in time on a signal x(t) can be expressed as:

$$y(t) = x(at - b), \quad a, b \in R$$

- There are two methods to describe the output signal:
  - Method A: “shift, then scale”
    - Define v(t)=x(t-b)
    - Define y(t)=v(at)=x(at-b)
  - Method B: “scale, then shift”
    - Define v(t)=x(at)
    - Define y(t)=v(t-b/a)=x(at-b)

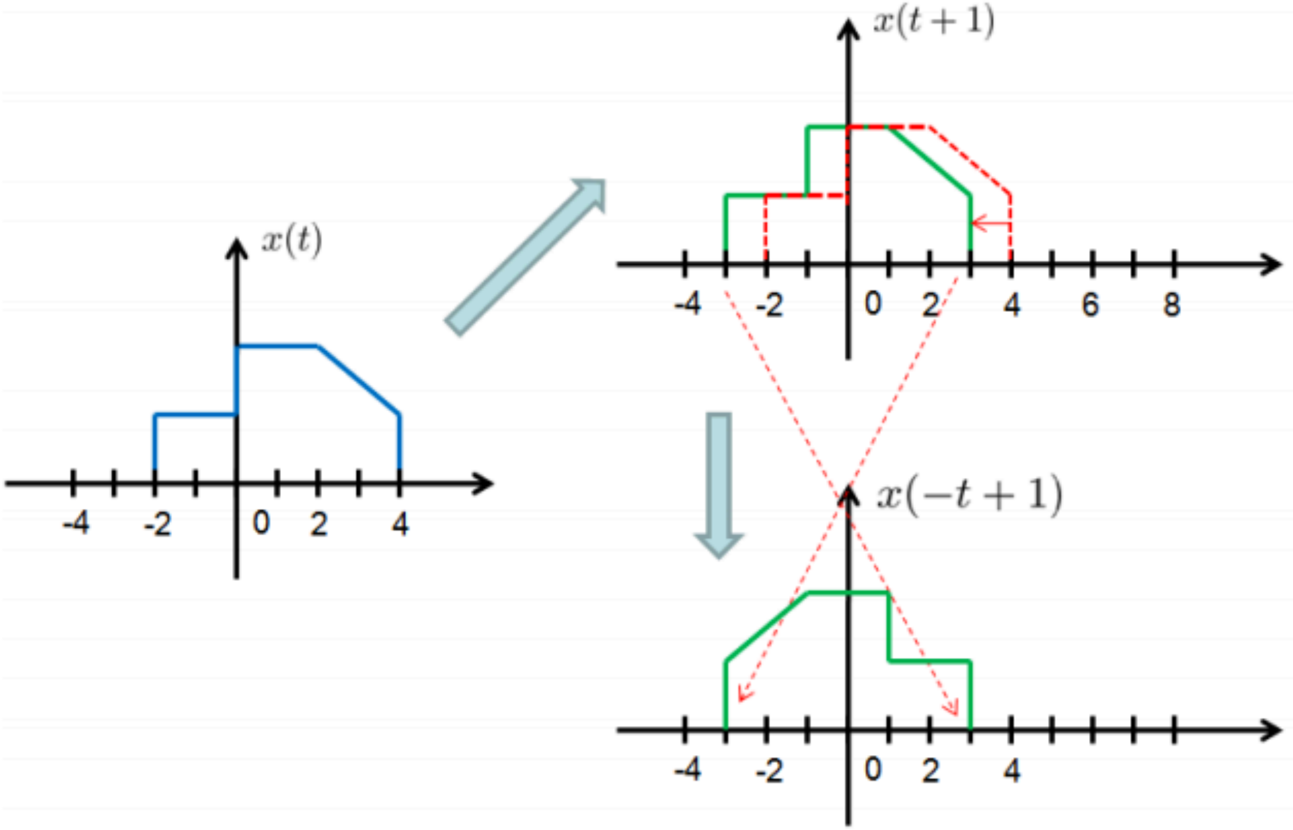
Combination of Operations

❖ Example-1:



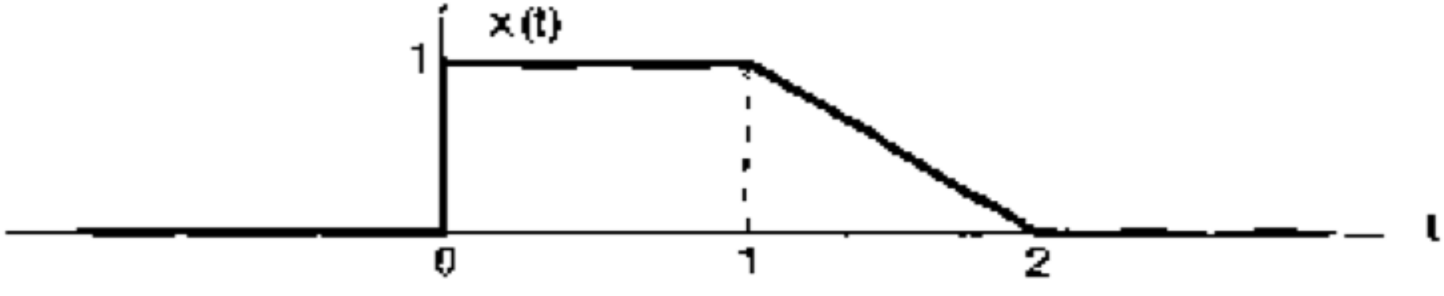
Combination of Operations

❖ Example-2:



Example #1

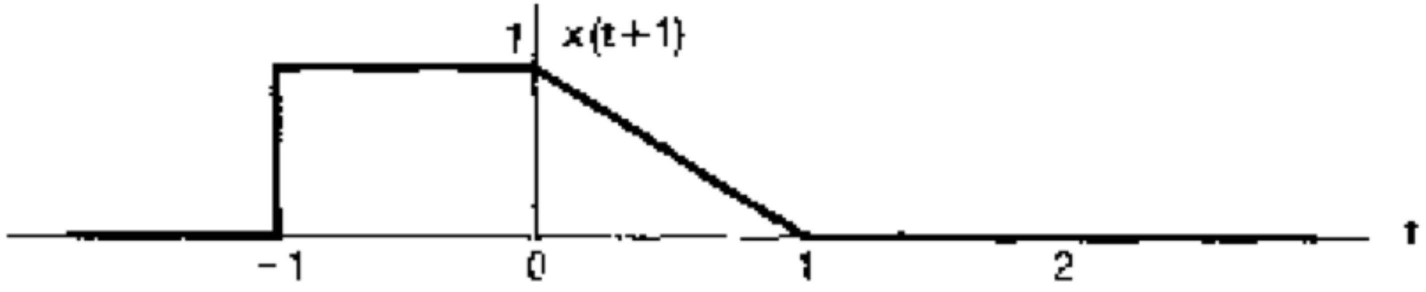
❖ Given the signal x(t) as shown below.



(a)

❖ Solution:

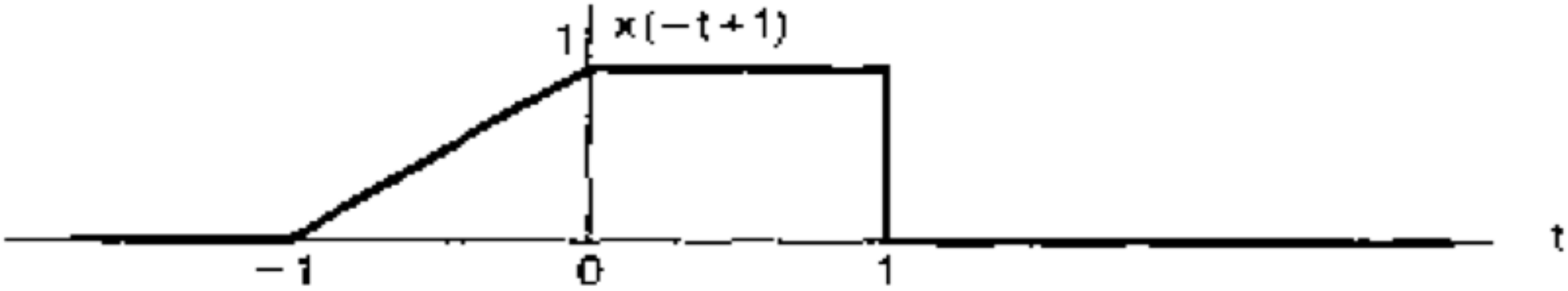
- ❖ (a): Draw the signal x (t+1):



(b)

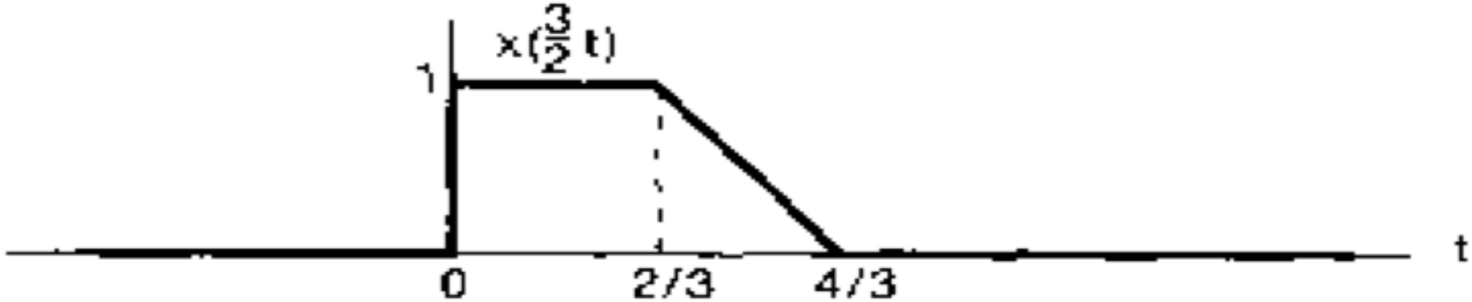
Example #1 (cont.)

- (b): Draw the signal x (-t+1) obtained by a time shift and a time reversal:



(c)

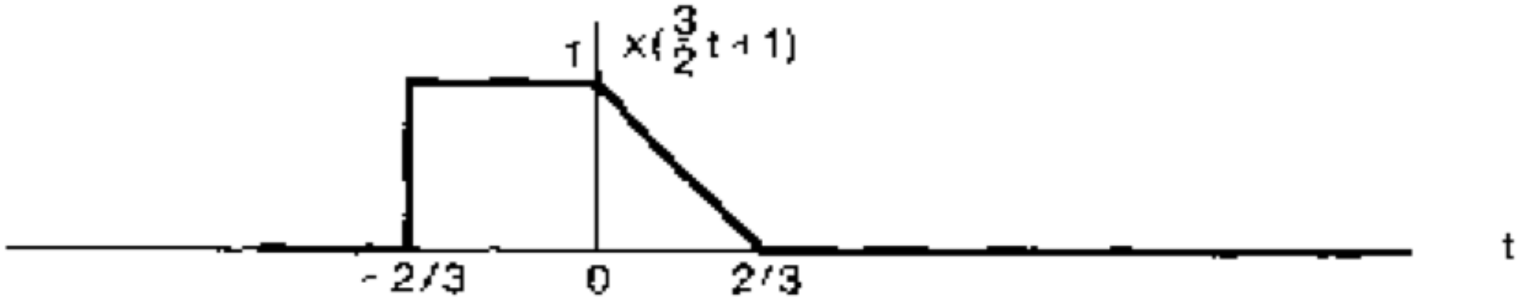
- (c): Draw the time scaled signal x (3/2 t):



(d)

Example #1 (cont.)

❖ (d): Draw the signal x (3/2 t+1) obtained by a time shift and scaling:



(e)

Decimation & Expansion

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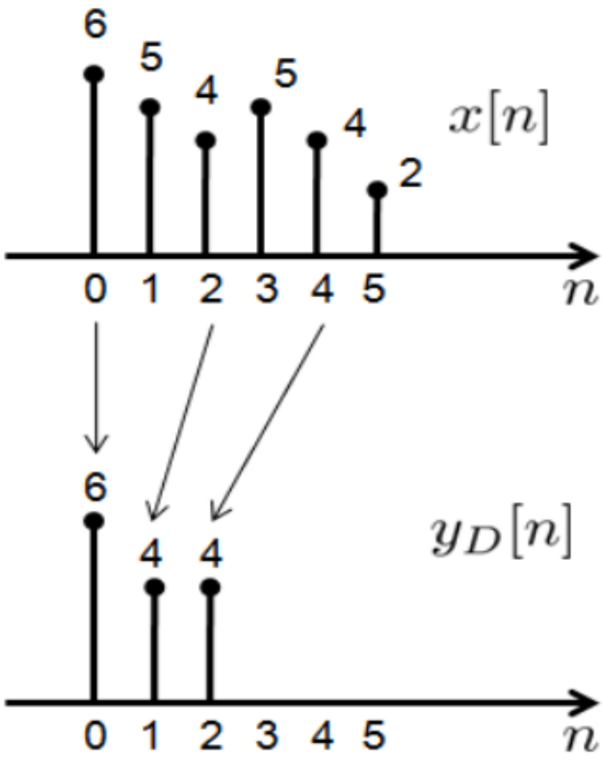
Decimation

- Decimation is defined as:

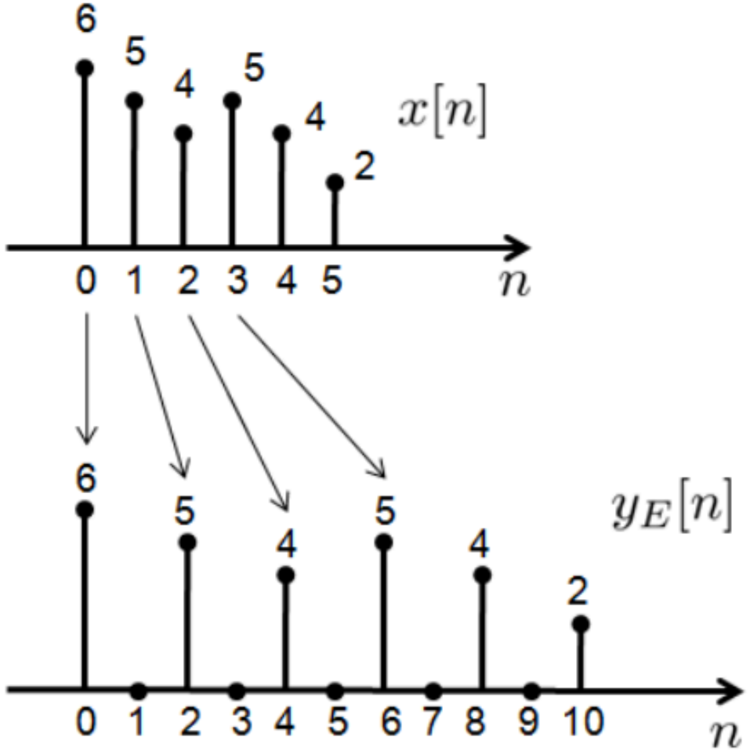
$$y_D[n] = x[Mn]$$

- For some integers M. M is called the decimation factor.
- When M=2.



Expansion

- Expansion is defined as: $$y_E[n] = \begin{cases} x\left[\frac{n}{L}\right], & n = \text{integer} \quad multiple \quad of \quad L \\ 0, & otherwise \end{cases}$$
- L is called the expansion factor.
- When L=2.



Periodicity

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Definitions

- Definition-1: A continuous time signal x(t) is periodic if there is a constant T > 0 such that:

$$x(t) = x(t+T), \quad for \quad all \quad t \in R$$

- Definition-2: A discrete time signal x[n] is periodic if there is an integer constant N > 0 such that:

$$x[n] = x[n+N], \quad for \quad all \quad n \in Z$$

- Signals do not satisfy the periodicity conditions are called aperiodic signals.
- T_0 is called the fundamental period of x(t) if it is the smallest value of T > 0 satisfying the periodicity condition. The number $\omega_0 = \dfrac{2\pi}{T_0}$ is called the fundamental frequency of x(t).

Definitions (cont.)

- N_0 is called the fundamental period of x[n] if it is smallest value of N > 0 where N ε Z satisfying the periodicity condition. The number $\frac{\Omega_0}{2\pi} = \frac{m}{N}$ is called the fundamental frequency of x[n].

Example #2

- Determine the fundamental period of the following signals:

$$(a): \quad e^{j3\pi t/5}$$

$$(b): \quad e^{j3\pi n/5}$$

Even & Odd Signals

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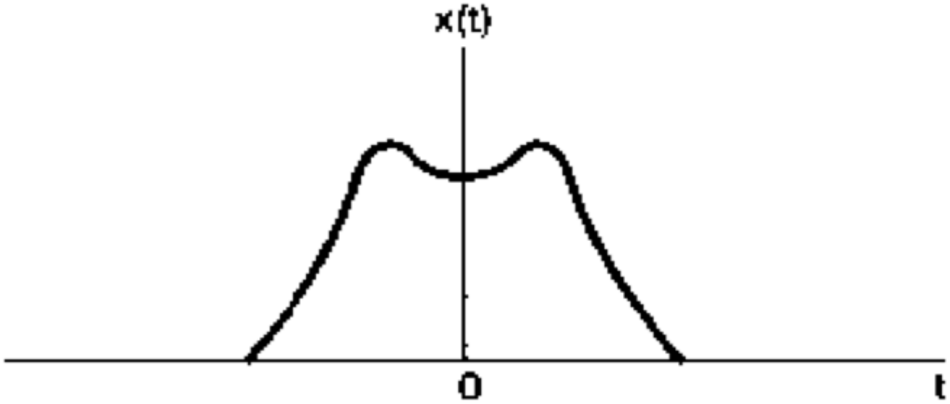
Even Signal

- A signal x(t) or x[n] is referred to as an even signal if it is identical to its time-reversed counterpart, i.e., with its reflection about the origin.
- In continuous time a signal is even if:

$$x(-t) = x(t)$$

- While a discrete time signal is even if:

$$x[-n] = x[n]$$



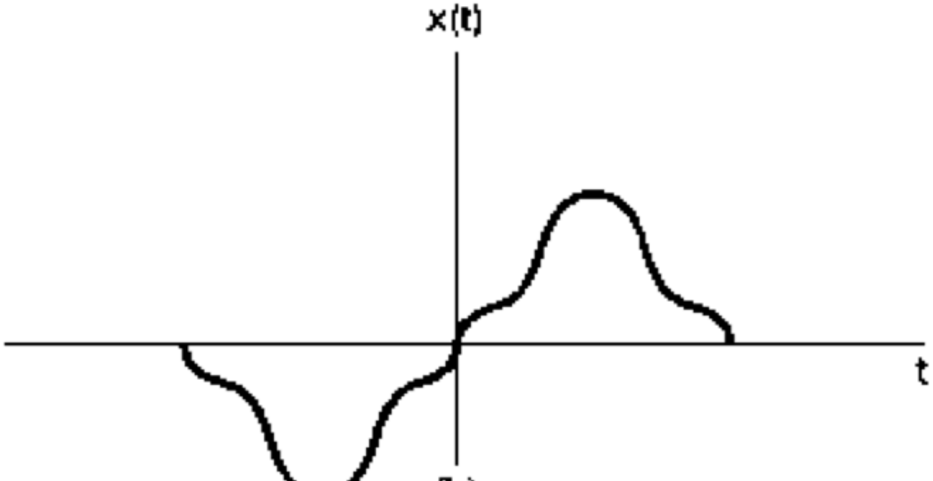
Odd Signal

- A signal is referred to as odd if:

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

- An odd signal must necessarily be 0 at t=0 or n=0, since above equations require that x(0) = -x(0) and x[0] = -x[0].



(b)

Even & Odd Signal

- The all-zero signal is both even and odd. Any other signal cannot be both even and odd, but may be neither.
- An important fact is that any signal can be broken into a sum of two signals, one of which is even and one of which is odd.
- Consider the signal : $Ev\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\}$
  - Which is referred to as the even part of x(t). Similarly the odd part of x(t) is given by:

$$Od\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

Thankyou

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