Lecture Notes
19th October 2016

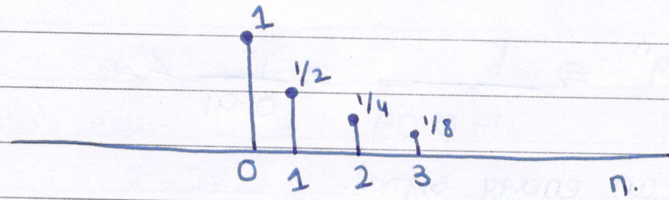
/Date 19-OCT-2016 / WEDNESDAY

PROBLEM #1:-

Sketch the following signals:-

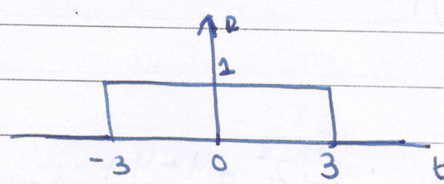
a) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + \left(\frac{1}{2}\right)^2 \delta[n-2] + \left(\frac{1}{2}\right)^3 \delta[n-3]$

SOL:-



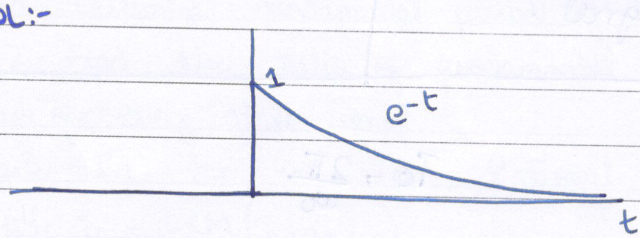
b) $x(t) = u(t+3) - u(t-3)$

SOL:-



c) $x(t) = e^{-t}\, u(t)$

SOL:-



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Energy & Power Signal:-

Example #1:-

a) $x[n] = (-0.3)^n u[n]$

Sol:-

$$E = \sum_{n=0}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{\infty} 0.09^n \Rightarrow \frac{1}{1-0.09} \Rightarrow \frac{1}{0.91} < \infty$$

Therefore, it is an energy signal.

b) $x[n] = 2u[n]$ (Pending)

Sol:-

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} 2^2 \qquad \therefore \sum_{n=0}^{\infty} 2^2 = 4+4+\cdots (N+1) \text{ times}$$

$$= 4(N+1)$$

$$= \lim_{N \to \infty} \frac{4\cancel{(2N+1)}}{\cancel{2N+1}} \Rightarrow 4 < \infty \qquad = \lim_{N \to \infty} \frac{1}{2N+1} 4(N+1)$$

$x(n)$ is a power signal.

c) $y(t) = e^{-bt} u(t)$

Sol:-

$$T_0 = \frac{2\pi}{\omega_0}$$

$$E = \int_{-\infty}^{\infty} [y(t)]^2 dt$$

$$= \int_0^{\infty} e^{-2bt} dt \Rightarrow \cancel{\frac{1}{b}}$$

Applying integral substitution $\int f(g(x)) \cdot g'(x) dx = \int f(u) du$

$u = g(x)$

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c) Continued:-

$$u = -2bt \qquad du = -2b\,dt$$

$$dt = -\frac{1}{2b}\,du$$

$$= \int_0^{\infty} e^u \left(-\frac{1}{2b}\right) du$$

taking constant out

$$= -\frac{1}{2b}\int_0^{\infty} e^u\,du \qquad \therefore \int e^u\,du = e^u$$

$$= -\frac{1}{2b}\left[e^u\right]_0^{\infty}$$

$$u = -2bt$$

substituting

$$E = -\frac{e^{-2bt}}{2b}\Big|_0^{\infty}$$

$$= \cancel{-\frac{e^{-2b\infty}}{2b}} + \cancel{\frac{e^0}{2b}} = -\frac{e^{-2b\infty}}{2b} + \frac{e^0}{2b}$$

$$E = \frac{-e^{-\infty}}{2b} + \frac{1}{2b} \Rightarrow \frac{1}{2b} < \infty$$

This is an energy signal.

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d) $y(t) = A\sin(w_0 t + \theta)$ $\qquad \therefore T_0 = \frac{2\pi}{w_0}$

$\cancel{P_{av} = \frac{1}{2T_0} \int_0}$

$\cancel{P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 \sin^2(w_0 t + \theta)\,dt}$

$\cancel{= \lim_{T\to\infty}}$

$$P_{av} = \frac{1}{T_0}\int_0^{T_0} \left[A\sin(w_0 t + \theta)\right]^2 dt$$

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$$= \frac{\omega_0}{2\pi}\int_0^{T_0} A^2 \sin^2(\omega_0 t+\theta)$$

$$= \frac{A^2\omega_0}{2\pi}\int_0^{T_0} \sin^2(\omega_0 t+\theta)\, dt$$

$$\sin(A) = \pm\sqrt{(1-\cos(2A)/2)}$$

$$= \frac{A^2\omega_0}{2\pi}\int_0^{T_0} \frac{1}{2}\left[1-\cos 2(\omega_0 t+\theta)\right] dt$$

$$= \frac{A^2\omega_0}{2\pi}\left[\frac{1}{2}\int_0^{T_0} 1\, dt - \frac{1}{2}\int_0^{T_0} \cos 2(\omega_0 t+\theta)\, dt\right]$$

$$= \frac{A^2\omega_0}{2\pi}\left[\frac{1}{2}\left[t\right]_0^{T_0}\right] - \frac{A^2\omega_0}{4\pi}\underbrace{\int_0^{T_0} \cos 2(\omega_0 t+\theta)\, dt}_{a} \rightarrow ①$$

$$a \Rightarrow \int_0^{T_0} \cos 2(\omega_0 t+\theta)\, dt \rightarrow ②$$

let $\omega_0 t+\theta = \tau$

$d\tau = \omega dt$ put in ② $\Rightarrow \frac{d\tau}{\omega} = dt$

$$= \frac{1}{\omega}\int_0^{T_0} \cos(2\tau)\, d\tau$$

$$= \frac{1}{\omega}\left[\sin 2\tau\right]_0^{T_0}$$ put back in equ ①

$$= \cancel{\frac{1}{\omega}}$$

$$P = \frac{A^2\omega_0}{2\pi}\left[\frac{1}{2}\left[t\right]_0^{T_0}\right] - \frac{A^2\cancel{\omega_0}}{4\pi\cancel{\omega}}\left[\sin 2\tau\right]_0^{T_0}$$

$$= \frac{A^2\omega_0}{4\pi}\left[-0+\frac{2\pi}{\omega_0}\right] - \frac{A^2}{4\pi}\left[-\sin(0)+\sin\frac{4\pi}{\omega_0}\right]$$

$$= \frac{A^2\cancel{\omega_0}}{\cancel{4\pi}}\left[-\frac{\cancel{2\pi}}{\cancel{\omega_0}}\right] - \frac{A^2}{4\pi}(0) + \frac{A^2}{4\pi}(0) \Rightarrow +\frac{A^2}{2} < \infty$$

Hence it is a power signal.

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Periodic / A-periodic Signals:-

$T_0 = ?$

$\Rightarrow$ $x(t) = A_0 e^{j\omega_0 t}$

$x(t+T_0) = A_0 e^{j\omega_0 (t+T_0)}$

$\cancel{A_0} e^{j\omega_0 t} = \cancel{A_0} e^{j\omega_0 (t+T_0)}$

$\cancel{e^{j\omega_0 t}} = \cancel{e^{j\omega_0 t}} \cdot e^{j\omega_0 T_0}$

$e^{j\omega_0 T_0} = 1$

Euler's equ:- $e^{jx} = \cos x + j\sin x$

$x = 2\pi k$

$e^{j2\pi k} = \underbrace{\cos 2\pi k}_{1} + j\underbrace{\sin 2\pi k}_{0}$

$e^{j2\pi k} = 1$

so, $e^{j\omega_0 T_0} = e^{j2\pi k}$

$\omega_0 T_0 = 2\pi k$

$T_0 = \dfrac{2\pi k}{\omega_0}$ $(\because k=1)$

$T_0 = \dfrac{2\pi}{\omega_0}$

$\Rightarrow$ $x(t) = x_1(t) + x_2(t)$. [composite signal as composed of 2 signals]

Step1: Determine fundamental period of individual signals

Step2: Find the ratio of fundamental period of 1st signal to fundamental period of every other signal.

Step3: If the ratio's are Rational, the composite signal is periodic.

Step4: $T_0 = LCM(T_1, T_2 \ldots)$

- $x(t) = x_1(t) \times x_2(t)$

$\rightarrow$ If any one signal is aperiodic $x(t)$ is aperiodic.

$\rightarrow$ The resultant signal may or may not be periodic.

$x_1(t) \rightarrow T_1$, $x_2(t) \rightarrow T_2$

$\dfrac{T_1}{T_2}$ If we have 3rd signal then also $\dfrac{T_1}{T_3}$

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→ step 4 only when ratio's are rational.

$$T_0 = HCF\ (T_1, T_2 \ldots\ldots)$$

→ $x(t) = \sin 4\pi t + \cos 2t$.

$x_1(t) = \sin 4\pi t$

$x_2(t) = \cos 2t$

Step 1:-

$T_1 = \frac{2\pi}{\omega_1} \Rightarrow \omega_1 = 4\pi$ $\qquad$ $T_2 = \frac{2\pi}{\omega_2} \Rightarrow \omega_2 = 2$

$T_1 = \frac{\cancel{2}\cancel{\pi}}{\cancel{2}4\cancel{\pi}} \Rightarrow \frac{1}{2}$ $\qquad$ $T_2 = \frac{2\pi}{\cancel{2}} \Rightarrow \pi$

Step 2:-

$$\frac{T_1}{T_2} \Rightarrow \frac{\frac{1}{2}}{\pi} \Rightarrow \frac{1}{2\pi}$$

Step 3:

$\frac{1}{2\pi} \Rightarrow$ Irrational so no Step 4.

$x(t) =$ Non periodic.

→ $x(t) = 5\cos\pi t \sin 3\pi t$

Step 1:-

$T_1 = \frac{2\pi}{\omega_1} \Rightarrow \omega_1 = \pi$ $\qquad$ $T_2 = \frac{2\pi}{\omega_2} \Rightarrow \omega_2 = 3\pi$

$T_1 = \frac{2\cancel{\pi}}{\cancel{\pi}} \Rightarrow 2$ $\qquad$ $T_2 = \frac{2\cancel{\pi}}{3\cancel{\pi}} \Rightarrow \frac{2}{3}$

Step 2:-

$$\frac{T_1}{T_2} = \frac{2}{2/3} = \frac{\cancel{2}}{\cancel{2}} \times 3 \Rightarrow 3$$

Step 3:- 3 is rational. so, $x(t)$ is periodic

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Step 4:-

$$T_o = LCM\left(2, \frac{2}{3}\right)$$

$$LCM \text{ of } \cancel{AVM} \text{ frac} = \frac{LCM \text{ of numerators}}{HCF \text{ of denominators}}$$

$$T_o = \frac{LCM(2,2)}{HCF(1,3)}$$

$$= \frac{2}{1} \Rightarrow 2 \text{ sec}$$

- irrational numbers

decimal form are neither terminating nor repeating decimals

$\sqrt{3}$, π irrational

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General Complex Exponential:-

→ The dashed lines in the figure correspond to the functions $\pm|C|e$

→ $|C|e^{\gamma t}$ is the magnitude of the complex exponential.

→ Thus the dashed curves act as an envelope for the oscillat curve. The peaks of the oscillations just reach these curves and in this way the envelope provides us with a convenient way to visualize the general trend in the amplitude of the oscillations.

Discrete-time Real Exponential Signals:-

→ If α is positive all the values of $C\alpha^n$ are of the same sign.

→ If α is negative then the sign of $x[n]$ alternates.

→ If $\alpha = 1$ then $x[n]$ is constant.

→ If $\alpha = -1$ $x(n)$ alternates in value b/w $-C$ & $-C$.