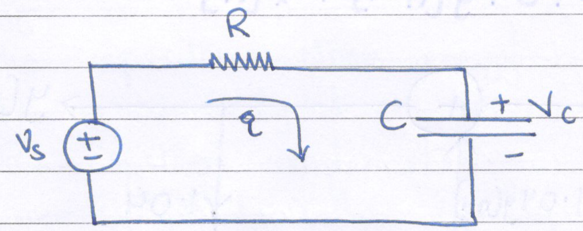
Lecture Notes
24th October 2016

Day/Date MONDAY / 24-OCT-16 LECTURE#3 + MAKEUP

EXAMPLE #1 :-



$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

$$i(t) = C\frac{dv_c}{dt}$$

$$\frac{v_s(t) - v_c(t)}{R} = C\frac{dv_c}{dt}$$

$$\frac{v_s(t) - v_c(t)}{RC} = \frac{dv_c}{dt}$$

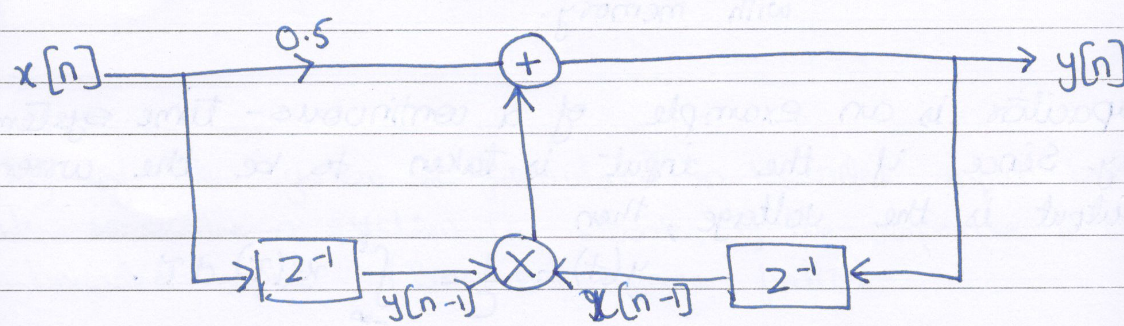
$$\frac{v_s(t)}{RC} - \frac{v_c(t)}{RC} = \frac{dv_c}{dt}$$

$$\frac{v_s(t)}{RC} = \frac{dv_c}{dt} + \frac{v_c(t)}{RC}.$$

REPRESENTATION OF DISCRETE TIME SYSTEM:-

EXAMPLE:-

★ $y[n] = y[n-1]\,x[n-1] + 0.5\,x[n]$



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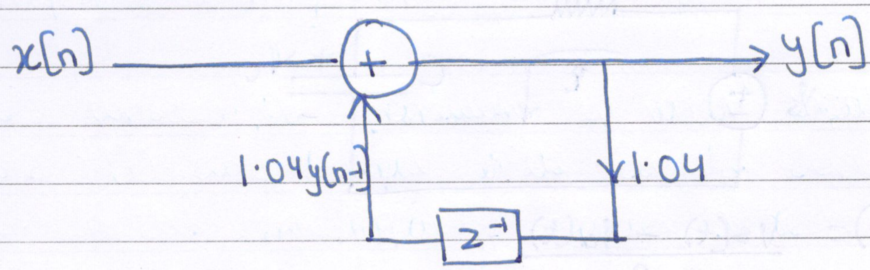
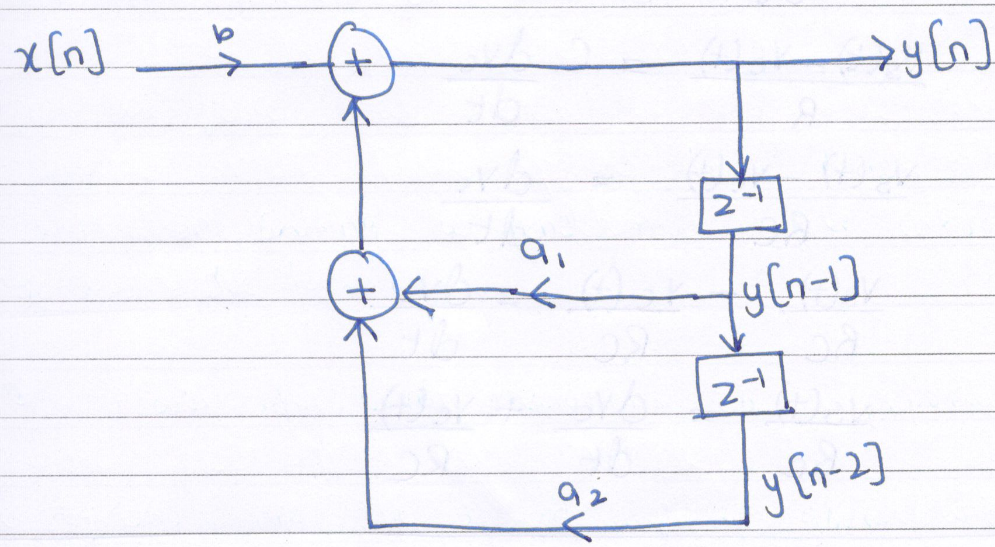
EXAMPLE :-

$$y[n] = 1.04\,y[n-1] + x[n]$$



EXAMPLE :-

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b x[n]$$



Memoryless System:-

→ $y[n] = \sum_{k=-\infty}^{\infty} x[k]$ is an accumulator or summer which is with memory.

→ A capacitor is an example of a continuous-time system with memory. Since if the input is taken to be the current and the output is the voltage, then

$$y(t) = \frac{1}{C} \int_{-\infty}^{t} x(\tau)\, d\tau .$$

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→ In many physical systems, memory is directedly associated with the storage of energy.

→ For example, the capacitor stores energy by accumulating electrical charge, represented as an integral of current.

→ In discrete-time systems implemented with computers or digital microprocessors, memory is typically directly associated with storage registers that retain values b/w clock pulses.

CAUSALITY:-

→ The motion of an automobile is causal, since it does not anticipate future actions of the driver.

EXAMPLE:-

$$y[n] = x[-n]$$

→ output $y[n_0]$ at a positive time n_0 depends only on the value of the input signal $x[-n_0]$ at time $[-n_0]$ which is negative and therefore in the past of n_0.

→ We may conclude at this point that the given system is causal.

→ However, we should always be careful to check the input-output relation for all times.

→ e.g $n = -4$ then $y[-4] = x[4]$, so the output at this time depends on a future value of the input. Hence, the system is not causal.

STABILITY:-

→ Stability of the system is formulated in bounded input bounded output sense i-e a system is stable if its response is bounded for a bounded input. (Bounded means finite).

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→ An unstable system is one in which the output of the system is unbounded for a bounded input. The response of an unstable system diverges to infinity

→ For example, consider the pendulum in which input is the applied force $x(t)$ and output is the angular deviation $y(t)$ from the vertical. In this case gravity applies a restoring force that tends to return the pendulum to the vertical position and frictional losses due to drag tend to slow it down. Consequently, if a small force $x(t)$ is applied, the resulting deflection from vertical will also be small.
In contrast for the inverted pendulum the effect of gravity is to apply a force that tends to increase the deviation from vertical. Thus a small force applied force leads to a large vertical deflection causing the pendulum to topple over, despite any retarding forces due to friction.

→ Example :- Models for chain reactions or for population growth with unlimited food supplies and no predators are examples of unstable systems, since the system response grows without bound in response to small iputs.

EXAMPLE :-

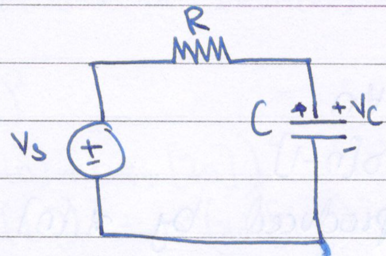
$$y(t) = t\,x(t)$$

$$x(t) = 1 \Rightarrow y(t) = t$$

which is unbounded as $|y(t)|$ will exceed that constant for some t. so above system is unstable.

TIME INVARIANCE:-

→ For Example:-



→ R & C are constant over tim

→ R & C fluctuate over time.

→ For Example:- Frictional coefficient b and mass m. of the automobile are constant. Or trunk is full changing the mass m.

EXAMPLE # 6:-

1) $y(t) = \sin|x(t)|$.

SOL:-

To check that this system is time invariant, we must determine whether the time invariance property holds for any input and any time shift t_0. Thus let $x_1(t)$ be an arbitrary input to this system and let:

$$y_1(t) = \sin|x_1(t)| \rightarrow (1)$$

be the corresponding output.

Then consider a second iput obtained by shifting $x_1(t)$ in time.

$$x_2(t) = x_1(t-t_0). \rightarrow (2)$$

The output corresponding to this input is:-

$$y_2(t) = \sin|x_2(t)| = \sin|x_1(t-t_0)| \rightarrow (3)$$

Similarly,

$$y_1(t-t_0) = \sin|x_1(t-t_0)|. \rightarrow (4)$$

Comparing equations (3) & (4) we see that

$$y_2(t) = y_1(t-t_0)$$

thus the system is time invariant.

2) $y[n] = n\,x[n]$.

SOL:-

To show that the system is not time-invariant, we can constru a counter example.

Let $x[n] = \delta[n]$

then $y[n] = n\delta[n] = 0 \quad \forall n$

Now let $x_1[n] = x[n-1] = \delta[n-1]$

If $y_1[n]$ is the output produced by $x_1[n]$, it is easy to show that:-

$$y_1[n] = n x_1[n]$$
$$= n\delta[n-1] \Rightarrow \delta[n-1]$$

However,

$$y[n-1] = (n-1)\cancel{x}\delta[n-1] \Rightarrow (n-1)\,\delta[n-1] = 0 \text{ for all } n.$$

So, $y_1[n] \neq y[n-1]$.

LINEARITY:-

EXAMPLE #7:-

1) $y(t) = 2\pi x(t)$

SOL:-

Let's consider a signal.

$$x(t) = a x_1(t) + b x_2(t).$$

where $y_1(t) = 2\pi x_1(t)$ & $y_2(t) = 2\pi x_2(t)$.

then:

$$a y_1(t) + b y_2(t) = a(2\pi x_1(t)) + b(2\pi x_2(t))$$
$$= 2\pi[a x_1(t) + b x_2(t)]$$
$$= 2\pi x(t) \Rightarrow y(t)$$

$$\longleftrightarrow$$

2) $y[n] = (x[2n])^2$

SOL:-

Let's consider the signal.

$$x[n] = a x_1[n] + b x_2[n]$$

where $y_1[n] = (x_1[2n])^2$ & $y_2[n] = (x_2[2n])^2$

then $y[n] = a y_1[n] + b y_2[n]$.

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$$a y_1[n] + b y_2[n] = a\left(x_1[2n]\right)^2 + b\left(x_2[2n]\right)^2$$

However,

$$y[n] = \left(x[2n]\right)^2$$
$$= \left(a x_1[2n] + b x_2[2n]\right)^2$$
$$= a^2\left(x_1[2n]\right)^2 + b^2\left(x_2[2n]\right)^2 + 2ab\,x_1[n]\,x_2[n].$$

Non-linear.

3) $y[n] = 2x[n] + 3$

Sol:- let's say the system violates the additive property.

If $x_1[n] = 2$ and $x_2[n] = 3$ then

$x_1[n] \Rightarrow y_1[n] = 2x_1[n] + 3 = 2(2) + 3 \Rightarrow 7$

$x_2[n] \Rightarrow y_2[n] = 2x_2[n] + 3 = 2(3) + 3 \Rightarrow 9$

However, the response $x_3[n] = x_1[n] + x_2[n]$

$y_3[n] = 2[x_1[n] + x_2[n]] + 3 = 13.$

which does not equal $y_1[n] + y_2[n] = 16.$

Alternatively since $y[n] = 3$ if $x[n] = 0$, we see that the system violates the "zero in / zero out" property of linear systems.