Signal & Systems

Continuous & Discrete Systems

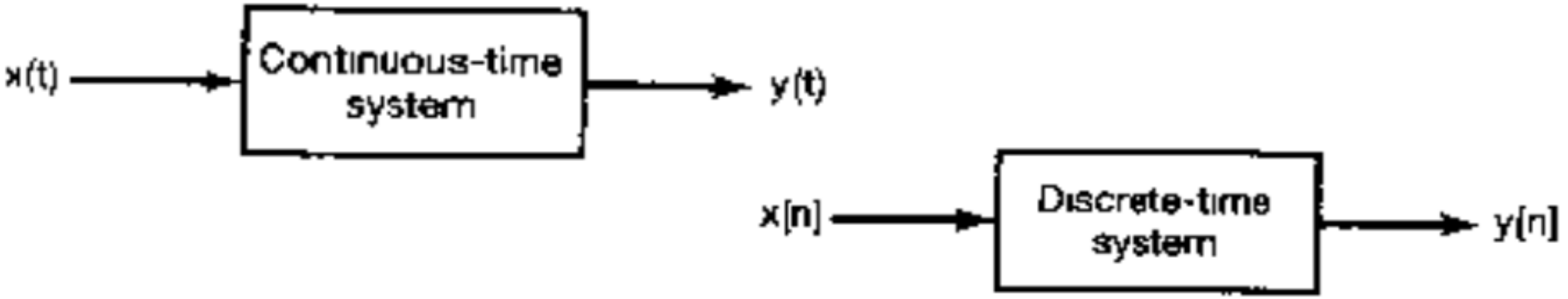
24TH October 16

Fundamentals of Systems

24TH October 16

Systems

- A system in the broadcast sense are an interconnection of components, devices or subsystems.
- A system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way resulting in other signals as output.
- A continuous time system is a system in which continuous time input signals are applied and result in continuous time output signals. The input-output relation is represented by the following notation: $x(t) \rightarrow y(t)$.

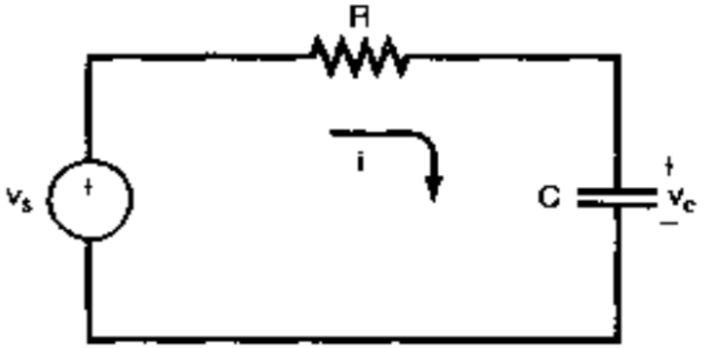


Systems (cont.)

- Similarly a discrete time system is a system that transforms discrete time inputs into discrete time outputs and represented symbolically as: $x[n] \rightarrow y[n]$.

Example #1

❖ Consider the RC circuit depicted below:



❖ If we regard $v_s(t)$ as the input signal and $v_c(t)$ as the output signal, then we can use simple circuit analysis to derive an equation describing the relationship between the input and output.

❖ Specifically from Ohm's law the current i(t) through the resistor is proportional (with proportionality constant 1/R) to the voltage drop across the resistor; i.e.,

Example #1 (cont.)

$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

- Similarly, using the defining constitutive relation for a capacitor, we can relate i(t) to the rate of change with tome of the voltage across the capacitor:

$$i(t) = C\frac{dv_c}{dt}$$

- Equating the right hand side of above two equations, we obtain a differential equation describing the relationship between the input $v_s(t)$ and the output $v_c(t)$:

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

Interconnection of Systems

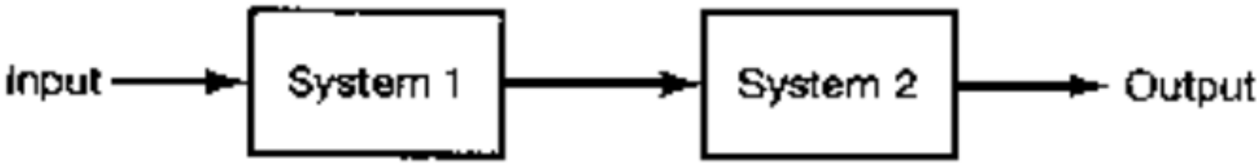
24TH October 16

Interconnection

- Many real systems are built as interconnection of several subsystems.
- For example an audio system which involves the interconnection of a radio receiver, compact disc player or tapo deck with an amplifier and one or more speakers.
- By viewing a system as an interconnection of its components we can use our understanding of the component systems and of how they are interconnected in order to analyse the operation and behaviour of the overall system.

Series Interconnection

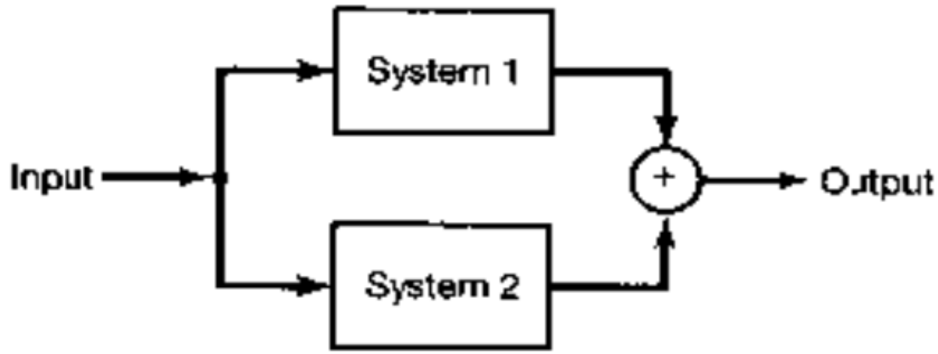
- A series or cascade interconnection of two systems shown below is referred to as a block diagram.



- An example of a series interconnection is a radio receiver followed by an amplifier.
- Similarly one can define a series interconnection of three or more systems.

Parallel Interconnection

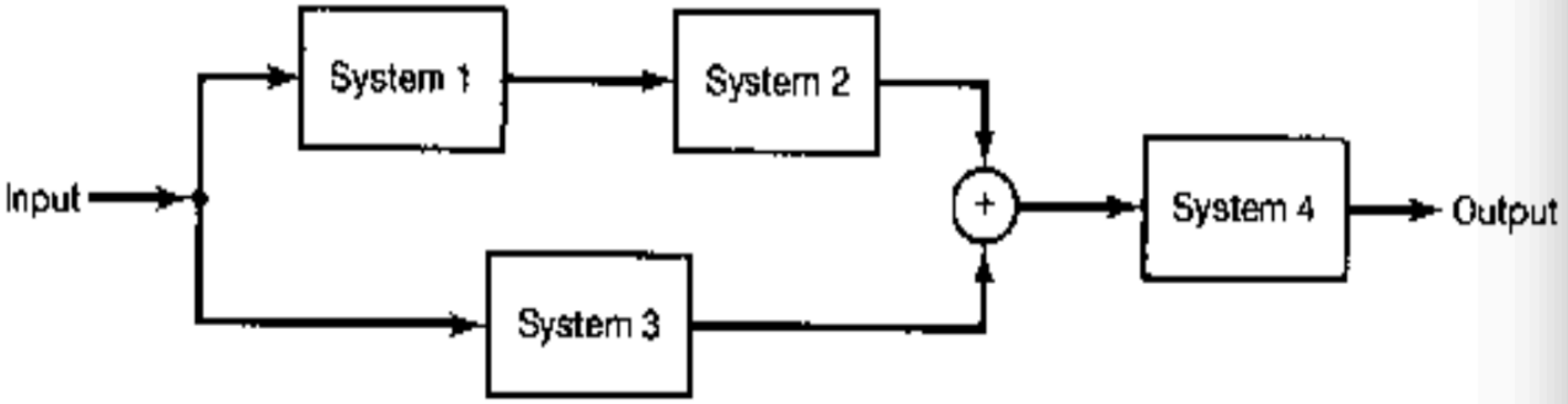
- A parallel interconnection of two systems is shown below:



- Here the same input signal is applied to Systems 1 and 2.
- The symbol "$\oplus$" denotes addition, so that the output of the parallel interconnection is the cum of the outputs of systems 1 and 2.
- We can define parallel interconnections of more than two systems, and we can combine both cascade and parallel interconnections to obtain more complicated interconnections.

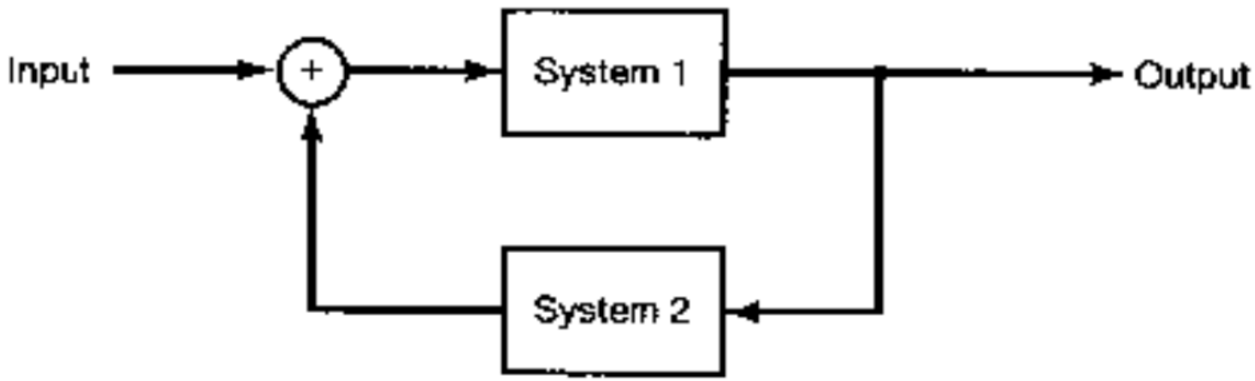
Parallel Interconnection (cont.)

- An example of such an interconnection is given below:



Feedback Interconnection

- Feedback interconnection is shown below:



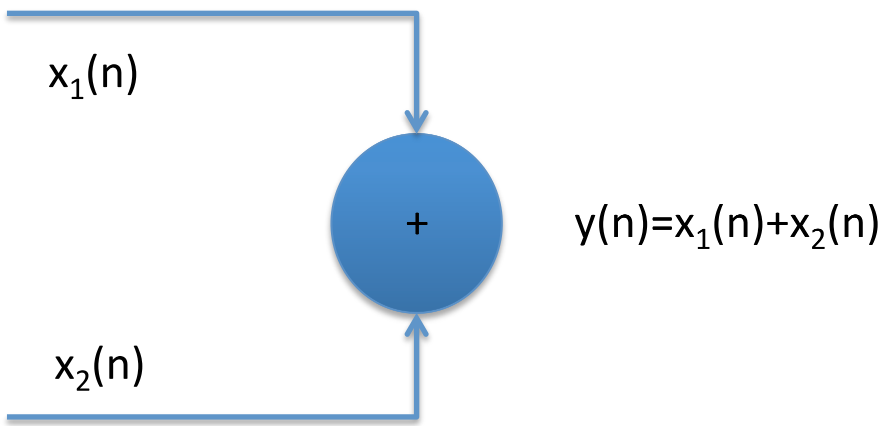
- Here the output of system 1 is the input to system 2 while the output of system 2 is fed back and added to the external input to produce the actual input to system 1.
- Feedback systems arise in a wide variety of applications.
- For example a cruise control system on an automobile senses the vehicle's velocity and adjusts the fuel flow in order to keep the speed at the desired level.

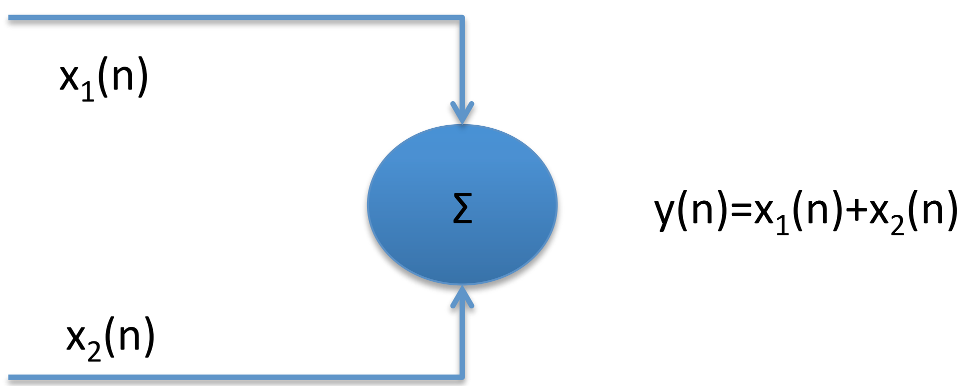
Representation of Discrete Time System

- The fundamental building blocks are :
  - Adders
  - Multipliers
  - Delay
  - Advance

Adders

- It performs the addition of two sequences and generates an output which is the sum of input sequences.



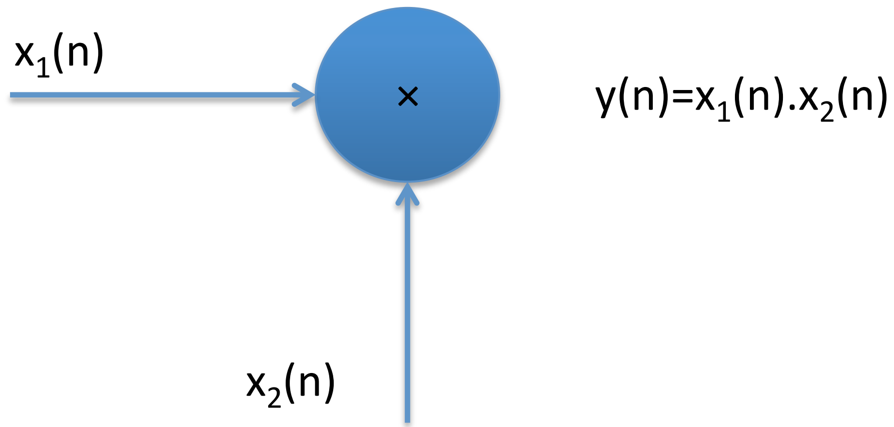


Constant Multiplier

- This is used for scaling operation and it does not need any memory storage.

Signal Multiplier

- It is used to indicate the multiplication of two sequences $x_1(n)$ and $x_2(n)$ to produce the output y(n).
- Any memory or storage is not required for this operation.

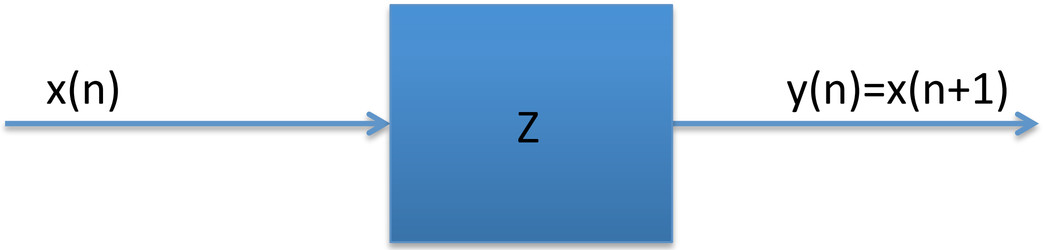


Unit Delay Block

- It delays the input signal by one sample.

Unit Advance Block

- It advances the input signal by one sample.



System Properties

24TH October 16

System With & Without Memory

- A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent only on the input at that same time.
- For example, the system specified by the relationship below is memoryless as the value of y[n] at any particular time n_0 depends only on the value of x[n] at that time.

$$y[n] = \left(2x[n] - x^2[n]\right)^2$$

- The concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time.

Examples of Memoryless Systems

- $y(t) = (2x(t) - x^2(t))^2$ is memoryless, because y(t) depends on x(t) only. There is no x(t-1) or x(t+1) terms, for example.
- y[n] = x[n] is memoryless. In fact this syetm is passing the input to output directly, without any processing.
- y[n] = x[n] + y[n-1] is not memoryless. To see this we consider:

$$y[n-1] = x[n-1] + y[n-2]$$

  - Substituting into:

$$y[n] = x[n] + y[n-1] \quad \textit{yeilds}$$

$$y[n] = x[n] + \left(x[n-1] + y[n-2]\right)$$

- By repeating the calculation, we have

$$y[n] = x[n] + x[n-1] + x[n-2] + \ldots$$

$$= \sum_{k=-\infty}^{\infty} x[k]$$

- Clearly, y[n] depends on more than just x[n].

Invertibility & Inverse Systems

- A system is said to be invertible if distinct inputs lead to distinct outputs.
- In other words, a system is invertible if there exists an one-to0one mapping from the set of input signals to the set of output signals.
- For example in systems for encoding used in a wide variety of communications applications. In such a system a signal that we wish to transmit is first applied as the input to a system known as an encoder.
- There are many reasons to do this, ranging from the desire to encrypt the original message for secure or private communication to the objective of providing some redundancy in the signal.
- So that if any error occur in transmission can be detected and possibly corrected. For lossless coding the input to the encoder must be exactly recoverable from the output, i.e., the encoder must be invertible.

Invertibility & Inverse Systems (cont.)

- Two rules of showing an invertible system are as follows:
  - To show that a system is invertible, one has to show the inversion formula.
  - To show that a system is not invertible, one has to give a counter example.

Example #2

- The system y(t) = (cos (t) +2) x(t) is invertible.
- Proof:
- To show that the system is invertible, we need to find an inversion formula.
- This is easy: y(t) = (cos (t) +2) x(t) by rearranging terms implies that:

$$x(t) = \frac{y(t)}{\cos(t) + 2}$$

- Which is the inversion formula. Note that the denominator is always positive, thus the division is valid.

Example #3

- The system $y(t) = x^2(t)$ is not invertible.
- Proof:
- To show that a system is not invertible, we construct a counter example. Let us consider two signals:

$$x_1(t) = 1, \quad \forall t$$

$$x_2(t) = -1, \quad \forall t$$

- Clearly,

$$x_1(t) \neq x_2(t)$$

$$\left(x_1(t)\right)^2 = \left(x_2(t)\right)^2$$

- Therefore, we have found a counter example such that different inputs give the same output. Hence the system is not invertible.

Causality

- A system is causal if the output at any time depends only on values of the input at the present time and in the past.
- Such a system is often referred to as being non-anticipative, as the system output does not anticipate future values of the input.
- Examples:
- y[n] = x[n-1] is causal, because y[n] depends on the past sample x[n-1].
- y[n] = x[n] +x[n+1] is not causal, because x[n+1] is a future sample.

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

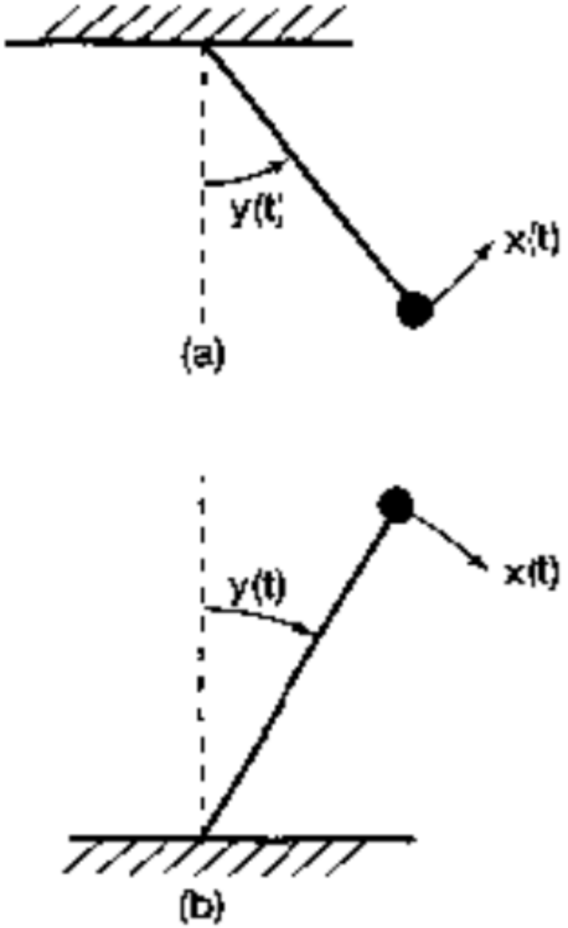
- Is causal, because the integral evaluates τ from -∞ to t which are all in the past.
- Note: All memoryless systems are causal, since the output responds only to the current value of the input.

Stability

- It is another important system property.
- Informally a stable system is one in which small inputs lead to responses that do not diverge.
- To describe a stable system, we first need to define the boundedness of a signal.
- Definition: A signal x(t) (and x[n]) is bounded if there exists a constant $B < \infty$ such that $|x(t)| < B$ for all t.
- Definition: A system is stable if a bounded input, always produces a bounded output signal. That is, if $|x(t)| \leq B$ for some $B < \infty$, then $|y(t)| < \infty$.

Stability Example

- (a): A stable pendulum
- (b): An unstable pendulum



Example #4

- The system $y(t) = 2x^2(t-1) + x(3t)$ is stable.
- Proof:
- To show the system is stable, let us consider a bounded signal x(t), that is $|x(t)| \leq B$ for some $B < \infty$. Then:

$$|y(t)| = |2x^2(t-1) + x(3t)|$$

$$\leq |2x^2(t-1)| + |x(3t)|, \quad \textit{by Triangle Inequality}$$

$$\leq 2|x^2(t-1)| + |x(3t)|$$

$$\leq 2B^2 + B < \infty$$

- Therefore, for any bounded input x(t), the output y(t) is always bounded. Hence the system is stable.

Example #5

- The system $y[n] = \sum_{k=-\infty}^{\infty} x[k]$ is not stable.
- Proof:
- To show that the above system is not stable, we can construct a bounded input signal x[n] and show that the output signal y[n] is not bounded.
- Let x[n] = u[n]. It is clear that |x[n]| ≤ 1 (i.e., bounded). Consequently,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]$$

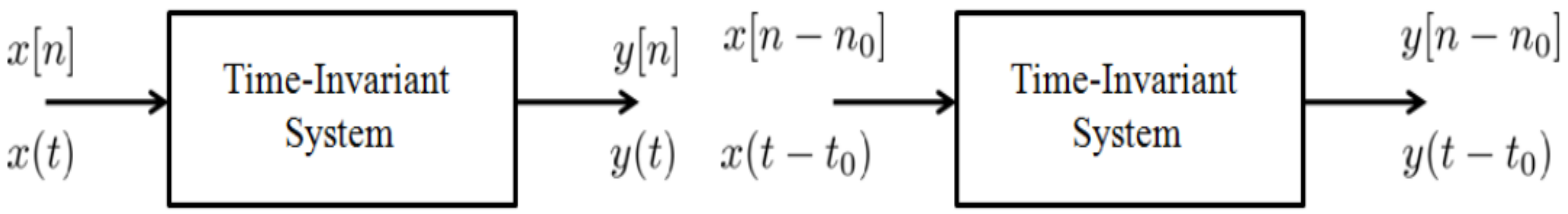
$$|y[n]| = \left| \sum_{k=-\infty}^{n} u[k] \right|$$

$$= \sum_{k=0}^{n} u[k] \le \sum_{k=0}^{n} 1 = n+1$$

- Which approaches ∞ as n→∞. Therefore, |y[n]| is not bounded.

Time Invariance

- A system is time invariant if the behaviour and characteristics of the system are fixed over time.
- A system is time-invariant if a time-shift of the input signal results in the same time-shift of the output signal. That is if: x (t) ➔ y(t), then the system is time invariant if: x $(t-t_0)$ ➔ $y(t-t_0)$ for any t_0 belonging to R.
- Illustration of a time-invariant system:



Example #6

- The system y(t) = sin [x(t)] is time invariant.
- The system y[n] = nx[n] is not time-invariant.

Linearity

- A linear system in continuous time or discrete time is a system that possess the important property of superposition.
- A system is linear if it is additive and scalable. That is:

$$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$$

$$for \quad all \quad a, b \in C$$

$$a x_1[n] + b x_2[n] \rightarrow a y_1[n] + b y_2[n]$$

Example #7

- The system $y(t) = 2\pi\, x(t)$ is linear.
- The system $y[n] = (x[2n])^2$ is not linear.
- The system $y[n] = 2x[n] + 3$ is not linear.

Thankyou

24TH October 16