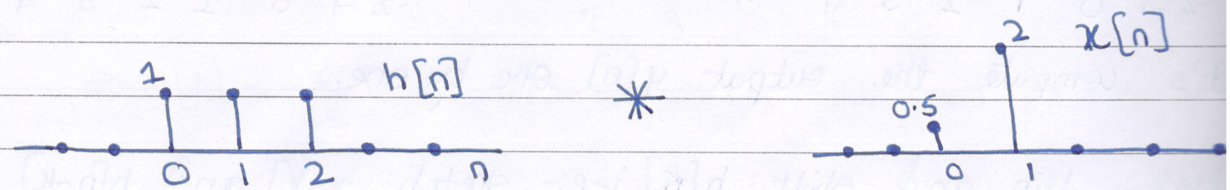
Lecture Notes
26th October 2016

Day/Date WEDNESDAY / 26th OCT, 2016 LECTURE # 4 + MAKEUP

CONVOLUTION:-

CONVOLUTION SUM:-

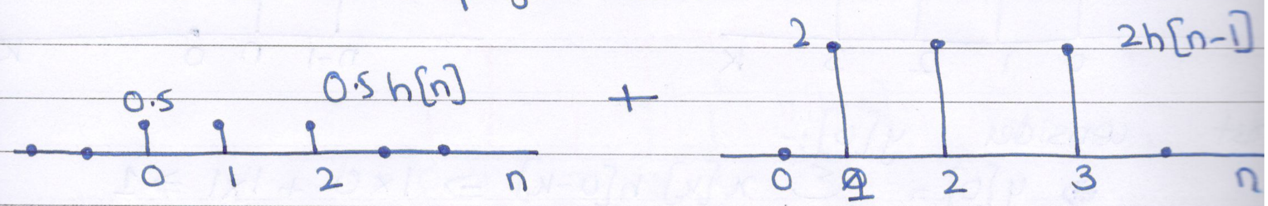
→ EXAMPLE # 1:-



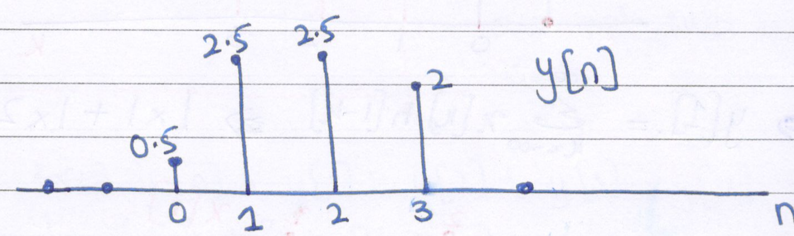
$$y[n] = 0.5\,h[n] + 2\,h[n-1].$$

→ The sequences $0.5\,h[n]$ and $2h[n-1]$ are the two echoes of the impulse response needed for the superposition involved in generating $y[n]$.

→ These echoes are displayed below.



→ By summing the two echoes for each value of n, we obtain $y[n]$, which is shown below:-



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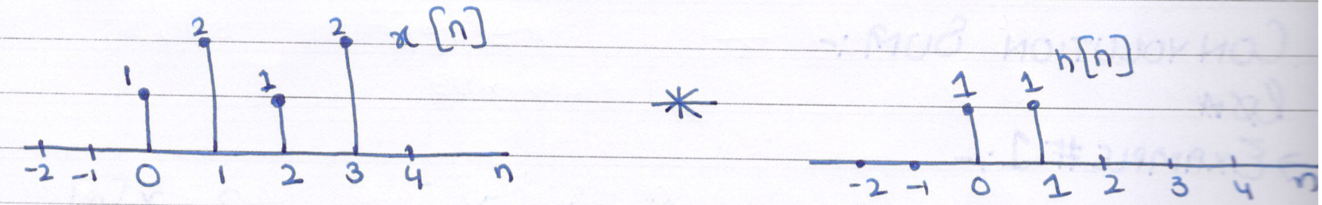
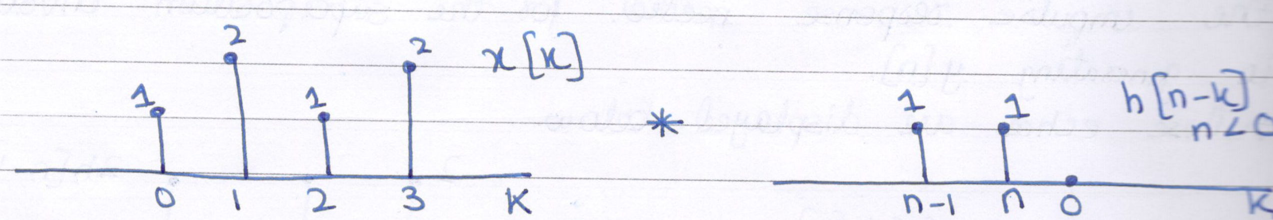
⇒ EXAMPLE #2:-



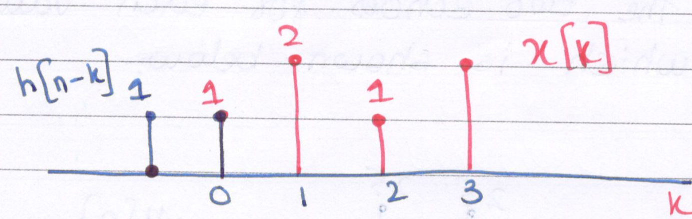
⇒ let's compute the output $y[n]$ one by one.

⇒ First flip and shift $h[k]$, i-e:- sketch $x[k]$ and $h[n-k]$ for any particular value of n, we multiply the two signals and sum over all values of k.

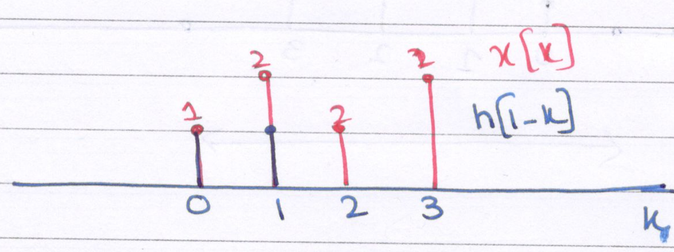


→ First consider $y[0]$:-

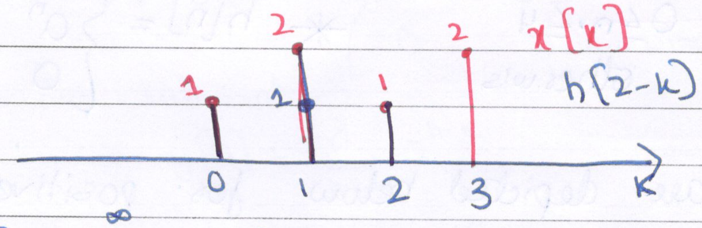
$$\Rightarrow y[0] = \sum_{k=-\infty} x[k]\, h[0-k] \Rightarrow 1\times 0 + 1\times 1 \Rightarrow 1$$



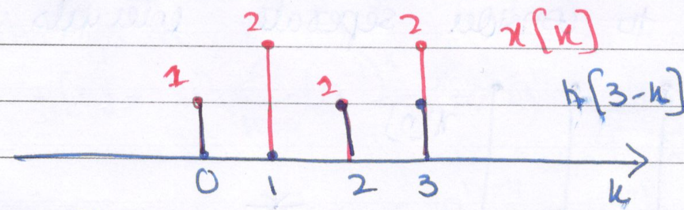
$$\Rightarrow y[1] = \sum_{k=-\infty}^{\infty} x[k]\, h[1-k] \Rightarrow 1\times 1 + 1\times 2 \Rightarrow 3$$



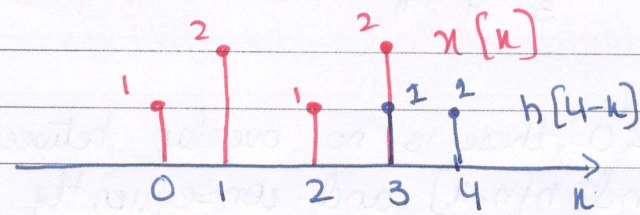
$$\Rightarrow y[2] = \sum_{k=-\infty}^{\infty} x[k]\, h[2-k] \Rightarrow 0\times1 + 1\times2 + 1\times1 \Rightarrow 3$$



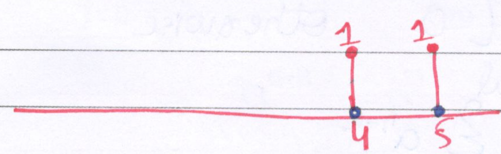
$$\Rightarrow y[3] = \sum_{k=-\infty}^{\infty} x[k]\, h[3-k] \Rightarrow 0\times1 + 0\times2 + 1\times1 + 2\times1 \Rightarrow 3$$



$$\Rightarrow y[4] = \sum_{k=-\infty}^{\infty} x[k]\, h[4-k] \Rightarrow 1\times2 + 1\times0 \Rightarrow 2$$



$\Rightarrow$ $n > 4$ then result is 0.



here $x[k]$ and $h[n-k]$ are not over lapping so, res is 0.

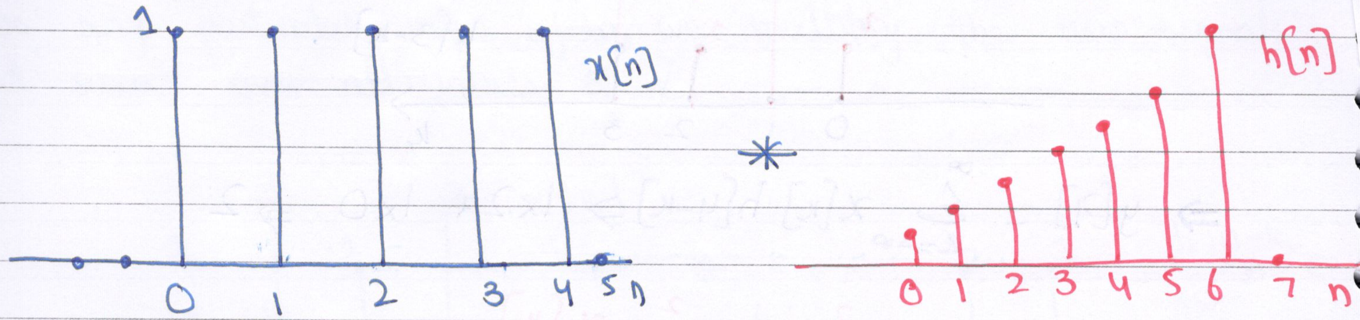
$$y[n] = \cancel{443}\; y[0] + y[1] + y[2] + y[3] + y[4]$$
$$= 1 + 3 + 3 + 3 + 2 \Rightarrow 12$$

→ EXAMPLE #3:-

$$x[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwis} \end{cases} \quad * \quad h[n] = \begin{cases} a^n & 0 \le n \le 6 \\ 0 & \text{otherwise.} \end{cases}$$

⇒ These signals are depicted below for positive value of $a > 1$.

⇒ In order to calculate the convolution of the two signals, it is convenient to consider seperate intervals for n.



Interval 1:- For $n < 0$, there is no overlap between the non-zero portions of $x[k]$ and $h[n-k]$ and consequently $y[n] = 0$.

Interval 2:- For $0 \le n \le 4$

$$x[k]\,h[n-k] = \begin{cases} a^{n-k} & 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}$$

Thus in this interval

$$y[n] = \sum_{k=0}^{n} a^{n-k}$$

We can evaluate this sum using the finite sum formula. Specifically changing the variable of summation in above equ. from k to $r = n-k$ we obtain

$$y[n] = \sum_{r=0}^{n} a^{r} \Rightarrow \frac{1-a^{n+1}}{1-a}$$

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Interval 3:- For $n>4$ but $n-6 \le 0$ (i.e $4<n\le 6$)

$$x[k]h[n-k] = \begin{cases} a^{n-k} & 0\le k\le 4 \\ 0 & \text{otherwise} \end{cases}$$

Thus in this interval:-

$$y[n] = \sum_{k=0}^{4} a^{n-k}$$

Once again we can use the geometric sum formula to evaluate above equation. Specifically factoring out the constant a^n from the summation yields,

$$y[n] = a^n \sum_{k=0}^{4} (a^{-1})^k \Rightarrow a^n \frac{1-(a^{-1})^5}{1-a^{-1}} \Rightarrow$$

$$= \frac{a^{n-4} - a^{n+1}}{1-a}.$$

Interval 4:- For $n>6$ but $n-6\le 4$ (i.e for $6<n\le 10$)

$$x[k]h[n-k] = \begin{cases} a^{n-4} & (n-6)\le k\le 4 \\ 0 & \text{other wise} \end{cases}$$

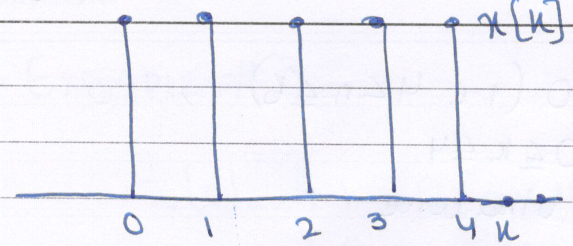
so $$y[n] = \sum_{k=n-6}^{4} a^{n-k}$$

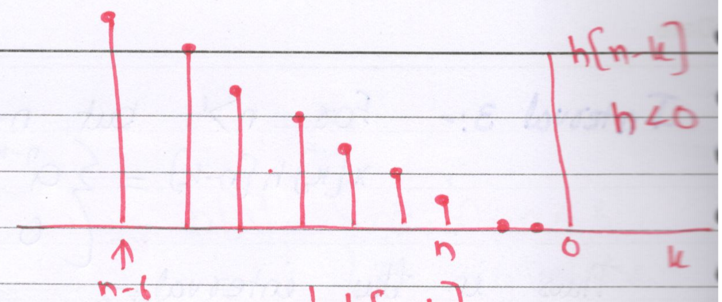
again using summation formula - letting $r = K-n+6$ we obtain:-

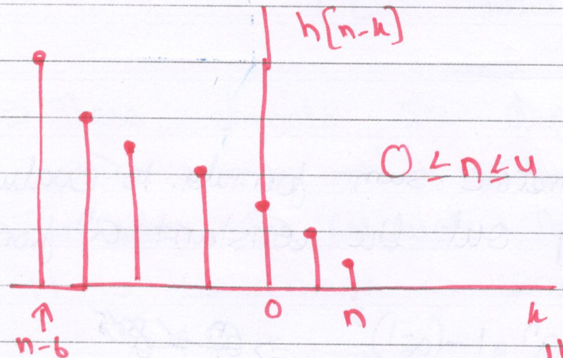
$$y[n] = \sum_{r=0}^{10-n} a^{6-r} = a^6 \sum_{r=0}^{10-n} (a^{-1})^r = a^6 \times \frac{1-a^{n-11}}{1-a^{-1}}$$

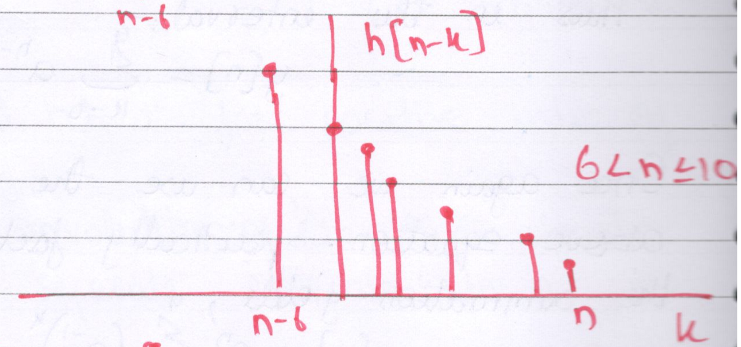
$$\Rightarrow \frac{a^{n-4} - a^7}{1-a}.$$

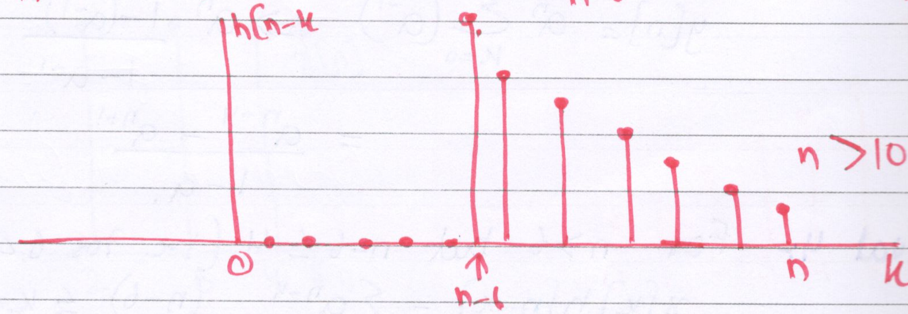
Interval 5:- For $n-6>4$ or equivalently $n>10$ there is no overlap between the non zero portions of $x[k]$ and $h[n-k]$ and hence, $y[n]=0$.





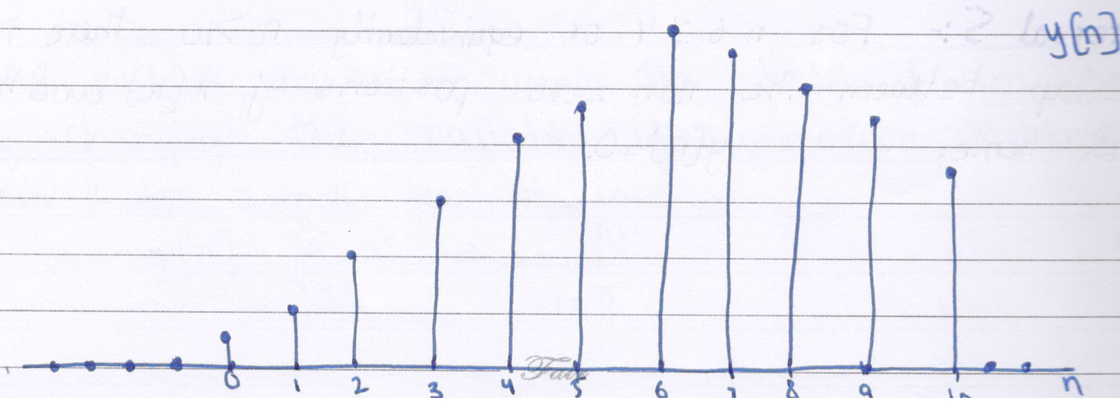






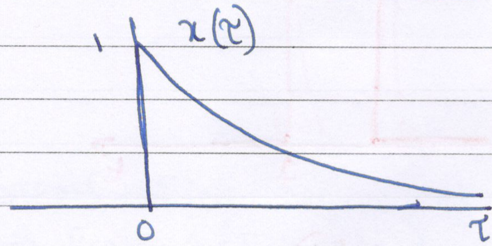
Summarizing then we obtain,

$$y[n] = \begin{cases} 0, & n < 0 \\ \dfrac{1-a^{n+1}}{1-a}, & 0 \leq n \leq 4 \\ \dfrac{a^{n-4} - a^{n+1}}{1-a}, & 4 \leq n \leq 6 \\ \dfrac{a^{n-4} - a^{7}}{1-a}, & 6 < n \leq 10 \\ 0, & 10 < n \Rightarrow n > 10 \end{cases}$$



=> EXAMPLE #4:-

Solved in slides.



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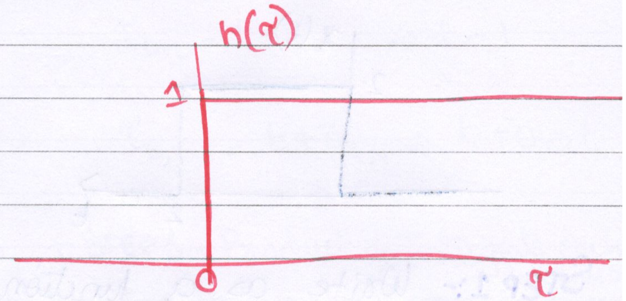

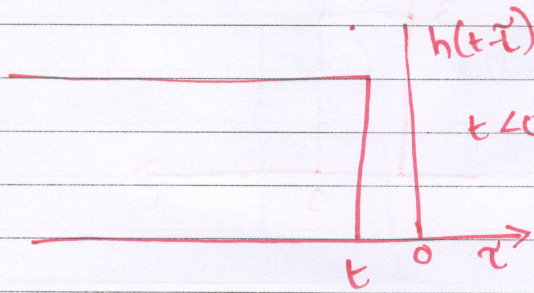

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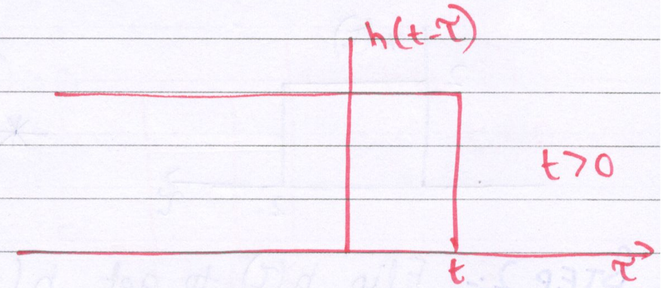
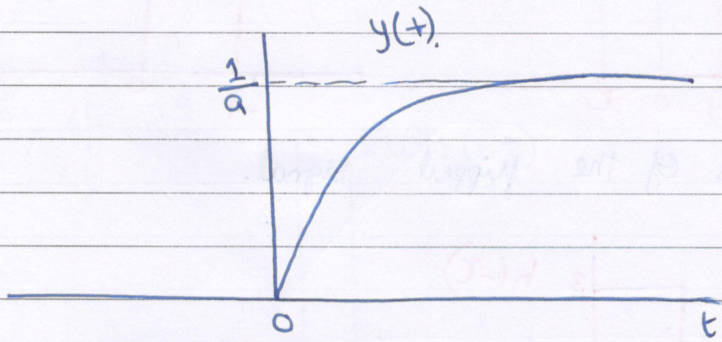
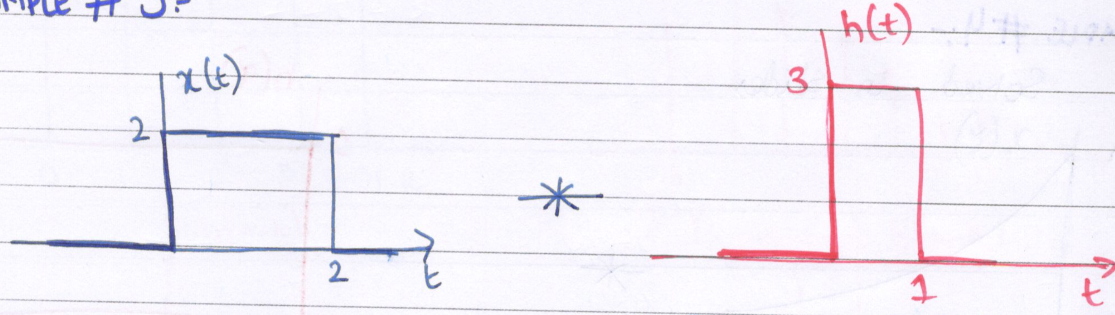


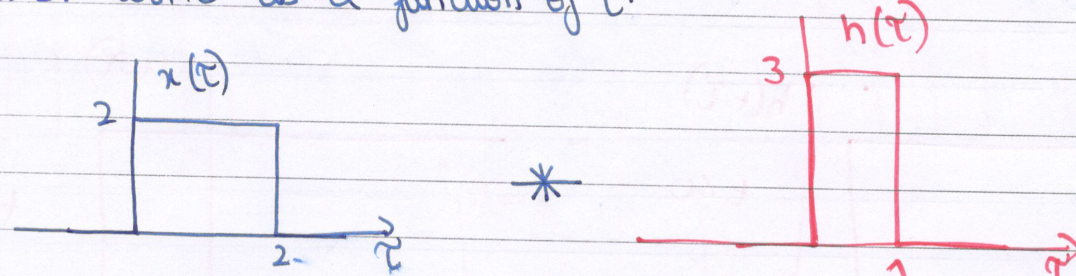
Finally, $y(t) = \frac{1}{a}\left[1-e^{-at}\right]u(t)$



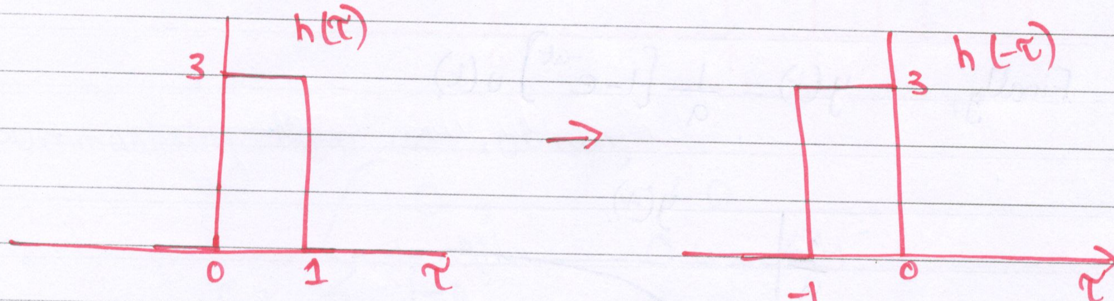
EXAMPLE #5:-



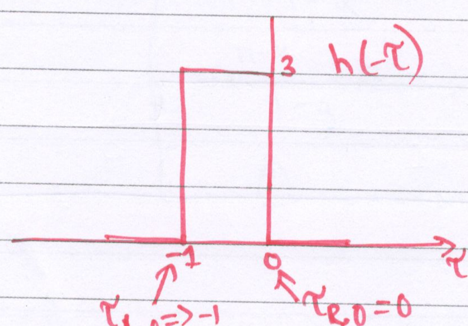
STEP 1:- Write as a function of τ.



STEP 2:- Flip $h(\tau)$ to get $h(-\tau)$



STEP 3:- Find edges of the flipped signal.



STEP 4:- Shift by t to get $h(t-\tau)$ and it edges

→ let's consider 2 for t values.

→ In each case we get $\tau_{L,t} = t + \tau_{L,0}$ and $\tau_{R,t} = t + \tau_{R,0}$

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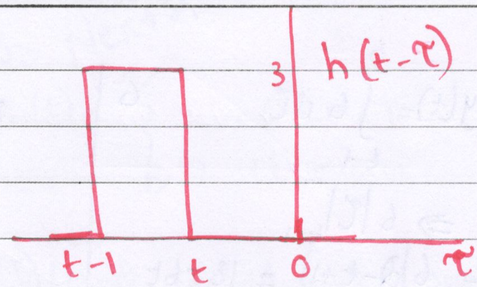
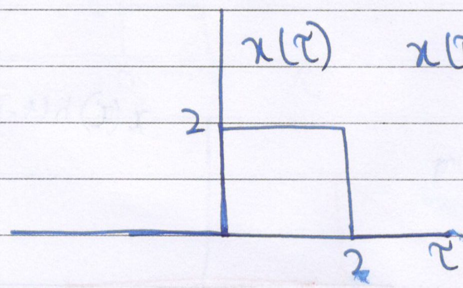


$\tau_{L,t} = t + \tau_{L,0} \Rightarrow t-1$

$\tau_{R,t} = t + \tau_{R,0} \Rightarrow t + 0 \Rightarrow t$

STEP 5:- Find regions of τ-Overlap:- + STEP 6:- Product & Integrate each interval o region

$\Rightarrow$ Region 1 (Interval 1) : no. τ-overlap $t < 0$.



$x(\tau) * h(t-\tau) \Rightarrow 0$

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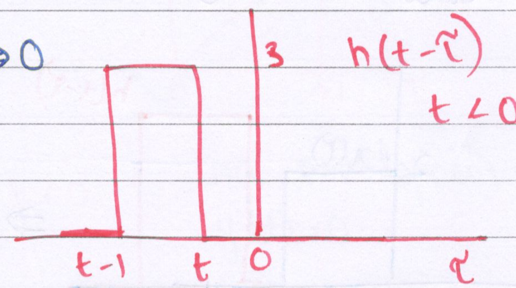
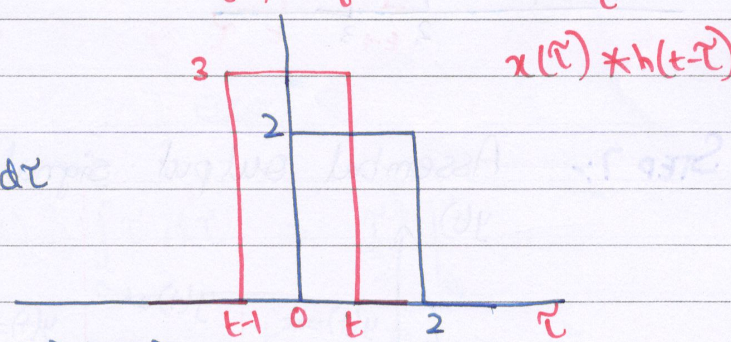


$\Rightarrow$ Interval 2 :- $0 \le t \le 1$

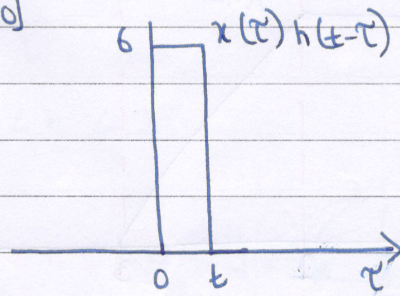


$\cancel{y(t) = \int_0^1 x(\tau)\, h(t-\tau)}$ $\quad y(t) = \int_0^t 6 \, d\tau$

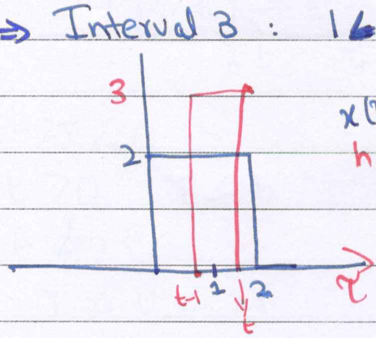
$= \cancel{\int_0^1} = |6\tau|_0^t$

$= 6[t - 0]$

$y(t) = 6t$



I $\Rightarrow$ Interval 3 : $1 \le t \le 2$



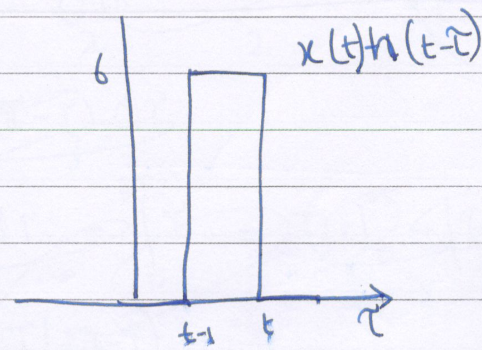
x(τ)

h(t-τ)

$y(t) = \int_{t-1}^{t} 6 \, d\tau$

$= 6 \, |\tau|_{t-1}^{t}$

$= 6[t - t + 1] \Rightarrow 6$



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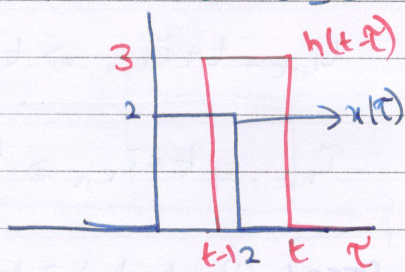
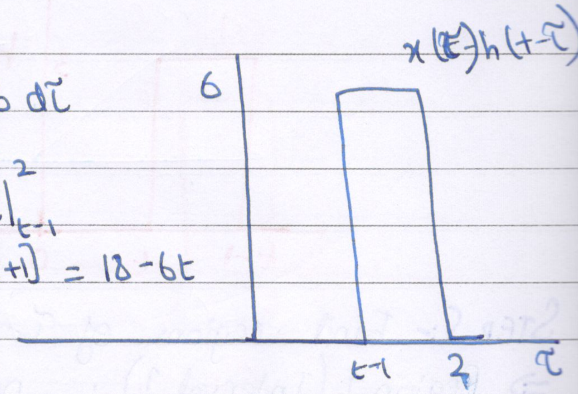
⇒ Interval 4: $2 < t \leq 3$.



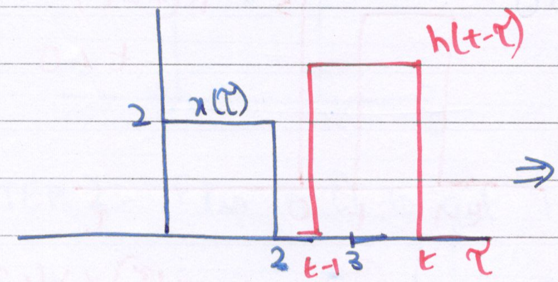
$$y(t) = \int_{t-1}^{2} 6 \, d\tau$$

$$\Rightarrow 6 \left|\tau\right|_{t-1}^{2}$$

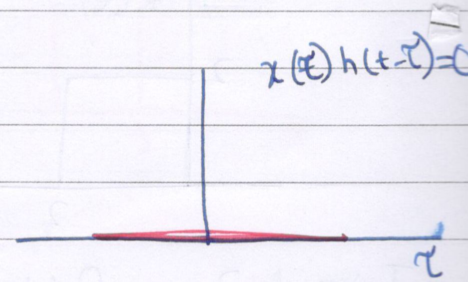
$$= 6[2 - t + 1] = 18 - 6t$$



⇒ Interval 5: $t > 3$



No overlapping ⇒



STEP 7:- Assembal output signal.