ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2016

EL313- Signal & Systems

Assignment – 1 Solution **Due Date: 26/10/2016**
Marks: 15 **Handout Date: 18/10/2016**

Question # 1:

Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period:

1. $x(t) = je^{j10t}$
2. $x[n] = 1 + e^{\frac{j4\pi n}{7}} - e^{\frac{j2\pi n}{5}}$

Solution:

1. $x(t) = je^{j10t}$

   If x (t) is periodic then there exists T>0 such that x (t) = x (t+T).

   The fundamental period of x (t) is as follows:

$$T = \frac{2\pi}{\omega_0}\ ,\ where\ \omega_0 = 10$$

$$T = \frac{2\pi}{10} \Rightarrow \frac{\pi}{5}$$

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2. $x[n] = 1 + e^{\frac{j4\pi n}{7}} - e^{\frac{j2\pi n}{5}}$

   Step #1: Determine the fundamental period of individual signals.

   Period of the first term in the RHS $N_1 = 1$

   Period for the second term in the RHS is:

$$\frac{N}{m} = \frac{2\pi}{\omega_0}\ ,\ where\ \omega_0 = \frac{4\pi}{7}$$

$$\frac{N}{m} = \frac{2\pi}{\frac{4\pi}{7}} \Rightarrow \frac{7}{2}$$

   Where N_2 (fundamental period) = 7 and m = 2.

   Period for the third term in the RHS is:

$$\frac{N}{m} = \frac{2\pi}{\omega_0}\ ,\ where\ \omega_0 = \frac{2\pi}{5}$$

$$\frac{N}{m} = \frac{2\pi}{\frac{2\pi}{5}} \Rightarrow 5$$

Where N_3 (fundamental period) = 5 and m = 1.
Step #2: Find the ratio of fundamental period of 1st signal to fundamental period of every other signal.

$$\frac{N_1}{N_2} \Rightarrow \frac{1}{7}, \ \frac{N_1}{N_3} \Rightarrow \frac{1}{5}$$

Step #3: If the ratios are rational, the composite signal is periodic. Hence above two ratios are rational so the signal x[n] is periodic.

Step #4: $N_0 = LCM(N_1, N_2, N_3)$

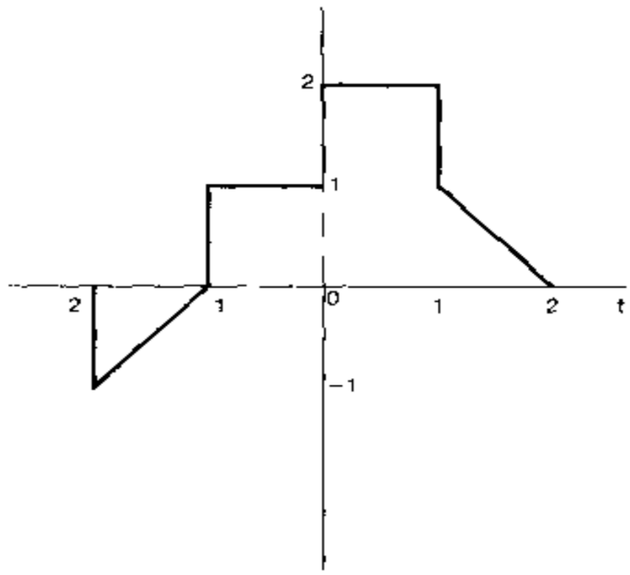
$$LCM = 1\times7\times5 \Rightarrow 35$$

Therefore, the overall signal x [n] is periodic with a period which is least common multiple of the period of the three terms in x [n]. This is equal to 35.

Question # 2:

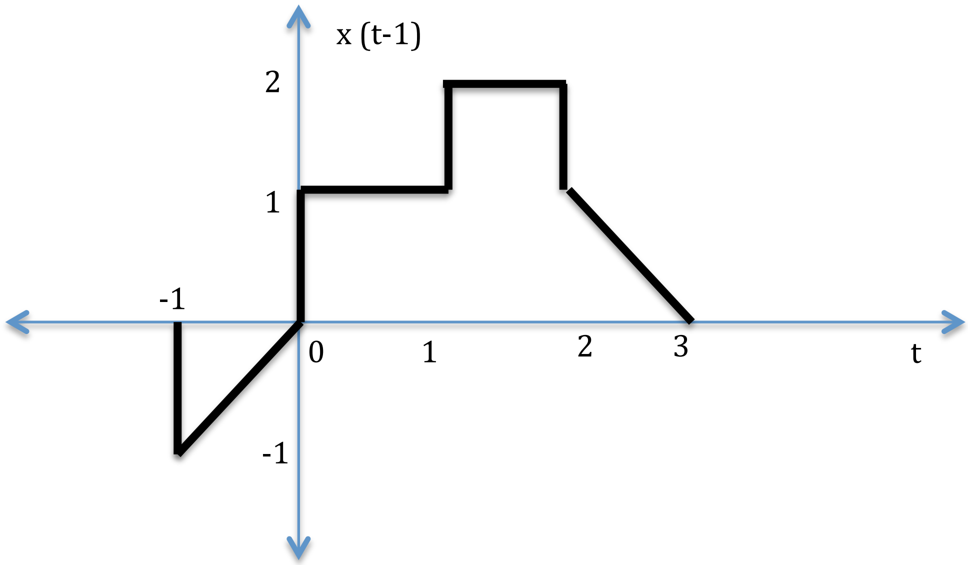
A continuous-time signal x (t) is shown in figure below. Sketch and label each of the following signals:

1. $x(t-1)$
2. $x(2t+1)$
3. $x(2-t)$

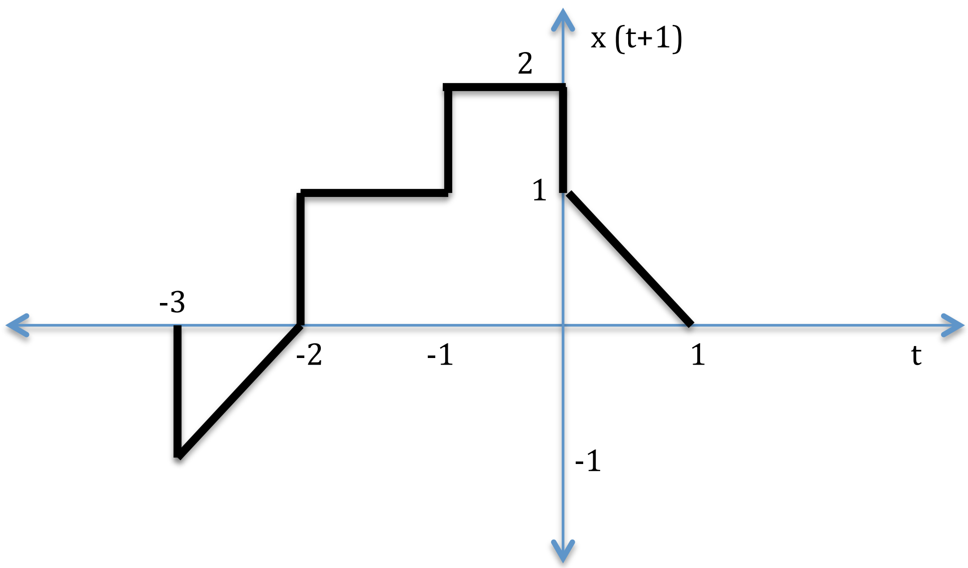


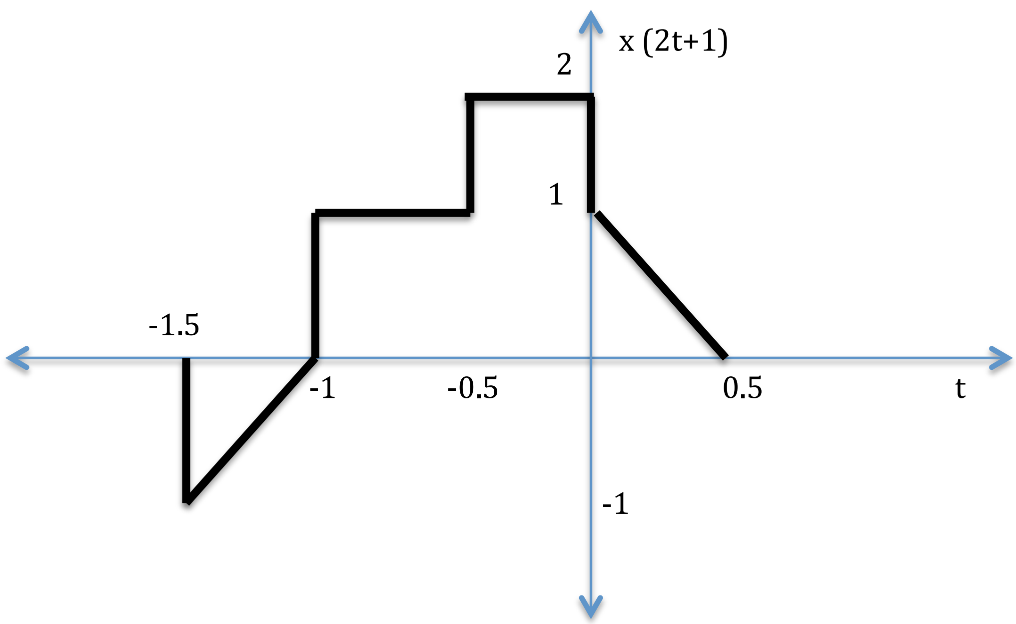
Solution:

1. $x(t-1)$



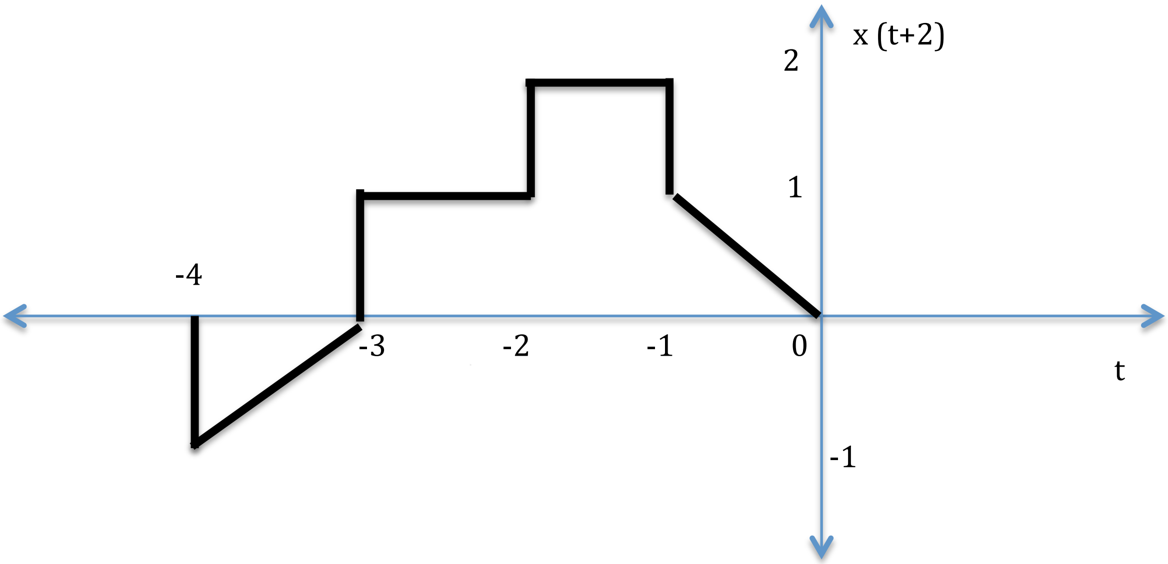
2. $x(2t+1)$

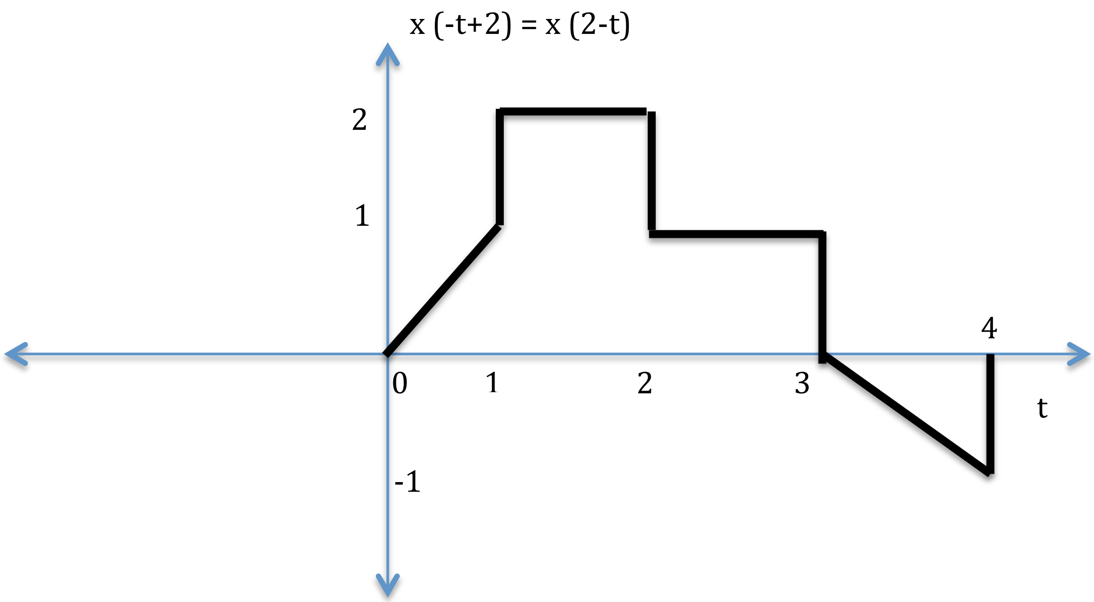




3. $x(2-t)$

$x(2-t) \Longrightarrow x(-t+2)$





Question # 3:

Determine whether the following signals are energy signal or power signal:

1. $x(t) = e^{-t}u(t)$
2. $x[n] = 3e^{j2n}$

Solution:

1. $x(t) = e^{-t}u(t)$

$$E = \int_{-\infty}^{\infty} [x(t)]^2 \, dt$$

$$= \int_{0}^{\infty} (e^{-t})^2 dt = \int_{0}^{\infty} e^{-2t} dt = -\left.\frac{e^{-2t}}{2}\right|_0^{\infty}$$

$$= -\frac{e^{-2(\infty)}}{2} + \frac{e^{-2(0)}}{2} = \frac{1}{2} < \infty$$

Hence, x (t) is an Energy signal and its Power =0.

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2. $x[n] = 3e^{j2n}$

$$\therefore |x[n]|^2 = \left|3e^{j2n}\right|^2 = 9\left|e^{j2n}\right|^2 = 9[\cos^2 2n + \sin^2 2n] \Longrightarrow 9$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} 9 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} 9$$

$$= \lim_{N\to\infty} \frac{9(2N+1)}{2N+1} = 9 < \infty$$

Hence, x [n] is a Power signal and its Energy is $E = \infty$

Good Luck