ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2016

EL313- Signal & Systems

Assignment – 2 Solution **Due Date: 07/11/2016**
Marks: 20 **Handout Date: 28/10/2016**

Question # 1:

For the discrete time system given below:

$$y(n) = x(n) + nx(n-2)$$

Check the following:

1. System with/ Without Memory
2. Invertible/ Non-invertible
3. Causality
4. Time Invariant or not
5. Linearity

Solution:

1. System with/ Without Memory
   The output y (n) depends on the past value as well so the system is not memoryless.

2. Invertible/ Non-invertible
   The system is non-invertible.

3. Causality
   The output y (n) depends on the present input x(n) and the past input x(n-2). Therefore the system is causal.

4. Time Invariant or not
   If the input is delayed by 'k' samples the output will be:
   $$y(n,k) = x(n-k) + nx(n-k-2) \rightarrow (1)$$
   Now if we delay y (n) by 'k' samples, we get:
   $$y(n-k) = x(n-k) + (n-k)x(n-k-2) \rightarrow (2)$$
   Hence, $y(n,k) \neq y(n-k)$. Therefore the system is Time Variant.

5. Linearity
   Let's consider a signal:
   $$x(n) = ax_1(n) + bx_2(n)$$
   Where $y_1(n) = x_1(n) + nx_1(n-2)$ & $y_2(n) = x_2(n) + nx_2(n-2)$

Then:

$$ay_1(n) + by_2(n) = ax_1(n) + anx_1(n-2) + bx_1(n) + bnx_1(n-2)$$

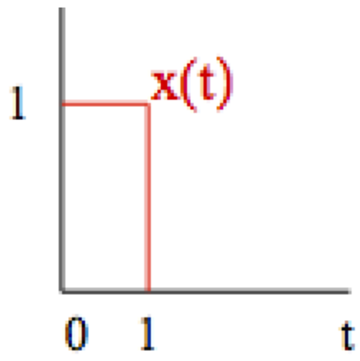
$$ay_1(n) + by_2(n) = a\big(x_1(n) + nx_1(n-2)\big) + b\big(x_1(n) + nx_1(n-2)\big)$$

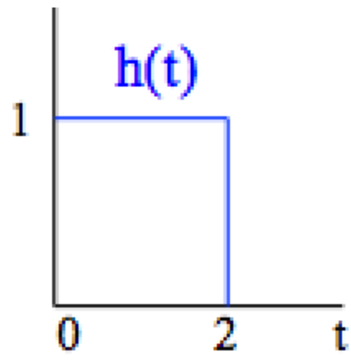
$$y_3(n) = ay_1(n) + by_2(n)$$

Hence the system is linear.

<u>Question # 2:</u>

Convolve the two Continuous-Time Signals given below:

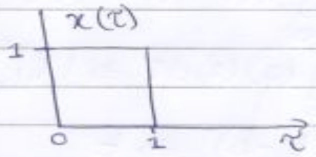




<u>Solution:</u>

STEP #1 :-

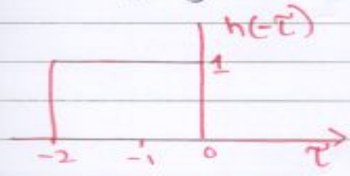
Change the subscript $t \Rightarrow \tau$

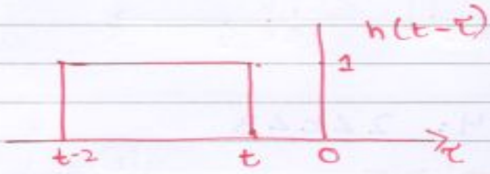


STEP # 2 :-

Flip and shift any one of the above signals

Flipping & Shifting $h(\tau)$.

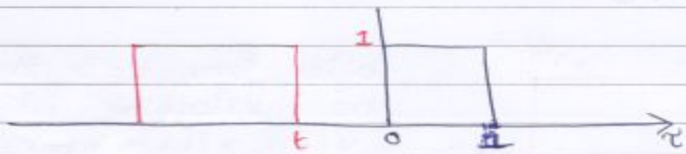




STEP # 3 :-

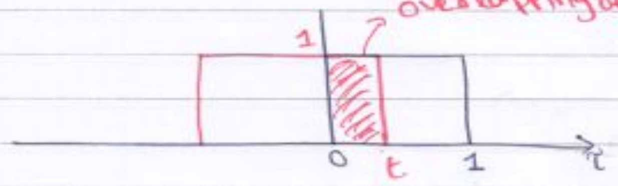
Start sliding $h(t-\tau)$ on $x(\tau)$ and integrate.

- INTERVAL #1 : $t < 0$



No overlapping so $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \Rightarrow 0$

- INTERVAL #2: $0 < t < 1$.

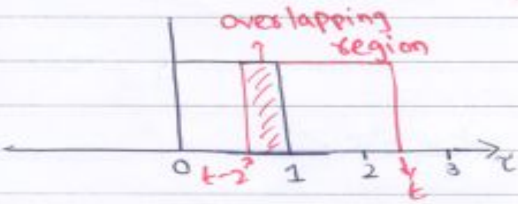


$$y(t) = \int_0^t x(\tau)h(t-\tau)\,d\tau$$
$$= \int_0^t (1)(1)\,d\tau \Rightarrow |\tau|_0^t$$
$$y(t) = (t-0) \Rightarrow t$$

- INTERVAL #3: $1 < t < 2$

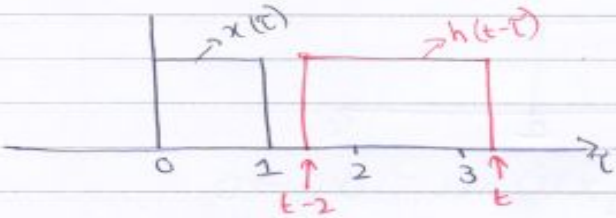
$$y(t) = \int_0^1 (1 \times 1)\,d\tau$$
$$= |\tau|_0^1 = 1-0$$
$$y(t) \Rightarrow 1$$

- INTERVAL #4: $2 < t < 3$



$$y(t) = \int_{t-2}^{1} (1 \times 1)\,d\tau$$
$$= |\tau|_{t-2}^{1} = 1-(t-2)$$
$$y(t) = 1-t+2 \Rightarrow 3-t$$

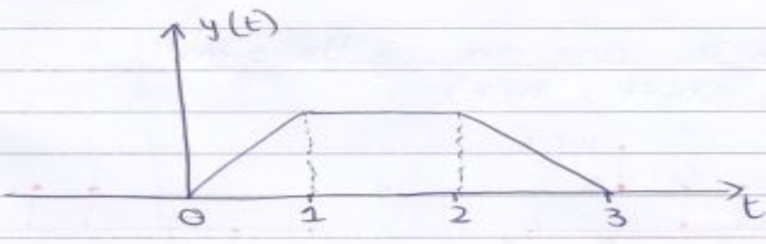
- INTERVAL #5: $t > 3$



after time $t=3$ there is no overlapping. integral is 0.
$y(t) = x(t) * h(t) \Rightarrow 0$

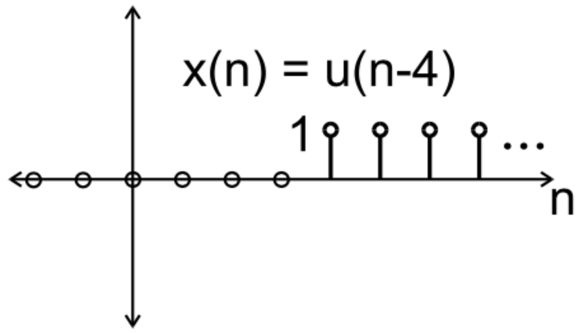
STEP #4:-
Draw the final signal.

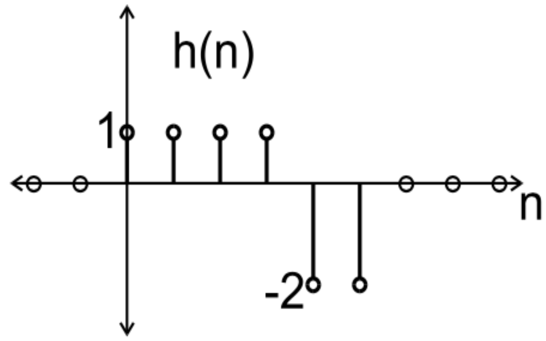
$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 3-t & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$



Question # 3:

Convolve the two Discrete-Time Signals given below:

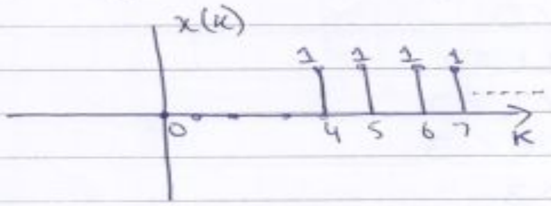


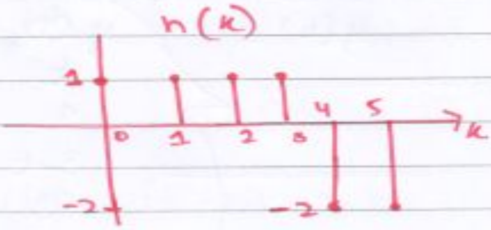


Solution:

STEP #1:-

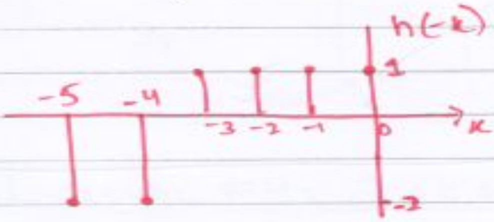
Change the subscripts. $n \Rightarrow k$

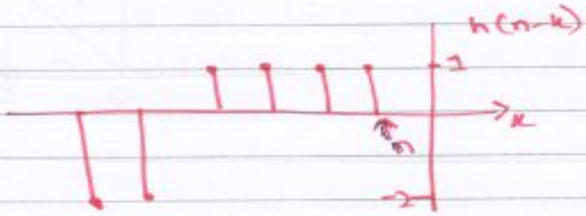




STEP # 2:-

Flip & Shift any one of the signal.
let's flip & shift h(k).



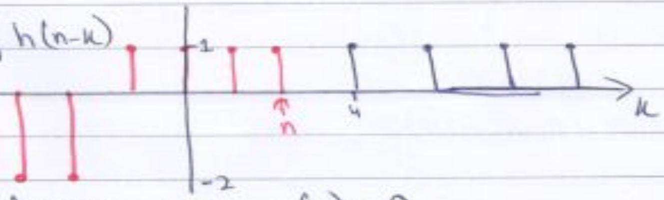


STEP # 3:-

Start sliding $h(n-k)$ on $x(k)$ and solve the convolution sum.
As we know that $x(k)$ starts from $k=4$ and continues to infinity as its unit step function, hence our overlapping will starts from $n=4$ and onwards.

8. $n < 4$

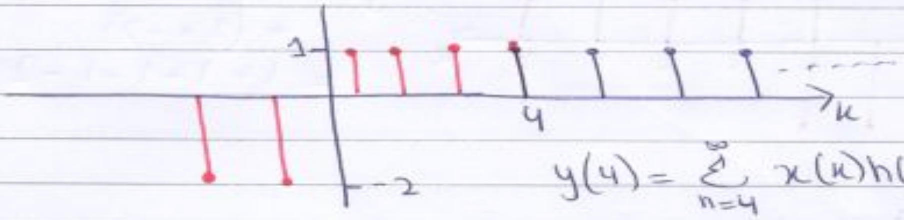
For example shifting $h(n-k)$ to the right.



There is no overlapping so $y(n) = 0$.

⇒ Now when $n \geq 4$, we see that $h(n-k)$ slides and ovelapp with $x(k)$ and hence overlapping starts.

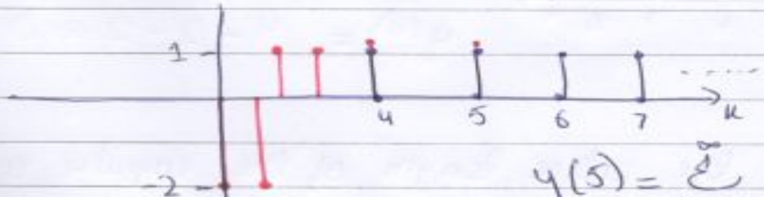
- when $n=4$:



$$y(4) = \sum_{n=4}^{\infty} x(k)h(n-k)$$

$$= (1) \times (1) \Rightarrow 1$$

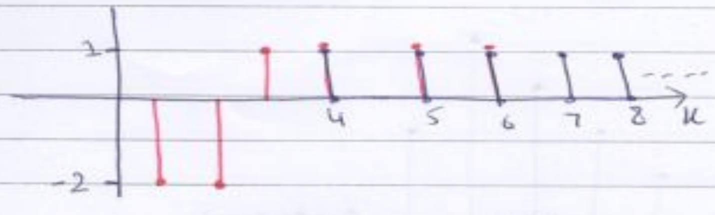
- when $n=5$:



$$y(5) = \sum_{n=4}^{\infty} x(k)h(n-k)$$

$$= (1\times1) + (1\times1) \Rightarrow 1+1=2$$

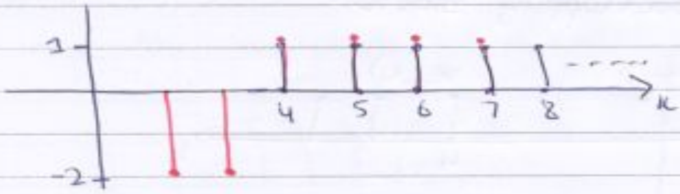
- when $n=6$:



$$y(6) = (1\times1) + (1\times1) + (1\times1)$$

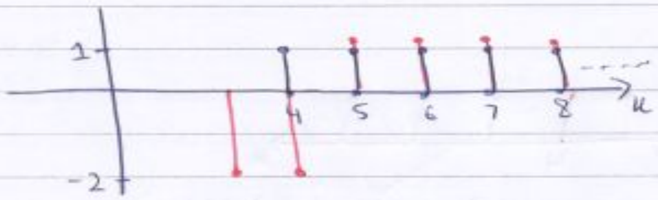
$$= 1+1+1 \Rightarrow 3$$

• when $n = 7$:



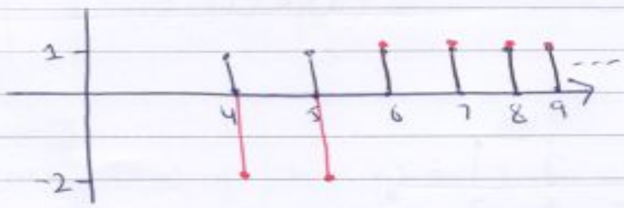
$$y(7) = (1\times1) + (1\times1) + (1\times1) + (1\times1)$$
$$= 1 + 1 + 1 + 1 \Rightarrow 4$$

• when $n = 8$:



$$y(8) = (1\times1) + (1\times1) + (1\times1) + (1\times1) + (1\times -2)$$
$$= 1 + 1 + 1 + 1 - 2 \Rightarrow 2$$

• when $n \geq 9$:

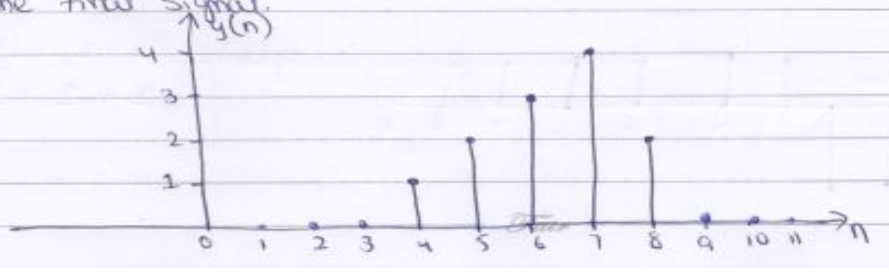


$$y(9) = (1\times1) + (1\times1) + (1\times1) + (1+1) + (1\times-2) + (1\times-2)$$
$$= 1 + 1 + 1 + 1 - 2 - 2$$
$$y(9) = 4 - 2 - 2 \Rightarrow 0$$

As its summing over the entire length of the impulse response so $y(n) = 1$ from $n \geq 9$.

Step #4:-

Plot the final signal.



Good Luck