Name:____________________________________ **Regd. No.**______________________

Course Title: Signal & Systems **Course Code:** EL-313

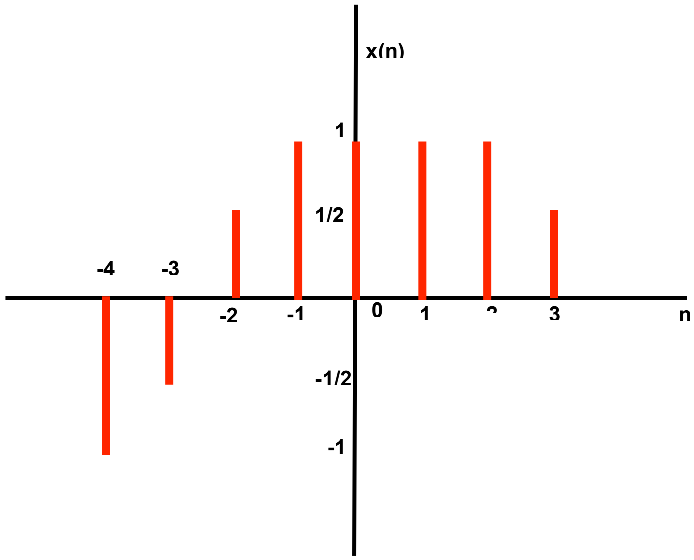
MID SEMESTER EXAMINATION – FALL 2015
Program: B.E. (Electrical)

Solution

SECTION-II: 30 MARKS
Time: 1hr 30min

Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

Q1. A discrete time signal is shown below. Sketch the following signals:

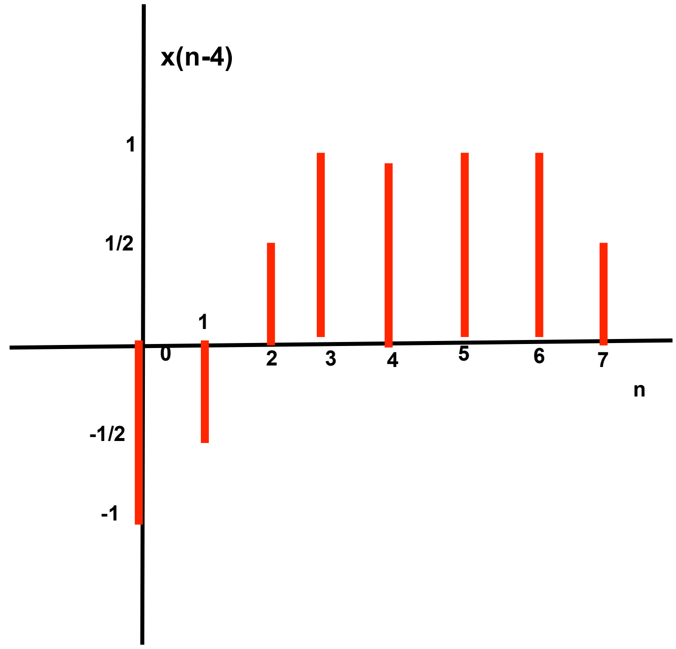


1. $x[n-4]$
2. $x[-n-4]$
3. $x[3n]$
4. $x[3n+1]$
5. $x[-3n]$

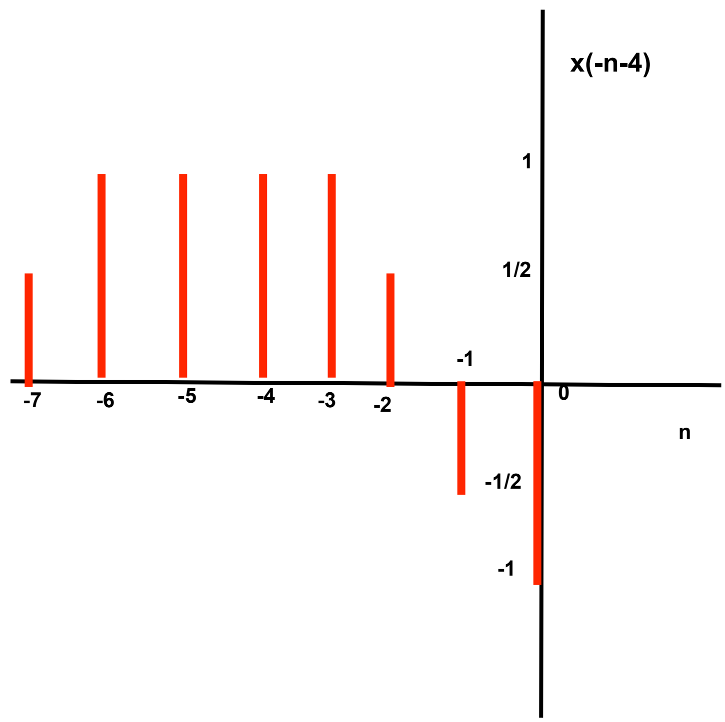
(05 Marks)

Solution:

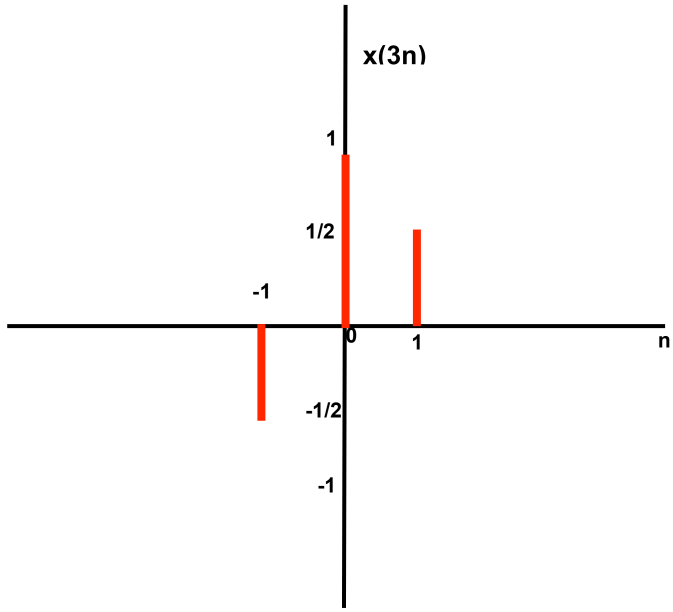
1. $x[n-4]$



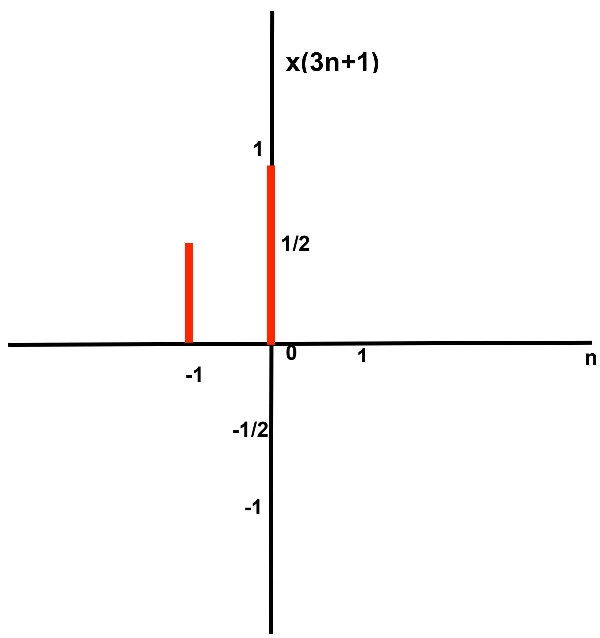
2. $x[-n-4]$



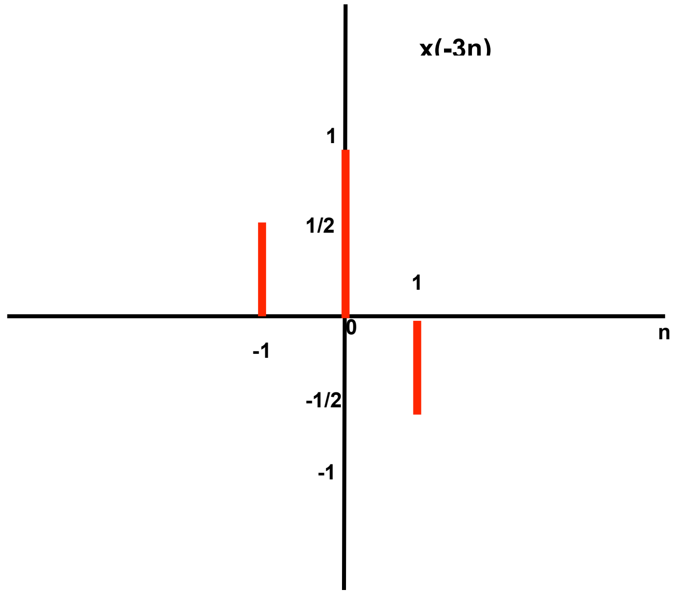
3. $x[3n]$



4. $x[3n+1]$



5. $x[-3n]$



Q2. Determine the values of P_∞ and E_∞ for each of the following signals:

1. $x(t) = e^{-t}u(t)$
2. $x(t) = e^{-4t}u(t)$
3. $x(t) = t^4u(t)$

(05 Marks)

Solution:

1. $x(t) = e^{-t}u(t)$

Let's solve it for Energy signal:

$$E = \int_0^\infty (e^{-t})^2\,dt = \int_0^\infty (e^{-2t})\,dt$$
$$= \left[-\frac{1}{2}e^{-2t}\right]_0^\infty = \left[-\frac{1}{2}e^{-2(\infty)} + \frac{1}{2}e^{-2(0)}\right] = \frac{1}{2} < \infty$$

Hence, it is an Energy Signal and Power of the signal is $P_\infty = 0$.

2. $x(t) = e^{-4t}u(t)$

Let's solve it for Energy signal:

$$E = \int_0^\infty (e^{-4t})^2\,dt = \int_0^\infty (e^{-8t})\,dt$$
$$= \left[-\frac{1}{8}e^{-8t}\right]_0^\infty = \left[-\frac{1}{8}e^{-8(\infty)} + \frac{1}{8}e^{-8(0)}\right] = \frac{1}{8} < \infty$$

Hence, it is an Energy Signal and Power of the signal is $P_\infty = 0$.

3. $x(t) = t^4u(t)$

Let's solve it for Energy signal first:

$$E = \int_0^\infty (t^4)^2\,dt = \int_0^\infty t^8\,dt$$
$$= \left[\frac{1}{9}t^9\right]_0^\infty = \left[\frac{1}{9}(\infty)^9 - \frac{1}{9}(0)^9\right] = \infty$$

Hence, its Energy is equals to infinity so it is not an energy signal.

Let's solve it for Power signal now:

$$P = \frac{1}{T}\int_0^{\infty}(t^4)^2\,dt = \frac{1}{T}\int_0^{\infty} t^8\,t$$

$$= \frac{1}{T}\left[\frac{1}{9}t^9\right]_0^{\infty} = \frac{1}{T}\left[\frac{1}{9}(\infty)^9 - \frac{1}{9}(0)^9\right] = \infty$$

Hence, its Power is equals to infinity so it is not power signal.

Q3. Determine which of the following properties hold for the following systems given below:

1. Memory less
2. Time Invariant
3. Linear
4. Casual
5. Stable

- $y[n] = x[-n]$
- $y[n] = x[n]u[n]$

(05 Marks)

Solution:

- $y[n] = x[-n]$

1. Memory less:
   Hence, the input signal depends on the past value, so it is not a Memory less system.
2. Time Invariant:

$$y[n] = x[-n]$$

Delay the input by 'k' samples and denote the output by y[n,k].

$$y[n,k] = x[-n-k]$$

Here n of x(n) has not been replaced by n-k. Here we are delaying x(n) and x(-n) will be delayed by the same amount.

Now delay the output y(n) by k samples.

$$y[n-k] = x[-(n-k)] = x[-n+k]$$

Hence, both $y[n,k] \neq y[n-k]$. Thus the system is Time variant.

3. Linear:

Let the system produce $y_1(n)$ and $y_2(n)$ for two separate inputs $x_1(n)$ and $x_2(n)$.

Therefore, $y_1(n) = x_1(-n)$ and $y_2(n) = x_2(-n)$

The response $y_3(n)$ due to linear combination of inputs is given by

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$
$$= T\{a_1x_1(n)\} + T\{a_2x_2(n)\}$$
$$= a_1T\{x_1(n)\} + a_2\,T\{x_2(n)\}$$
$$= a_1x_1(-n) + a_2x_2(-n)$$
$$= a_1y_1(n) + a_2y_2(n)$$

The response $y'_3(n)$ of the system due to linear combination of two outputs will be:

$$y'_3(n) = a_1y_1(n) + a_2y_2(n)$$

Hence; $y_3(n) = y'_3(n)$. Therefore the system is linear.

4. Casual

It is non-casual system as if value of n = -1 then the system will have the future input value.

5. Stable

It is stable system as x(n)<∞.

- $y[n] = x[n]u[n]$

1. Memory less:

The output y(n) depends on the present input only. Therefore, the system is static only i.e; memory less.

2. Time Invariant:

$$y[n] = x[n]u[n]$$

Delay the input by 'k' samples and denote the output by y[n,k].

$$y[n,k] = x[n-k]u[n]$$

Now delay the output y(n) by k samples.

$$y[n-k] = x[n-k]u[n-k]$$

Hence, both $y[n,k] \neq y[n-k]$. Thus the system is Time variant.

3. Linear:

Let the system produce $y_1(n)$ and $y_2(n)$ for two separate inputs $x_1(n)$ and $x_2(n)$.

Therefore, $y_1(n) = x_1(n)u(n)$ and $y_2(n) = x_2(n)u(n)$

The response $y_3(n)$ due to linear combination of inputs is given by

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$
$$= T\{a_1x_1(n)\} + T\{a_2x_2(n)\}$$
$$= a_1T\{x_1(n)\} + a_2T\{x_2(n)\}$$
$$= [a_1x_1(n) + a_2x_2(n)]u(n)$$
$$= a_1y_1(n) + a_2y_2(n)$$

The response $y'_3(n)$ of the system due to linear combination of two outputs will be :

$$y'_3(n) = a_1y_1(n) + a_2y_2(n) = a_1x_1(n)u(n) + a_2x_2(n)u(n)$$

Hence; $y_3(n) = y'_3(n)$. Therefore the system is linear.

4. Casual

In the given system the present output depends on the present value of input. So the system is casual.

5. Stable

The output y(n) has bounded value for any value of x(n). Hence the system is stable.

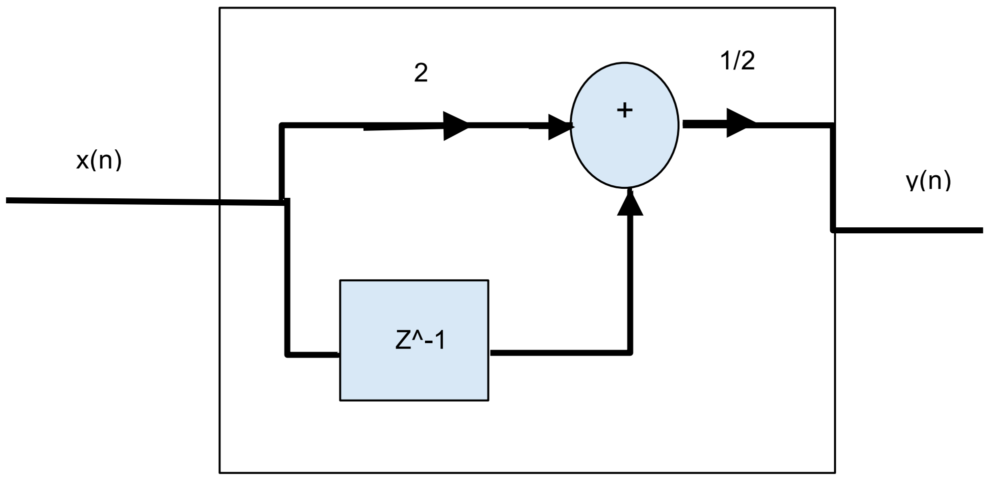
Q4. Draw a block diagram for the following input-output relation:

1. $y[n] = \frac{(2x[n]+x[n-1])}{2}$
2. $y[n] = (x[n-1] + 2x[n-2])\times y[n-1]$
3. $y[n] = y[n-1]x[n-1] + 0.5x[n]$
4. $y[n] = x[n] + 1.04y[n-1]$

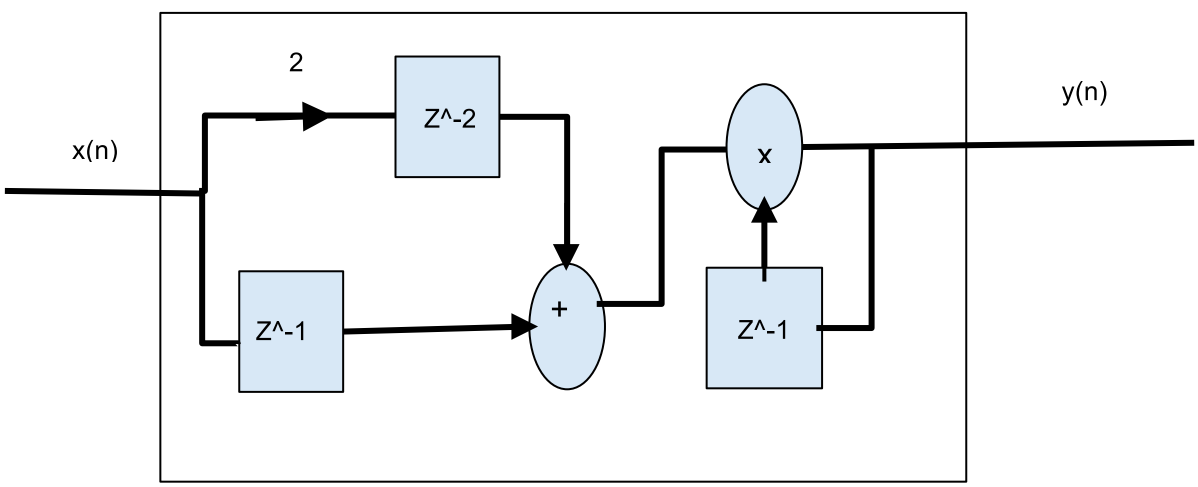
(05 Marks)

Solution:

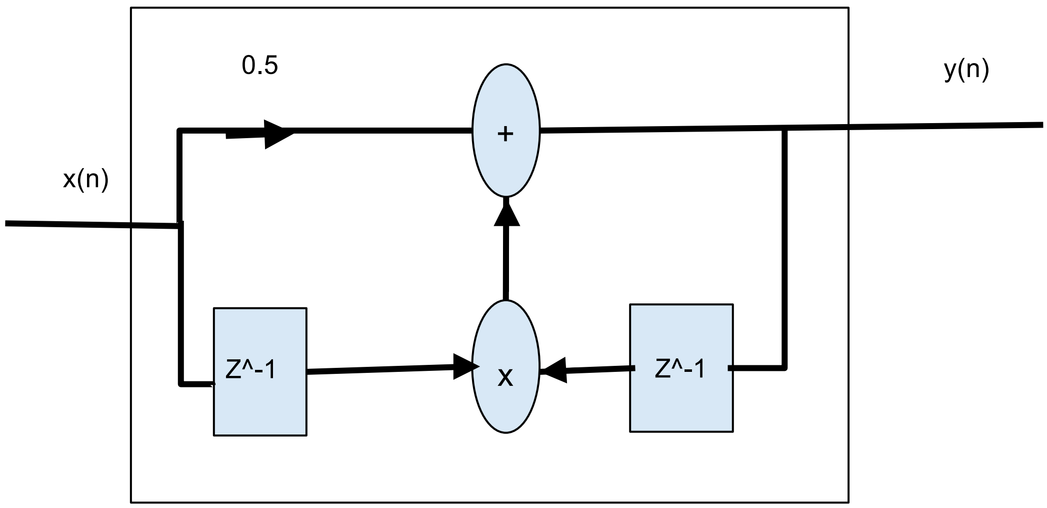
1. $y[n] = \frac{(2x[n]+x[n-1])}{2}$



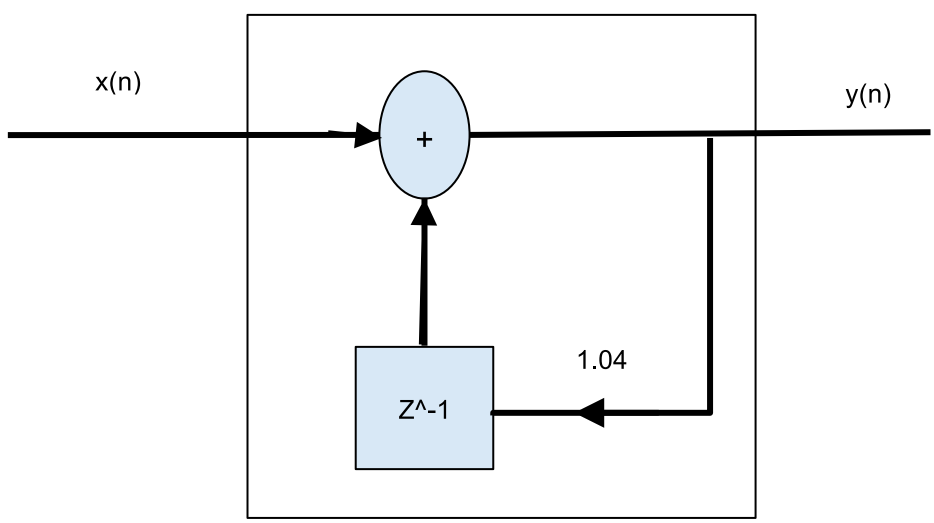
2. $y[n] = (x[n-1] + 2x[n-2])\times y[n-1]$



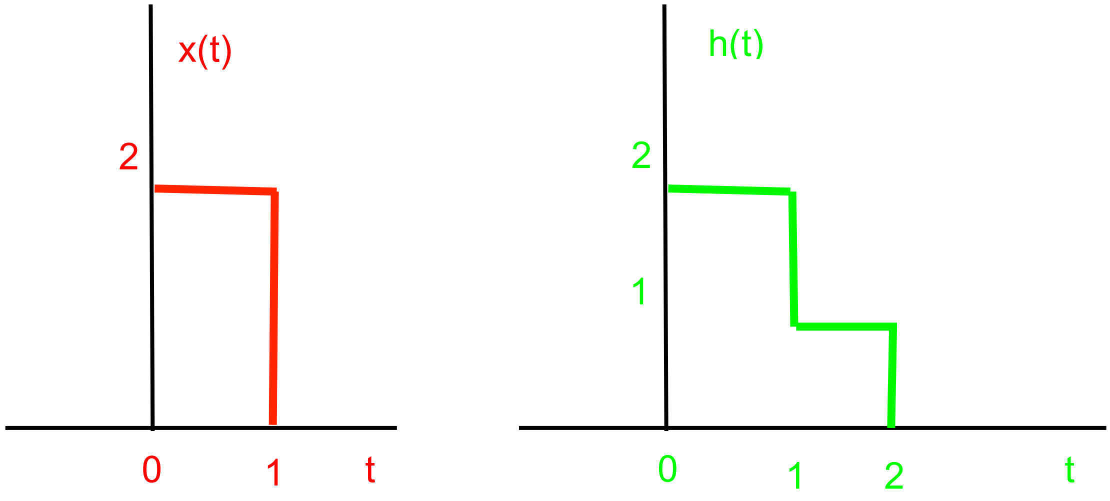
3. $y[n] = y[n-1]x[n-1] + 0.5x[n]$



4. $y[n] = x[n] + 1.04y[n-1]$



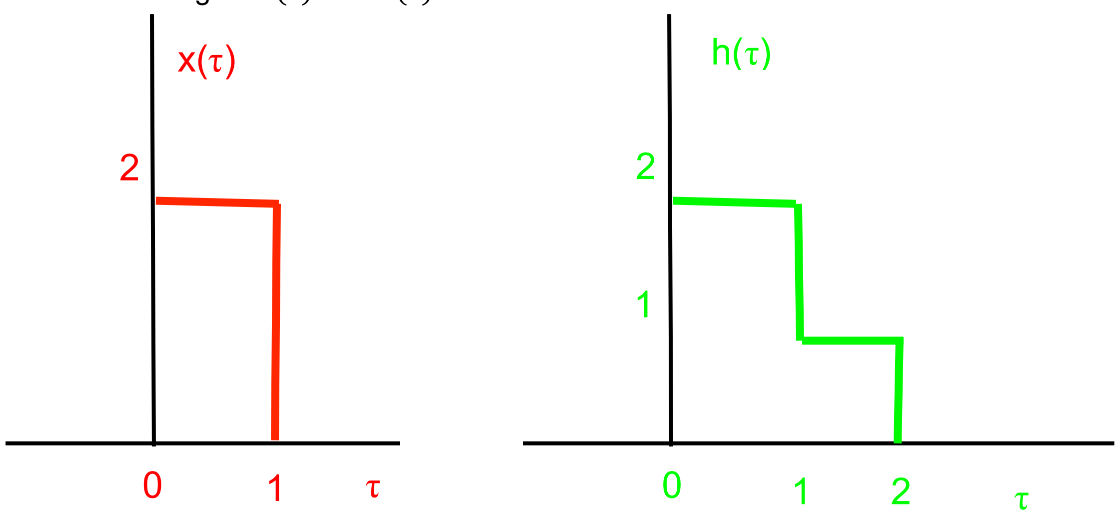
Q5. Graphically convolve the following signals shown below:



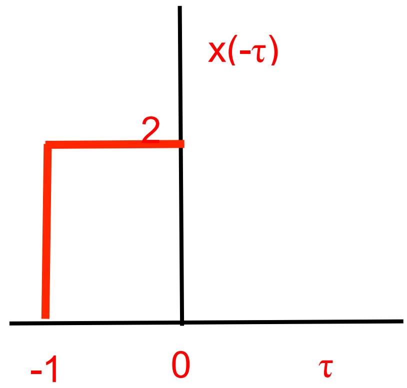
(10 Marks)

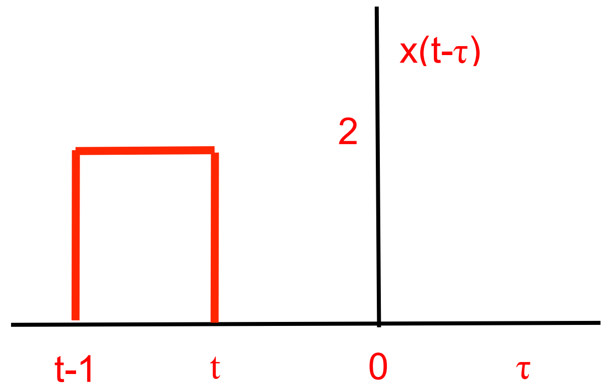
Solution:

Step1: First draw the signal $x(\tau)$ and $h(\tau)$.



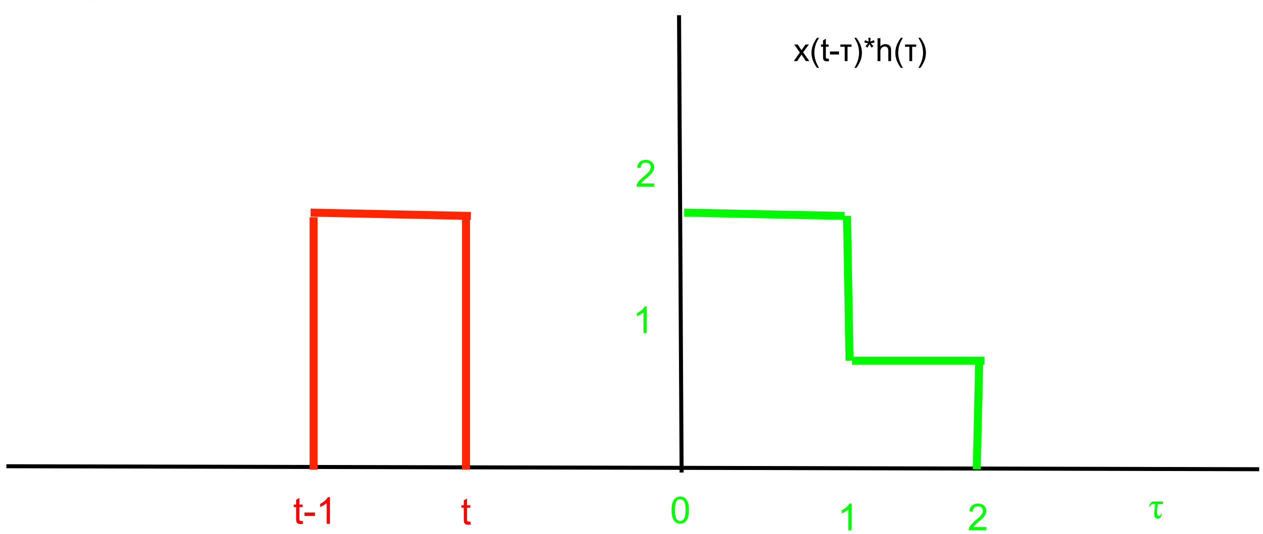
Step2: Choose to flip and slide the signal, e.g., $h(\tau)$.
Note: Any signal can be flipped and shifted.





Step3: Now we'll slide our shifted signal and will calculate the overlapping points.

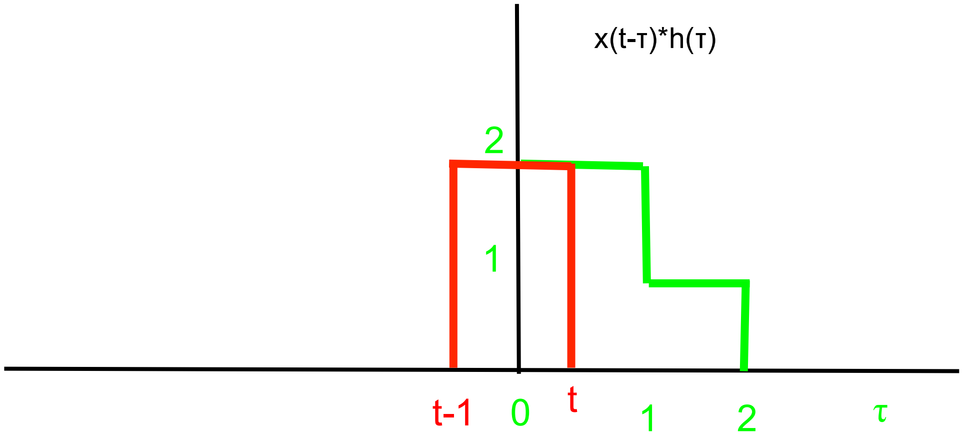
1. $t < 0$:



There is no overlapping so the area under the product of two functions will be zero.

$$y(0) = \int x(t-\tau) * h(\tau) = 0$$

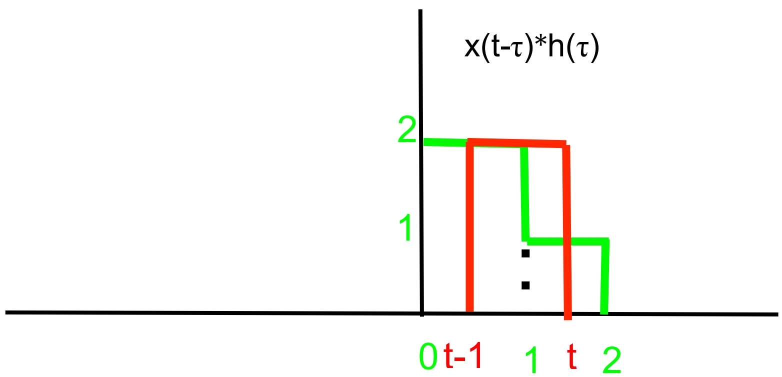
2. $0 < t < 1$:



Here part of h(τ) overlaps with x(τ) so the area under the product of the function is:

$$y(t) = \int_0^t 4d\tau = 4[\tau]_0^t = 4t - 0 \Rightarrow 4t$$

3. $1 < t < 2$:

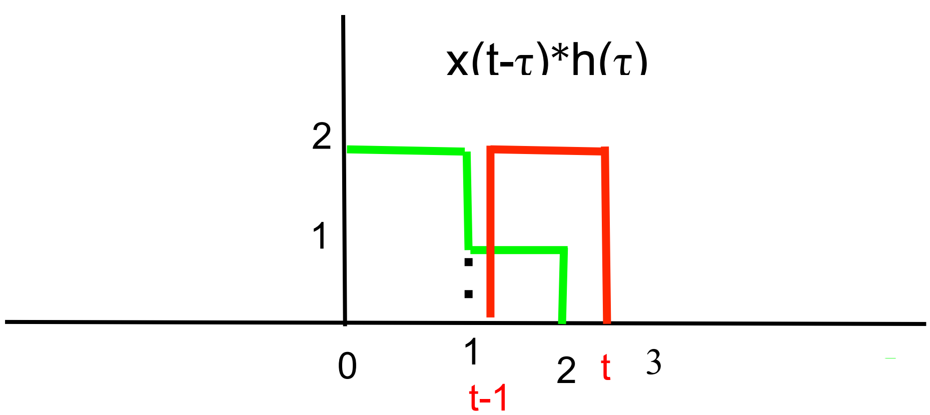


Here part of h(τ) overlaps with x(τ) so the area under the product of the function is:

$$y(t) = \int_{t-1}^{1} 4d\tau + \int_1^t 2d\tau = 4[\tau]_{t-1}^{1} + 2[\tau]_1^t = [4(1) - 4(t-1)] + [(2(t) - 2(1)]$$

$$= [4 - 4t + 4] + [2t - 2] = 4 - 4t + 4 + 2t - 2 \Rightarrow 6 - 2t$$

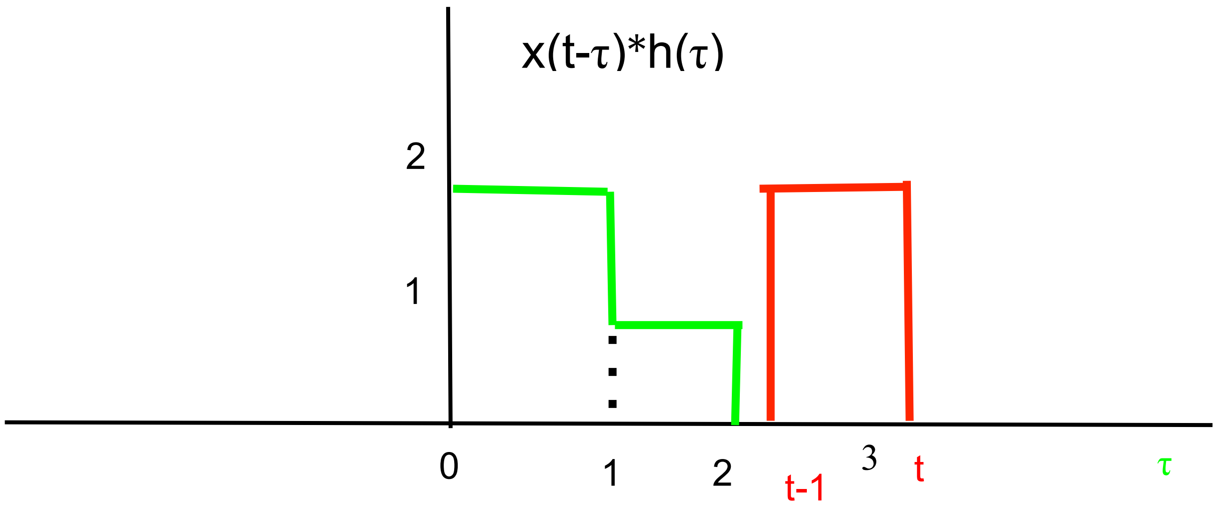
4. $2 < t < 3$:



Here part of h(τ) overlaps with x(τ) so the area under the product of the function is:

$$y(t) = \int_{t-1}^{2} 2d\tau = 2[\tau]_{t-1}^{2}$$
$$= 2(2) - 2(t-1)] = 4 - 2t + 2 \Rightarrow 6 - 2t$$

5. $t > 3$:



There is no overlapping so the area under the product of two functions will be zero.

$$y(0) = \int x(t-\tau) * h(\tau) = 0$$

Step4: Draw the final signal.

$$y(t) = x(t) * h(t) = \begin{cases} 0 & t < 0 \\ 4t & 0 < t < 1 \\ 6 - 2t & 1 < t < 2 \\ 6 - 2t & 2 < t < 3 \\ 0 & t > 3 \end{cases}$$