Lecture Notes
9th December 2016

Date Monday / 9th Dec 16 Lecture # 11

Example # 1:-

$$y[n] - a y[n-1] = x[n], \; |a| < 1 \rightarrow ①$$

Sol:-

The frequency response: $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1}{1-ae^{-j\omega}}$

$y[n] \Rightarrow Y(e^{j\omega})$
$y[n-1] \Rightarrow e^{-j\omega} Y(e^{j\omega})$ ∴ using time shifting property
$x[n] \Rightarrow X(e^{j\omega})$

Hence equ ① becomes.

$$Y(e^{j\omega}) - ae^{-j\omega} Y(e^{j\omega}) = X(e^{j\omega})$$

$$Y(e^{j\omega}) [1 - ae^{-j\omega}] = X(e^{j\omega})$$

Cross multiplying gives

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \Rightarrow \frac{1}{1-ae^{-j\omega}}$$

We know that it is the Fourier transform of $a^n u[n]$. Hence the impulse response of the system is :-

$$h[n] = a^n u[n].$$

Example # 2:-

$$y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = 2x[n]$$

$$x[n] = \left(\frac{1}{4}\right)^n u[n]$$

$Y(e^{j\omega}) = ?$, $y[n] = ?$

Sol:-

$$Y(e^{j\omega}) - \frac{3}{4} e^{-j\omega} Y(e^{j\omega}) + \frac{1}{8} e^{-2j\omega} Y(e^{j\omega}) = 2X(e^{j\omega})$$

$$Y(e^{j\omega}) \left[1 - \frac{3}{4} e^{j\omega} + \frac{1}{8} e^{-j2\omega}\right] = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \Rightarrow \frac{2}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j2\omega}}$$

$-\frac{1}{4} - \frac{1}{2} \Rightarrow \frac{-1-2}{4} \Rightarrow -\frac{3}{4}$

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Factor the denominator of $H(e^{j\omega})$ i.e.,

$$1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega} = \left[1 - \frac{1}{2}e^{-j\omega}\right]\left[1 - \frac{1}{4}e^{-j\omega}\right]$$

$$H(e^{j\omega}) = \frac{2}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)}$$

No need in this example just for information

Using partial fraction method

$$H(e^{j\omega}) = \frac{2}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right)} = \frac{A}{\left(1-\frac{1}{2}e^{-j\omega}\right)} + \frac{B}{1-\frac{1}{4}e^{-j\omega}}$$

let $s = e^{-j\omega}$

$$\frac{2}{\left(1-\frac{1}{2}s\right)\left(1-\frac{1}{4}s\right)} = \frac{A}{1-\frac{1}{2}s} + \frac{B}{1-\frac{1}{4}s}$$

Cross multiplication gives

$$2 = A\left(1-\frac{1}{4}s\right) + B\left(1-\frac{1}{2}s\right)$$

put $1 - \frac{1}{4}s = 0$, $1 - \frac{1}{2}s = 0$

$-\frac{1}{4}s = -1$ $\qquad -\frac{1}{2}s = -1$

$s = 4$ $\qquad s = 2$

when $s=4$.

$$2 = A\left[1 - \frac{1}{\cancel{4}}(\cancel{4})\right] + B\left[1 - \frac{1}{\cancel{2}}(\cancel{4})\right]$$

$$2 = A[0] + B[1-2]$$

$$2 = -B$$

$$B \Rightarrow -2$$

ate

just for information

when $s = 2$

$$2 = A\left[1-\frac{1}{\cancel{4}_2}(\cancel{2})\right] + B\left[1-\frac{1}{\cancel{2}}(\cancel{2})\right]$$

$$2 = A\left[1-\frac{1}{2}\right]$$

$$2 = A\left[\frac{2-1}{2}\right]$$

$$2 = \frac{A}{2} \Rightarrow A = 4$$

Hence

$$H(e^{jw}) = \frac{4}{1-\frac{1}{2}e^{-jw}} - \frac{2}{1-\frac{1}{4}e^{-jw}}$$

~~Hen~~ Now, $H(e^{jw}) = \dfrac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]}$

$$x[n] = \left[\frac{1}{4}\right]^n u[n] \qquad \because a^n u[n] = \frac{1}{1-ae^{-jw}}$$

$$X(e^{jw}) = \frac{1}{1-\frac{1}{4}e^{-jw}}$$

$$Y(e^{jw}) = H(e^{jw})\,X(e^{jw})$$

$$= \left[\frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]}\right]\left[\frac{1}{1-\frac{1}{4}e^{-jw}}\right]$$

$$Y(e^{jw}) = \frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]^2}$$

Using partial fraction method :-

$$Y(e^{jw}) = \frac{2}{\left[1-\frac{1}{2}e^{-jw}\right]\left[1-\frac{1}{4}e^{-jw}\right]^2} = \frac{A}{\left[1-\frac{1}{4}e^{-jw}\right]} + \frac{B}{\left[1-\frac{1}{4}e^{-jw}\right]^2} + \frac{C}{1-\frac{1}{2}e^{-jw}}$$

Cross multiplication gives

$$2 = A\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{1}{4}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right) + C\left(1-\frac{1}{4}e^{-j\omega}\right)^2$$

let $e^{-j\omega} = s$

$$2 = A\left[1-\frac{1}{2}s\right]\left[1-\frac{1}{4}s\right] + B\left[1-\frac{1}{2}s\right] + C\left[1-\frac{1}{4}s\right]^2$$

let $s=2$, $s=4$

when $s=2$,

$$2 = A\left[1-\frac{1}{\cancel{2}}(\cancel{2})\right]\left[1-\frac{1}{\cancel{4}_2}(\cancel{2})\right] + B\left[1-\frac{1}{\cancel{2}}(\cancel{2})\right] + C\left[1-\frac{1}{\cancel{4}_2}(\cancel{2})\right]^2$$

$$2 = A(0)\left(1-\frac{1}{2}\right) + B(0) + C\left[1-\frac{1}{2}\right]^2$$

$$2 = C\left[\frac{2-1}{2}\right]^2 \Rightarrow 2 = C\left[\frac{1}{4}\right]$$

$$C \Rightarrow 8$$

when $s=4$,

$$2 = A\left[1-\frac{1}{2}(4)\right]\left[1-\frac{1}{\cancel{4}}(\cancel{4})\right] + B\left[1-\frac{1}{\cancel{2}_1}(\cancel{4})\right] + C\left[1-\frac{1}{\cancel{4}}(\cancel{4})\right]^2$$

$$2 = A(0) + B(1-2) + C(0)$$

$$B \Rightarrow -2$$

for the value of A we will put $s=0$ as we don't have any other value for s from the equation above

Hence, when $s=0$

$$2 = A\left[1-\frac{1}{2}(0)\right]\left[1-\frac{1}{4}(0)\right] + B\left[1-\frac{1}{2}(0)\right] + C\left[1-\frac{1}{4}(0)\right]^2$$

$$2 = A[1][1] + (-2)(1) + (8)(1)$$

$$2 = A - 2 + 8$$

$$2 = A + 6$$

$$A = -6 + 2 \Rightarrow -4$$

So, $$Y(e^{j\omega}) = -\frac{4}{1-\frac{1}{4}e^{-j\omega}} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} + \frac{8}{1+\frac{1}{2}e^{-j\omega}}$$

Using the properties :-

$$a^n u[n] \doteq \frac{1}{1-ae^{-j\omega}} \quad \& \quad \frac{1}{(1-ae^{-j\omega})^2} = (n+1)a^n u$$

The inverse fourier transform $y[n]$ is :-

$$y[n] = \left[-4\left(\frac{1}{4}\right)^n - 2(n+1)\left(\frac{1}{4}\right)^n + 8\left(\frac{1}{2}\right)^n\right] u[n].$$

PROBLEMS:-

PROBLEM # 1:-

$$x[n] = \delta[n] + 2\delta[n-1] + 4\delta[n-2]$$
$$h[n] = \delta[n] + \delta[n-1]$$

SOL:-

The DTFT of $x[n]$ is:

$$X(e^{j\omega}) = 1 + 2e^{-j\omega} + 4e^{-2j\omega}$$

The Frequency response is :

$$H(e^{j\omega}) = 1 + e^{-j\omega}$$

Therefore, the output is :

$$\begin{aligned} Y(e^{j\omega}) &= H(e^{j\omega})\, X(e^{j\omega}) \\ &= (1+e^{-j\omega})(1+2e^{-j\omega}+4e^{-j2\omega}) \\ &= 1 + 2e^{-j\omega} + 4e^{-j2\omega} + e^{-j\omega} + 2e^{-2j\omega} + 4e^{-j3\omega} \end{aligned}$$

$$Y(e^{j\omega}) = 1 + 3e^{-j\omega} + 6e^{-j2\omega} + 4e^{-j3\omega}.$$

The inverse transform is :

$$y[n] \Rightarrow \delta[n] + 3\delta[n-1] + 6\delta[n-2] + 4\delta[n-3]$$

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$n = n+1$
$1 = n+1$
$n=0$
$n+1-1$

PROBLEM #2 :-

a) $\left[\frac{1}{2}\right]^{|n-1|}$

SOL:-

Using analysis equation : $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|n-1|} e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-(n-1)} e^{-j\omega n} + \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (1/2)^{(n+1)} e^{j\omega n} + \sum_{n=0}^{\infty} (1/2)^{n} e^{-j\omega(n+1)}$$

$$= \left(\frac{1}{2}\right) \frac{1}{1-(1/2)e^{j\omega}} + e^{-j\omega} \frac{1}{1-(1/2)e^{-j\omega}}$$

$$= \frac{(1/2)(1-1/2\, e^{-j\omega}) + e^{-j\omega}(1-1/2\, e^{j\omega})}{(1-\frac{1}{2}e^{j\omega})(1-\frac{1}{2}e^{-j\omega})}$$

$$= \frac{0.5 - 0.25e^{-j\omega} + e^{-j\omega} - 0.5e^{-j\omega+j\omega}}{1-0.5e^{-j\omega} - 0.5e^{j\omega} + 0.25e^{j\omega-j\omega}}$$

$$= \frac{\cancel{0.5} - 0.25e^{-j\omega} + e^{-j\omega} - \cancel{0.5}}{1-0.5e^{-j\omega} - 0.5e^{j\omega} + 0.25}$$

$\therefore\ -\frac{1}{2}e^{-j\omega} - \frac{1}{2}e^{j\omega} \Rightarrow -\left[\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right] \Rightarrow -\cos\omega$

$$X(e^{j\omega}) \Rightarrow \frac{0.75e^{-j\omega}}{1.25 - \cos\omega}$$

$\longleftrightarrow$

b) $\delta[n+2] - \delta[n-2]$

Sol:-

$$x[n] = \delta[n+2] - \delta[n-2]$$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} [\delta(n+2) - \delta(n-2)] e^{-jwn}$$

$$X(e^{jw}) = e^{2jw} - e^{-2jw} \Rightarrow 2j\sin(2w)$$

c) $\sin\left(\frac{\pi}{2}n\right) + \cos(n)$ (in detail at the end)

Sol:-

$$x[n] = \sin\left(\frac{\pi}{2}n\right) + \cos(n)$$

$$= \frac{1}{2j}\left[e^{j\pi/2n} - e^{-j\pi/2n}\right] + \frac{1}{2}\left[e^{jn} + e^{-jn}\right]$$

Therefore,

$$X(e^{jw}) = \frac{\pi}{j}\left[\delta(w - \pi/2) - \delta(w + \pi/2)\right] + \pi\left[\delta(w-1) + \delta(w+1)\right],$$

in $0 \le w < \pi$

PROBLEM #3:-

b) $X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega$

SOL:-

$$X(e^{j\omega}) = \cos^2\omega + \sin^2 3\omega$$
$$= \frac{1+\cos(2\omega)}{2} + \frac{1-\cos(6\omega)}{2}$$

$$\therefore \cos^2 x = \frac{1+\cos(2x)}{2}, \quad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$X(e^{j\omega}) = \frac{1}{2} + \frac{\cos(2\omega)}{2} + \frac{1}{2} - \frac{\cos(6\omega)}{2}$$
$$= \frac{1}{2} + \left(\frac{e^{j2\omega} + e^{-j2\omega}}{4}\right) + \frac{1}{2} - \left(\frac{e^{j6\omega} + e^{-j6\omega}}{4}\right)$$
$$= 1 + \frac{e^{j2\omega}}{4} + \frac{e^{-j2\omega}}{4} - \frac{e^{j6\omega}}{4} - \frac{e^{-j6\omega}}{4}$$

Inverse Fourier transform $x[n]$ is:

$$x[n] = \delta[n] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n+2] - \frac{1}{4}\delta[n-6] - \frac{1}{4}\delta[n+6]$$

PROBLEM #4:-

$$h_1[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$H(e^{j\omega}) = \frac{-12+5e^{-j\omega}}{12-7e^{-j\omega}+e^{-2j\omega}} \qquad h_2[n] = ?$$

SOL:-

When two LTI systems are connected in parallel, the impulse response, of the overall systems is the sum of the impulse responses of the individual systems. Therefore,

$$h[n] = h_1[n] + h_2[n]$$

Using the linearity property, we have

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

Now, $h_1[n] = \left(\frac{1}{3}\right)^n u[n]$ then

$$H_1(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$H_2(e^{j\omega}) = H(e^{j\omega}) - H_1(e^{j\omega})$$

$$= \frac{-12+5e^{-j\omega}}{\underbrace{3(e^{-j\omega}-3)(e^{-j\omega}-4)}_{a}} - \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$a \Rightarrow \frac{-12+5e^{-j\omega}}{(e^{-j\omega}-3)(e^{-j\omega}-4)} = \frac{A}{e^{-j\omega}-3} + \frac{B}{e^{-j\omega}-4}$$

Cross multiplication gives,

$$-12+5e^{-j\omega} = A(e^{-j\omega}-4) + B(e^{-j\omega}-3)$$

Let $e^{-j\omega} = 4$, $e^{-j\omega} = 3$

when $e^{-j\omega} = 4$

$$-12+5(4) = A(4-4) + B(4-3)$$

$$8 = B(1) \Rightarrow B = 8$$

when $e^{-j\omega} = 3$

$$-12+5(3) = A(3-4) + B(3-3)$$

$$3 = A(-1) \Rightarrow A = -3$$

$$a \Rightarrow \frac{-3}{e^{-j\omega}-3} + \frac{8}{e^{-j\omega}-4}$$

$$H_2(e^{j\omega}) = \frac{-3}{e^{-j\omega}-3} + \frac{8}{e^{-j\omega}-4} - \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$= \frac{-3}{e^{-j\omega}-3} - \frac{1}{\frac{3-e^{-j\omega}}{3}} + \frac{8}{e^{-j\omega}-4}$$

$$= \frac{-3}{e^{-j\omega}-3} - \frac{3}{3-e^{-j\omega}} + \frac{8}{e^{-j\omega}-4}$$

$$= \cancel{\frac{3}{3-e^{-j\omega}}} - \cancel{\frac{3}{3-e^{-j\omega}}} + \frac{8}{e^{-j\omega}-4}$$

$$= \frac{8}{e^{-j\omega}-4}$$ (taking 4 common from numerator & denominator)

$$= \frac{\cancel{4}}{\cancel{4}}\left(\frac{2}{\frac{e^{-j\omega}}{4}-1}\right)$$ (Multiplying dividing with "-")

$$H_2(e^{j\omega}) = \frac{-2}{1-\frac{1}{4}e^{-j\omega}}$$

Taking inverse Fourier transform,

$$h_2[n] = -2\left(\frac{1}{4}\right)^n u[n]$$

PROBLEM#5:-

$$y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$$

SOL:-

a) $H(e^{j\omega}) = ?$

$$Y(e^{j\omega}) - \frac{1}{6}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{6}e^{-j2\omega}Y(e^{-2j\omega}) = X(e^{j\omega})$$

$$Y(e^{j\omega})\left[1-\frac{1}{6}e^{-j\omega} - \frac{1}{6}e^{-2j\omega}\right] = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \Rightarrow \frac{1}{1-\frac{1}{6}e^{-j\omega}-\frac{1}{6}e^{-2j\omega}}$$

$$-\frac{1}{2}+\frac{1}{3} \Rightarrow \frac{-3+2}{6} \Rightarrow -\frac{1}{6}$$

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$$-\frac{1}{2}e^{j\omega} = -1$$
$$e^{-j\omega} = 2$$

$$H(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1+\frac{1}{3}e^{-j\omega}\right)}$$

b) $h[n] = ?$

Using Patial Fractions:

$$H(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1+\frac{1}{3}e^{-j\omega}\right)} = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1+\frac{1}{3}e^{-j\omega}}$$

Cross multiplication we get:

$$\frac{1}{1} = A\left(1+\frac{1}{3}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right)$$

let $e^{-j\omega} = -3$, $e^{-j\omega} = 2$

when $e^{j\omega} = 3$

$$1 = A\left[1+\frac{1}{3}(-3)\right] + B\left[1-\frac{1}{2}(-3)\right]$$

$$1 = A(0) + B\left[1-\frac{(-3)}{2}\right]$$

$$1 = B\left[\frac{2+3}{2}\right]$$

$$B\left[\frac{5}{2}\right] = 1 \Rightarrow B = 2/5$$

when $e^{-j\omega} = 2$

$$1 = A\left[1+\frac{1}{3}(2)\right] + B\left[1-\frac{1}{2}(2)\right]$$

$$1 = A\left[\frac{3+2}{3}\right]$$

$$A = 3/5$$

$$H(e^{j\omega}) = \frac{3/5}{1-\frac{1}{2}e^{-j\omega}} + \frac{2/5}{1+\frac{1}{3}e^{-j\omega}}$$

taking the inverse Fourier transform:-

$$h[n] = \frac{3}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(\frac{-1}{3}\right)^n u[n]$$

PROBLEM #2:-

c) $\sin\left(\frac{\pi}{2}n\right) + \cos(n)$

SOL:-

$$x[n] = \sin\left[\frac{\pi}{2}n\right] + \cos[n]$$

$$= \frac{1}{2j}\left[\underset{\downarrow\ \omega_0 = \frac{\pi}{2}}{e^{j\pi/2 n} - e^{-j\pi/2 n}}\right] + \frac{1}{2}\left[\underset{\downarrow\ \omega_0 = 1}{e^{jn} + e^{-jn}}\right]$$

$$e^{j\omega_0 n} \overset{DTFT}{\longleftrightarrow} \sum_{l=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 - 2\pi l)$$

Now if $0 \leq \omega < \pi$ then i.e is periodic. (repeats periodically)

$$X(e^{j\omega}) = \frac{1}{2j}\left[2\pi\left\{\delta\left(\omega - \frac{\pi}{2}\right) - \delta\left(\omega + \frac{\pi}{2}\right)\right\}\right] + \frac{1}{2}\left[2\pi\left\{\delta(\omega - 1) + \delta(\omega + 1)\right.\right.$$

$$= \frac{\cancel{2}\pi}{\cancel{2}j}\left[\delta\left(\omega - \frac{\pi}{2}\right) - \delta\left(\omega + \frac{\pi}{2}\right)\right] + \frac{\cancel{2}\pi}{\cancel{2}}\left[\delta(\omega - 1) + \delta(\omega + 1)\right.$$

$$X(e^{j\omega}) \Rightarrow \frac{\pi}{j}\left[\delta\left(\omega - \frac{\pi}{2}\right) - \delta\left(\omega + \frac{\pi}{2}\right)\right] + \pi\left[\delta(\omega - 1) + \delta(\omega + 1)\right]$$

PROBLEM #3:-

a) $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}\left\{2\pi\delta(\omega - 2\pi k) + \pi\delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) + \pi\delta\left(\omega + \frac{\pi}{2} - 2\pi k\right)\right.$

SOL:-

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[2\pi\delta(\omega - 2\pi k) + \pi\delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) + \pi\delta\left(\omega + \frac{\pi}{2} - 2\pi k\right)\right]e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega)e^{j\omega n}\,d\omega + \frac{\pi}{2\pi}\int_{-\pi}^{\pi}\delta\left(\omega - \frac{\pi}{2}\right)e^{j\omega n}\,d\omega + \frac{\pi}{2\pi}\int_{-\pi}^{\pi}\delta\left(\omega + \frac{\pi}{2}\right.$$

$$= \int_{-\pi}^{\pi} \delta(\omega) e^{j\omega n}\, d\omega + \frac{1}{2}\int_{-\pi}^{\pi} \delta\left(\omega - \frac{\pi}{2}\right) e^{j\omega n}\, d\omega + \frac{1}{2}\int_{-\pi}^{\pi} \delta\left(\omega + \frac{\pi}{2}\right) e^{j\omega n}\, d\omega$$

$\omega_0 = 0$ $\quad$ $\omega_0 = \frac{\pi}{2}$ $\quad$ $\omega_0 = -\pi/2$

$$\therefore\; e^{j\omega_0 n} \overset{DTFT}{\longleftrightarrow} \sum_{l=-\infty}^{\infty} 2\pi\,\delta(\omega - \omega_0 2\pi l)$$

$$x[n] = e^{j0} + \frac{1}{2} e^{j(\pi/2)n} + \frac{1}{2} e^{-j(\pi/2)n}$$

$$x[n] = 1 + \frac{1}{2}\left[e^{j(\pi/2)n} + e^{-j(\pi/2)n}\right] \Rightarrow 1 + \cos\left(\frac{\pi}{2} n\right)$$