# Lecture Notes
# 19th December 2016

Date: Monday / 19th Dec. 16 — SAMPLING

⇒ Nyquist Rate :-

The minimum rate at which a signal can be sampled without introducing errors, which is twice the highest frequency present in the signal i.e $F_N = 2F_{max}$.

EXAMPLE #1:-

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t + 3\cos 100\pi t$$

$$F_N = \text{Nyquist rate} = ?$$

SOL:-

$$\omega = 2\pi F$$

$$F_1 = \frac{\omega}{2\pi} \Rightarrow \frac{50\pi}{2\pi} \Rightarrow 25Hz, \quad F_2 = \frac{300\pi}{2\pi} \Rightarrow 150Hz$$

$$F_3 = \frac{100\pi}{2\pi} \Rightarrow 50Hz$$

$$F_{max} = F_2 = 150\,Hz$$

Nyquist rate $\Rightarrow F_N = 2F_{max} \Rightarrow 300Hz$

EXAMPLE #2:-

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t - 10\cos 12000\pi t$$

1) What is the Nyquist rate for this signal?

SOL:-

$$F_1 = \frac{2k\pi}{2\pi} \Rightarrow 1KHz, \quad F_2 = \frac{6k\pi}{2\pi} \Rightarrow 3kHz, \quad F_3 = \frac{12k\pi}{2\pi} \Rightarrow 6kHz$$

The Nyquist rate is $F_N = 2F_{max} = 2 \times 6kHz \Rightarrow 12KHz$.

2) Using a sampling rate $F_s = 5000$ samples/s. What is the discrete-time signal obtained after sampling?

SOL:-

$$F_s = 5kHz$$

$$x[n] = x_a(nT) = x_a(n/F_s)$$
$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin\left(\frac{3}{5}\right)2\pi n - 10\cos 2\pi\left(\frac{6}{5}\right)n$$

$$= 3\cos 2\pi\left(\frac{1}{5}\right)n + 5\sin 2\pi\left(1-\frac{2}{5}\right)n - 10\cos 2\pi\left(1+\frac{1}{5}\right)n$$
$$= 3\cos 2\pi\left(\frac{1}{5}\right)n - 5\sin 2\pi\left(\frac{2}{5}\right)n - 10\cos 2\pi\left(\frac{1}{5}\right)n$$
$$x[n] = -7\cos 2\pi\left(\frac{1}{5}\right)n - 5\sin 2\pi\left(\frac{2}{5}\right)n$$

$F_S = 5kHz$

$$F_{max} = \frac{5k}{2} \Rightarrow 2.5kHz$$

Hence, $F_1 = 1kHz$ is not effected by aliasing

$F_2 = 3kHz$ is changed by the aliasing effect $F_2' = F_2 - F_S$
$= 3 - 5 \Rightarrow -2kHz$

$F_3 = 6kHz$ is changed by the aliasing effect $F_3' = F_3 - F_S$
$= 6k - 5k \Rightarrow 1kHz$

so that the normalize frequencies are: $f_1 = \frac{1}{5}$, $f_2 = \frac{2}{5}$, $f_3 = \frac{1}{5}$

3) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Sol:-

The analog signal that we can recover is:-

$y_a(t) = -7\cos 2000\pi t - 5\sin 4000\pi t$

which is different than the original signal $x_a(t)$.

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$F_s = 5kHz$

$F_1 = f_1 F_s = \frac{1}{5} \times 5\,kHz \Rightarrow 1kHz$

$F_2 = f_2 F_s = \frac{2}{5} \times 5\,kHz \Rightarrow 2\,kHz$