Lecture Notes
26th December 2016

Date MONDAY/26DEC16 LECTURE #14

PRACTICE PROBLEMS:-

"DTFT"

PROBLEM #1:-

a) $x[n] = \delta[n+2] - \delta[n-2]$

SOL:-

Using the Fourier transform analysis equation, the Fourier transform of this signal is:-

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} [\delta[n+2] - \delta[n-2]]\, e^{-j\omega n}$$

$$= e^{2j\omega} - e^{-2j\omega}$$

Using Euler's identity :- $\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$

$$X(e^{j\omega}) = 2j\left[\frac{e^{2j\omega} - e^{-2j\omega}}{2j}\right] \Rightarrow 2j\sin(2\omega)$$

b) $x[n] = \sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$, $-\pi \leq \omega < \pi$

SOL:-

The given signal is periodic so, it has a Fourier transform :

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta\left(\omega - \frac{2\pi k}{N}\right)$$

The fundamental period of given signal is:-

$$\frac{N}{m} = \frac{2\pi}{\omega} = \frac{2\pi \times 3}{\pi} \Rightarrow 6$$

$$N \Rightarrow 6.$$

The signal may be written as:-

$$x[n] = \frac{1}{2j}e^{j(\pi/3 n + \pi/4)} - \frac{1}{2j}e^{-j(\pi/3 n + \pi/4)}$$

$$= \frac{1}{2j}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{3}n} - \frac{1}{2j}e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{3}n}$$

From this, we obtain the non-zero Fourier series coefficients a_k of $x[n]$ in the range $-2 \le k \le 3$ as

$$a_1 = \left(\frac{1}{2j}\right) e^{j\frac{\pi}{4}} \quad , \quad a_{-1} = -\left(\frac{1}{2j}\right) e^{-j\frac{\pi}{4}}$$

Therefore in the range $-\pi \le \omega \le \pi$ we obtain,

$$X(e^{j\omega}) = 2\pi a_1 \delta\left(\omega - \frac{2\pi}{6}\right) + 2\pi a_{-1} \delta\left(\omega + \frac{2\pi}{6}\right)$$

$$\Rightarrow \cancel{2}\pi \left(\frac{1}{\cancel{2}j} e^{j\pi/4}\right) \delta(\omega - 2\pi/6) + \cancel{2}\pi \left(-\frac{1}{\cancel{2}j} e^{-j\pi/4}\right) \delta(\omega + 2\pi/6)$$

$$X(e^{j\omega}) \Rightarrow \frac{\pi}{j} e^{j\frac{\pi}{4}} \delta(\omega - 2\pi/6) - \frac{\pi}{j} e^{-j\pi/4} \delta(\omega + 2\pi/6)$$

$\longleftrightarrow$

c) $x[n] = (a^n \sin \omega_0 n) u[n]$

Sol:-

Using modulation property:

$$\sin \omega_0 n \overset{DTFT}{\longleftrightarrow} \frac{2\pi}{2j} \left[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right]$$

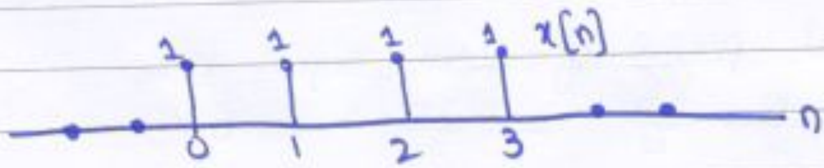
periodically repeated then:

$$X(e^{j\omega}) = \frac{1}{2j} \left[\frac{1}{1 - a e^{-j(\omega - \omega_0)}} - \frac{1}{1 - a e^{-j(\omega + \omega_0)}}\right]$$

periodically repeated.

$\longleftrightarrow$

d) $x[n]$:



Sol:-

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{3} e^{-j\omega n} = \sum_{n=0}^{3} \left(e^{-j\omega}\right)^n$$

Using the identity $\sum_{n=0}^{N-1} a^n \Rightarrow \frac{1 - a^N}{1 - a}$

$$X(e^{j\omega}) \Rightarrow \frac{1-e^{-j4\omega}}{1-e^{-j\omega}}$$

→ Alternatively, we can use the fact that $x[n] = u[n] - u[n-4]$, so

$$X(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} - \frac{e^{-j\omega 4}}{1-e^{-j\omega}} \Rightarrow \frac{1-e^{-j4\omega}}{1-e^{-j\omega}}$$

PROBLEM # 2 :-

a) $$X(e^{j\omega}) = \begin{cases} 1 & , \pi/4 \le |\omega| \le 3\pi/4 \\ 0 & , 3\pi/4 \le |\omega| \le \pi , 0 \le |\omega| < \pi/4 \end{cases}$$

SOL:-

Using Fourier transform synthesis equation:

$$x[n] = \frac{1}{2\pi}\int_{-3\pi/4}^{-\pi/4}(1)e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\pi/4}^{3\pi/4}(1)e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-3\pi/4}^{-\pi/4} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\pi/4}^{3\pi/4}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\frac{\pi}{4}n} - e^{-j\frac{3\pi}{4}n}}{jn}\right] + \frac{1}{2\pi}\left[\frac{e^{j\frac{3\pi}{4}n} - e^{j\frac{\pi}{4}n}}{jn}\right]$$

$$= \frac{1}{2\pi n}\left[\frac{e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n} + e^{-j\frac{\pi}{4}n} - e^{j\frac{\pi}{4}n}}{j}\right]$$

$$= \frac{1}{\pi n}\left(\frac{e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}}{2j} - \frac{\left[e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right]}{2j}\right)$$ ∴ using euler's identity

$$x[n] \Rightarrow \frac{1}{\pi n}\left[\sin(3\pi n/4) - \sin(n\pi/4)\right]$$

$\longleftrightarrow$

b) $$X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

$1+\frac{1}{4}e^{-j\omega} = 0$

$\frac{1}{4}e^{-j\omega} = -1$

$e^{-j\omega} = -4$

Sol:-

Factoring denominator & expanding ~~equ~~ given transform using Partial fractions gives,

$$X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1+\frac{1}{4}e^{-j\omega}\right)} = \frac{A}{1-\frac{1}{2}e^{-j\omega}} + \frac{B}{1+\frac{1}{4}e^{-j\omega}} \to (1)$$

cross multiplication gives,

$$1-\frac{1}{3}e^{-j\omega} = A\left(1+\frac{1}{4}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right)$$

let $e^{-j\omega} = -4$

$$1-\frac{1}{3}(-4) = A\left[1+\frac{1}{4}(-4)\right] + B\left[1-\frac{1}{2}(-4)\right]$$

$$\cancel{1+2 = A(0)+B(1+2)} \qquad 1+\frac{4}{3} = A(0)+B(1+2)$$

$$\cancel{3 = B(3) \Rightarrow B = 1} \qquad \frac{3+4}{3} = B(3) \Rightarrow B = \frac{7}{9}$$

let $e^{-j\omega} = 2$

$$1-\frac{1}{3}(2) = A\left[1+\frac{1}{4}(2)\right] + B\left[1-\frac{1}{2}(2)\right]$$

$$1-\frac{2}{3} = A\left[1+\frac{1}{2}\right] + B(0)$$

$\frac{1}{3}\times\frac{2}{3}$

$$\frac{3-2}{3} = A\left[\frac{2+1}{2}\right] \Rightarrow \frac{1}{3} = A\left(\frac{3}{2}\right)$$

$$A \Rightarrow \frac{2}{9}$$

putting A & B in equ (1).

$$X(e^{j\omega}) = \frac{2/9}{1-\frac{1}{2}e^{-j\omega}} + \frac{7/9}{1+\frac{1}{4}e^{-j\omega}}$$

using transform pair $a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\omega}}$ gives,

$$x[n] = \frac{2}{9}\left(\frac{1}{2}\right)^n u[n] + \frac{7}{9}\left(-\frac{1}{4}\right)^n u[n] \quad \text{ans}$$

PROBLEM #3:-

$$\left(\frac{4}{5}\right)^n u[n] \rightarrow n\left(\frac{4}{5}\right)^n u[n] \Rightarrow \text{system S.}$$

a) $H(e^{j\omega}) = ?$

SOL:-

→ Since the LTI system is causal and stable, a single input-output pair is sufficient to determine the frequency response of the system.

→ In this case the input is $x[n] = \left(\frac{4}{5}\right)^n u[n]$ and the output is $y[n] = n\left(\frac{4}{5}\right)^n u[n]$.

→ The frequency response is given by $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$

→ Hence using transform pair $a^n u[n] \overset{DTFT}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}$

$$x[n] = \left(\frac{4}{5}\right)^n u[n] \overset{FT}{\longleftrightarrow} X(e^{j\omega}) = \frac{1}{1-\frac{4}{5}e^{-j\omega}}$$

→ Using the differentiation in frequency property :-

$$y[n] = n\left(\frac{4}{5}\right)^n u[n] \overset{DTFT}{\longleftrightarrow} Y(e^{j\omega}) = j\frac{dX(e^{j\omega})}{d\omega} \Rightarrow \frac{(4/5)e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2}$$

Therefore,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{(4/5)e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^2} \Big/ \left(1-\frac{4}{5}e^{-j\omega}\right)$$

$$= \frac{(4/5)e^{-j\omega}}{\left(1-\frac{4}{5}e^{-j\omega}\right)^{\cancel{2}}} \times 1 - \cancel{\frac{4}{5}e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{(4/5)e^{-j\omega}}{1-\frac{4}{5}e^{-j\omega}}$$

b) Difference Equation = ?

SOL:-

Since $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$ we may write above equation as:

$$Y(e^{j\omega})\left[1-\frac{4}{5}e^{-j\omega}\right] = X(e^{j\omega})\left((4/5)e^{-j\omega}\right)$$

Taking inverse Fourier transform of both sides:-

$$y[n]-\frac{4}{5}\,y[n-1] = \frac{4}{5}\,x[n]$$

PROBLEM # 4:-

a) $y[n] = ?$ $\quad h[n] = \left(\frac{1}{2}\right)^n u[n]$ $\quad , x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n]$

SOL:-

$$Y(e^{j\omega}) = H(e^{j\omega})\,X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \qquad \left(\because a^n u[n] \overset{\text{DTFT}}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}\right)$$

$$X(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

$$Y(e^{j\omega}) = \left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]\left[\frac{1}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}\right]$$

Expanding through partial fraction we get

$$Y(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}} - \frac{3}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2}$$

Taking Fourier transform, we obtain.

$$y[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n] - 3(n+1)\left(\frac{1}{4}\right)^n u[n]$$

b) $y[n] = x[n] * h[n] \Rightarrow ?$

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j\omega 3}$$
$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-j2\omega} + e^{-j4\omega}$$

SOL:-

$$Y(e^{j\omega}) = X(e^{j\omega})\,H(e^{j\omega})$$
$$= \left(3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j\omega 3}\right)\left(-e^{j\omega} + 2e^{-j2\omega} + e^{-j4\omega}\right)$$
$$= -3e^{j2\omega} + 6e^{-j\omega} + 3e^{-j3\omega} - e^{+j\omega} + 2\cancel{e^{-j2\omega}} + e^{-j4\omega} - e^{0} - 2e^{-j\omega} - 4\cancel{e^{-j3\omega}}$$
$$- 2\cancel{e^{-j\omega 2}} + 4e^{-j5\omega} + 2e^{-j7\omega}$$

$$Y(e^{j\omega}) = -3e^{j2\omega} - 4e^{-j\omega} + 3e^{-j3\omega} - e^{j\omega} + e^{-j4\omega} + 2e^{-j7\omega}$$

Therefore,

$$y[n] = -3\,\delta[n+2] - 4\,\delta[n-1] + 3\,\delta[n-3] - \delta[n+1] + \delta[n-4] + 2\,\delta[n-7]$$

"SAMPLING"

PROBLEM #6:-

ω_N = Nyquist rate = ?

a) $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$

SOL:-

maximum frequency $= \omega_{max} = 4000\pi$

Therefore, the Nyquist rate for this signal is $\omega_N = 2(4000\pi)$

$\omega_N \Rightarrow 8000\pi$.

b) $x(t) = \dfrac{\sin(4000\pi t)}{\pi t}$

SOL:-

$\Rightarrow$ We know that, $X(j\omega)$ is a rectangular pulse for which $X(j\omega) = 0$ for $|\omega| > 4000\pi$.

Therefore, the Nyquist rate $\omega_N = 2(4000\pi) \Rightarrow 8000\pi$

"Z - TRANSFORM"

PROBLEM #7:-

$$x[n] = \left(\frac{1}{5}\right)^n u[n-3]$$

$X(z) = ?$ ROC $= ?$

SOL:-

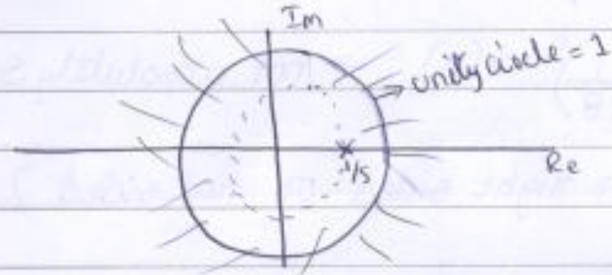
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$= \sum_{n-\infty}^{\infty} \left[\left(\frac{1}{5}\right)^n u[n-3]\right] z^{-n}$$

$$= \sum_{n=3}^{\infty} \left(\frac{1}{5}\right)^n z^{-n}$$

$$= \left(\frac{z^{-1}}{5}\right)^3 \sum_{n=0}^{\infty} \left(\frac{1}{5} z^{-1}\right)^n$$

$$X(z) = \left(\frac{z^{-3}}{125}\right)\left[\frac{1}{1-\frac{1}{5}z^{-1}}\right], \quad |z| > \frac{1}{5}$$



PROBLEM #8 :-

$$x[n] = \begin{cases} (1/3)^n \cos(\pi/4\, n) & n \le 0 \\ 0 & , n > 0 \end{cases}$$

poles & ROC => ?

Sol:-

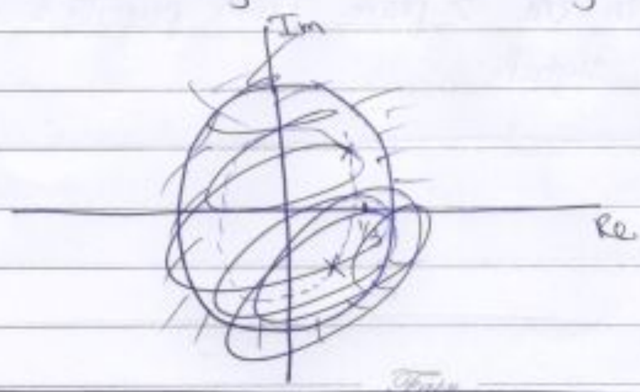
$$X(z) = \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right) z^{-n}$$

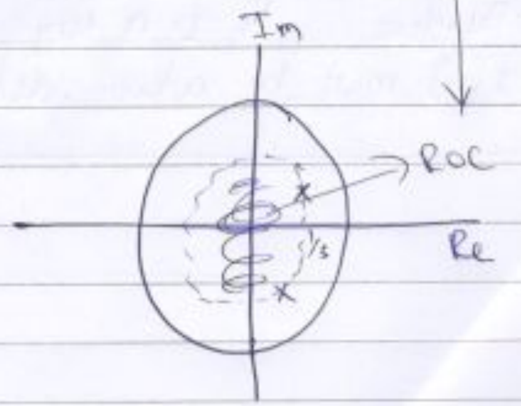
$$= \left(\frac{1}{2}\right) \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n e^{j\frac{\pi}{4}n} z^{-n} + \left(\frac{1}{2}\right) \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n e^{-j\frac{\pi}{4}n} z^{-n}$$

$$= \left(\frac{1}{2}\right) \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{-n} e^{-j\frac{\pi}{4}n} z^{n} + \left(\frac{1}{2}\right) \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{-n} e^{j\frac{\pi}{4}n} z^{n}$$

$$X(z) = \left(\frac{1}{2}\right) \frac{1}{1-3e^{-j\frac{\pi}{4}} z} + \left(\frac{1}{2}\right) \frac{1}{1-3e^{j\pi/4} z}, \quad |z| < \frac{1}{3}$$

The poles are at $z = \frac{1}{3}e^{j\frac{\pi}{4}}$ and $z = \frac{1}{3}e^{-j\frac{\pi}{4}}$





PROBLEM #9:-

$$x_1[n] = \left(\frac{1}{4}\right)^n x[n] \Rightarrow \text{absolutely summable}$$

$$x_2[n] = \left(\frac{1}{8}\right)^n x[n] \rightarrow \text{not absolutely summable}$$

$x[n]$ = left sided or right sided or two sided ?

SOL:-

if $x[n] \overset{z}{\longleftrightarrow} X(z) \quad R,$

$$\left(\frac{1}{4}\right)^n x[n] \overset{z}{\longleftrightarrow} X(4z) \quad , \frac{1}{4}R$$

$$\left(\frac{1}{8}\right)^n x[n] \overset{z}{\longleftrightarrow} X(8z) \quad , \frac{1}{8}R$$

Since $\frac{1}{4}$ includes the unit circle and $X(z)$ has a pole at $z = 1/2$, we may ~~include~~ conclude that R is definitely outside the circle with radius $\frac{1}{2}$.

→ The only question we have to answer is whether R extends to infinity outside this circle of radius 1/2.

→ Since 1/8 R does not include the unit circle, it is clear that this is not the case.

→ Therefore R is a ring in the z-plane From property 6 we know that $x[n]$ must be a two-sided signal.

PROBLEM # 10 :-

$$X(z) = \frac{1+z^{-1}}{1+\frac{1}{3}z^{-1}}$$

ROC to be $|z| > \frac{1}{3}$, $x[0] = x[1] = x[2] = ?$

SOL:-

$$\frac{2}{3}\left(1+\frac{1}{3}z^{-1}\right)$$

$$\frac{2}{3}z^{-1} + \frac{2}{9}z^{-2}$$

$$\begin{array}{r|l}
 & 1 + \frac{2}{3}z^{-1} + \frac{2}{9}z^{-2} + \cdots \\
\hline
1+\frac{1}{3}z^{-1} & 1 + z^{-1} \\
 & \ominus 1 \ominus \frac{1}{3}z^{-1} \\
\hline
 & \frac{2}{3}z^{-1} \\
 & \ominus \frac{2}{3}z^{-1} \ominus \frac{2}{9}z^{-2} \\
\hline
 & \frac{2}{9}z^{-2} \\
 & \frac{2}{9}z^{-2} + \frac{2}{27}z^{-3} \\
\hline
 & \vdots
\end{array}$$

$x[0] = 1$, $x[1] = \frac{2}{3}$, $x[2] = \frac{2}{9}$

PROBLEM # 11 :-

a) $$\frac{z-1/2}{z^2+\frac{1}{2}z-\frac{3}{16}} \qquad \frac{z-1/2}{\left(z+\frac{1}{4}\right)\left(z-\frac{3}{4}\right)}$$

SOL:-

For a system to be both causal and stable, the corresponding z-transform must not have any poles outside the unit circle.
The poles of this z-transform are at $z = \frac{1}{4}$, $z = -\frac{3}{4}$. Therefore, it is causal.

b) $$\frac{z+1}{z+\frac{4}{3}-\frac{1}{2}z^{-2}-\frac{2}{3}z^{-3}}$$

SOL:-

This z-transform has a pole at $-\frac{4}{3}$. Therefore it is not-causal.

PROBLEM #12 :-

Diagram in slides

a) Difference equation =?

SOL:-

$$H(z) = \frac{1-6z^{-1}+8z^{-2}}{1-\frac{2}{3}z^{-1}+\frac{1}{9}z^{-2}}$$

$$H(z) = \frac{Y(z)}{X(z)}$$ we may write

$$Y(z)\left[1-\frac{2}{3}z^{-1}+\frac{1}{9}z^{-2}\right] = X(z)\left[1-6z^{-1}+8z^{-2}\right]$$

Taking the inverse z-transform we obtain

$$y[n]-\frac{2}{3}y[n-1]+\frac{1}{9}y[n-2] = x[n]-6x[n-1]+8x[n-2]$$

b) Is this system stable?

SOL:-

$H(z)$ has only two poles. These are both at $z=\frac{1}{3}$. Since the system is causal, the ROC of $H(z)$ will be of the form $|z|>\frac{1}{3}$.

Since the ROC includes the unit circle, the system is stable.

PROBLEM # 13:-

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

a) $H(z) = ?$ plot poles & zeros of $H(z)$ and indicate ROC.

SOL:-

$$y[n] - y[n-1] - y[n-2] = x[n-1]$$

Taking z-transform on both sides we get

$$Y(z) - z^{-1}Y(z) - z^{-2}Y(z) = z^{-1}X(z)$$

$$Y(z)\left[1 - z^{-1} - z^{-2}\right] = z^{-1}(X(z))$$

$$H(z) = \frac{Y(z)}{X(z)} \Rightarrow \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

The poles of $H(z)$ are at:-

$$z = (1/2) \pm (\sqrt{5}/2)$$

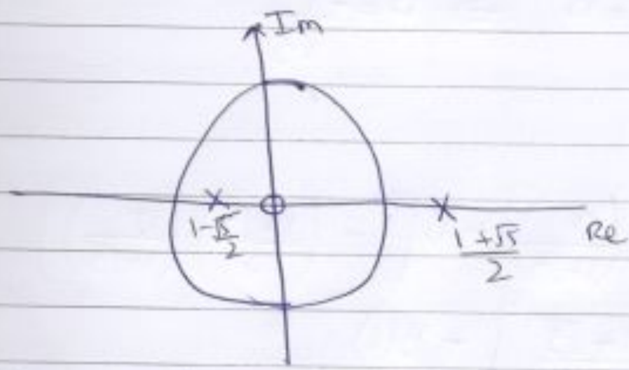
$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+1 \pm \sqrt{1 - 4(+1\times -1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+4}}{2} \Rightarrow \left(\frac{1}{2} \pm \sqrt{5}/2\right)$$

$H(z)$ has a zero at $z = 0$.

The pole-zero plot for $H(z)$ is shown below:-



Since $h[n]$ is causal, the ROC for $H(z)$ has to be $|z| > \left[(1/2) + (\sqrt{5}/2)\right]$.

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$$-\frac{1+\sqrt{5}}{2} z^{-1} = +1$$

$$z^{-1} = \frac{2}{1+\sqrt{5}}$$

b) $h[n] = ?$

Sol:-

$$H(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

Expanding through Partial fraction :-

$$H(z) = \frac{z^{-1}}{\left[1-\left(\frac{1+\sqrt{5}}{2}\right)z^{-1}\right]\left[1-\left(\frac{1-\sqrt{5}}{2}\right)z^{-1}\right]} = \frac{A}{1-\left(\frac{1+\sqrt{5}}{2}\right)z^{-1}} + \frac{B}{1-\left(\frac{1-\sqrt{5}}{2}\right)z^{-1}} \rightarrow (1)$$

Cross multiplication gives.

$$z^{-1} = A\left[1-\left(\frac{1-\sqrt{5}}{2}\right)z^{-1}\right] + B\left[1-\left(\frac{1+\sqrt{5}}{2}\right)z^{-1}\right]$$

put $z^{-1} = \frac{2}{1-\sqrt{5}}$

$$\frac{2}{1-\sqrt{5}} = A\left[1-\left(\frac{1-\sqrt{5}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{1-\sqrt{5}}}\right)\right] + B\left[1-\left(\frac{1+\sqrt{5}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{1-\sqrt{5}}\right)\right]$$

$$\frac{2}{1-\sqrt{5}} = A(0) + B\left[\frac{\cancel{1}-\sqrt{5}-\cancel{1}-\sqrt{5}}{1-\sqrt{5}}\right]$$

$$\frac{2}{1-\sqrt{5}} = B\left[\frac{-2\sqrt{5}}{1-\sqrt{5}}\right]$$

$$\frac{\cancel{2}}{\cancel{1-\sqrt{5}}} \times -\frac{\cancel{(1-\sqrt{5})}}{\cancel{2}\sqrt{5}} = B \Rightarrow B = -\frac{1}{\sqrt{5}}$$

put $z^{-1} = \frac{2}{1+\sqrt{5}}$

$$\frac{2}{1+\sqrt{5}} = A\left[1-\left(\frac{1-\sqrt{5}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{1+\sqrt{5}}\right)\right] + B\left[1-\left(\frac{\cancel{1+\sqrt{5}}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{1+\sqrt{5}}}\right)\right]$$

$$\frac{2}{1+\sqrt{5}} = A\left[\frac{\cancel{1}+\sqrt{5}-\cancel{1}+\sqrt{5}}{1+\sqrt{5}}\right] + B(0)$$

$$\frac{2}{1+\sqrt{5}} = A\left[\frac{2\sqrt{5}}{1+\sqrt{5}}\right]$$

$$\left(\frac{\cancel{2}}{\cancel{1+\sqrt{5}}}\right)\left(\frac{\cancel{1+\sqrt{5}}}{\cancel{2}\sqrt{5}}\right) = A \Rightarrow A = \frac{1}{\sqrt{5}}$$

putting A & B in equ ①.

$$H(z) = \frac{1/\sqrt{5}}{1-\left(\frac{1+\sqrt{5}}{2}\right)z^{-1}} - \frac{1/\sqrt{5}}{1-\left(\frac{1-\sqrt{5}}{2}\right)z^{-1}}$$

Taking inverse ~~[illegible]~~ Z-transform and using pair:

$$a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1-az^{-1}}$$

$$h[n] = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n u[n]$$