Signal & Systems

Practice Problems

26th December 16

Discrete-Time Fourier Transform

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Problem #1

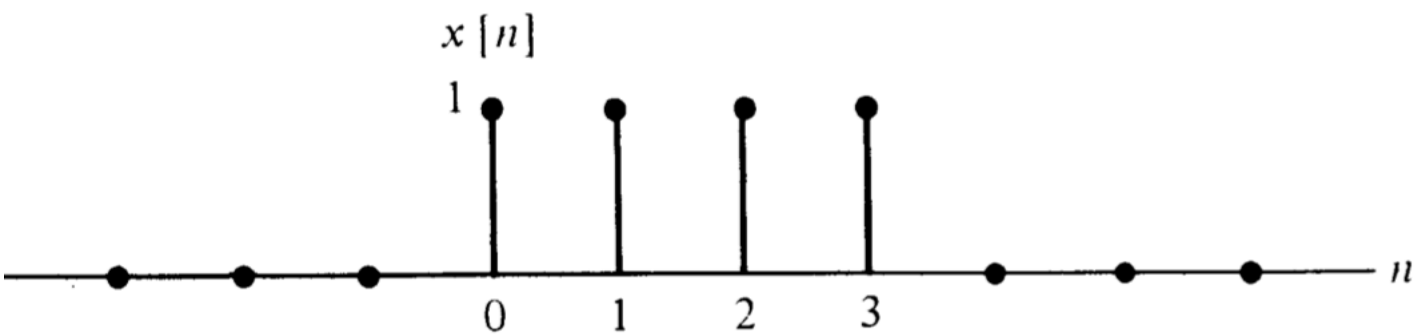
❖ Compute the Discrete-time Fourier transform of the following signals:

$$(a):\quad x[n] = \delta[n+2] - \delta[n-2]$$

$$(b):\quad x[n] = \sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right), \quad -\pi \le \omega < \pi$$

$$(c):\quad x[n] = \left(a^n \sin\omega_0 n\right)u[n]$$

$$(d):\quad x[n] \quad as \quad shown \quad below:$$



Problem #2

- Use the Fourier transform synthesis equation to determine the inverse Fourier transform of:

$$(a): \quad X\left(e^{j\omega}\right)=\begin{cases} 1, & \frac{\pi}{4} \leq |\omega| \leq \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \leq |\omega| \leq \pi, 0 \leq |\omega| < \frac{\pi}{4} \end{cases}$$

$$(b): \quad X\left(e^{j\omega}\right)=\frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-2j\omega}}$$

Problem #3

- A causal and stable LTI system S has the property that:

$$\left(\frac{4}{5}\right)^n u[n] \rightarrow n\left(\frac{4}{5}\right)^n u[n]$$

- (a): Determine the frequency response $H(e^{j\omega})$ for the system S.
- (b): Determine a difference equation relating any input x[n] and the corresponding output y[n].

Problem #4

- (a): Consider a discrete-time LTI system with impulse response:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

  - Use Fourier transform to determine the response of the following input signal:

$$x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n]$$

- (b): Let x[n] and h[n] be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j3\omega}$$

$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-j2\omega} + e^{-j4\omega}$$

  - Determine y[n] = x[n] * h[n].

Sampling

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Problem #6

- The frequency which, under the sampling theorem must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals:

$$(a): \quad x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$$

$$(b): \quad x(t) = \frac{\sin(4000\pi t)}{\pi t}$$

Z-Transform

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Problem #7

- Consider the signal:

$$x[n] = \left(\frac{1}{5}\right)^n u[n-3]$$

- Evaluate the z-transform of this signal and specify the corresponding region of convergence.

Problem #8

- Consider the signal:

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right), & n \le 0 \\ 0, & n > 0 \end{cases}$$

- Determine the poles and ROC for X(z).

Problem #9

- Let x[n] be a signal whose rational z-transform X(z) contains a pole at z=1/2. given that:

$$x_1[n] = \left(\frac{1}{4}\right)^n x[n]$$

- Is absolutely summable and

$$x_2[n] = \left(\frac{1}{8}\right)^n x[n]$$

- Is not absolutely summable, determine whether x[n] is left sided , right sided or two sided.

Problem #10

- Consider the following algebraic expression for the z-transform X(z) of a signal x[n]:

$$X(z) = \frac{1+z^{-1}}{1+\frac{1}{3}z^{-1}}$$

- Assuming the ROC to be |z|>1/3, use long division to determine the values of x[0], x[1] and x[2].

Problem #11

- Consider the following system functions for stable LTI systems. Without utilizing the inverse z-transform, determine in each case whether or not the corresponding system is causal:

$$(a): \quad \frac{z - \frac{1}{2}}{z^2 + \frac{1}{2}z - \frac{3}{16}}$$

$$(b): \quad \frac{z + 1}{z + \frac{4}{3} - \frac{1}{2}z^{-2} - \frac{2}{3}z^{-3}}$$

Problem #12

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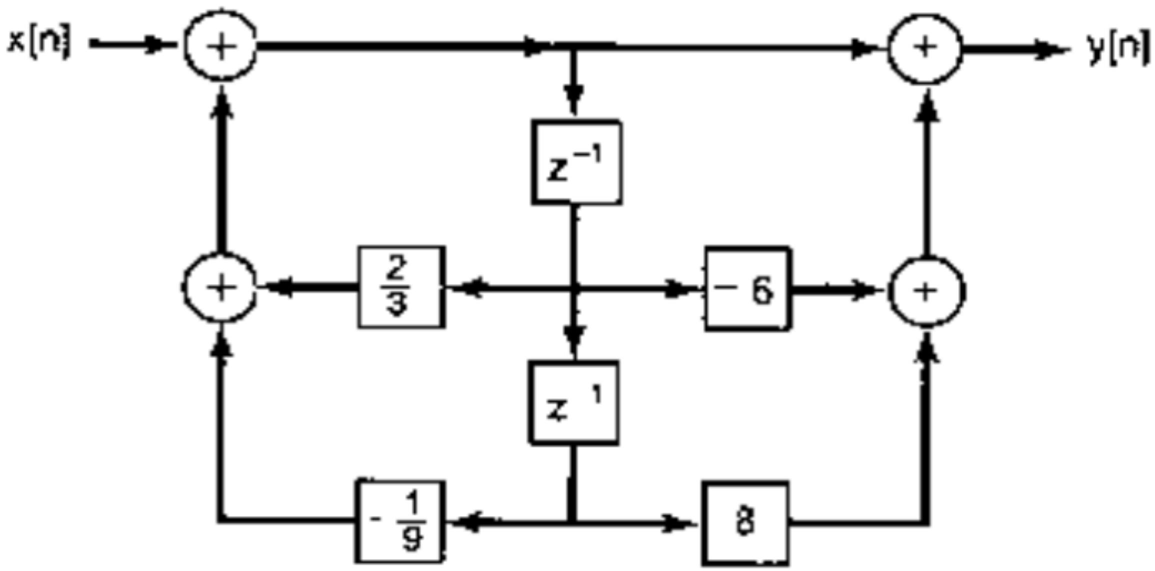
❖ Consider a causal LTI system whose input x[n] and output y[n] are related through the block diagram representation shown below:



❖ (a): Determine a difference equation relating y[n] and x[n].

❖ (b): Is this system stable?

Problem #13

- A causal LTI system is described by the difference equation:

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

- (a): Find the system function H(z) = Y(z) / X(z) for this system. Plot the poles and zeros of H(z) and indicate the ROC.
- (b): Find the unit sample response of the system.

Thankyou

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