ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2016

EL313- Signal & Systems

Assignment – 2 Solution | **Due Date: 21/12/2016**
Marks: 20 | **Handout Date: 14/12/2016**

Question # 1:

Consider a discrete-time LTI system with impulse response:

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

Use the Fourier transforms to determine the response y [n] to the given input:

$$x[n] = \left(\frac{3}{4}\right)^n u[n]$$

Solution:

Let the output of the system by y [n]. We know that:

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

Here:

$$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

And:

$$X(e^{j\omega}) = \frac{1}{1 - \frac{3}{4}e^{-j\omega}}$$

Therefore,

$$Y(e^{j\omega}) = \left[\frac{1}{1 - \frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right]$$

$$Y(e^{j\omega}) = \frac{1}{\left(1 - \frac{3}{4}e^{-j\omega}\right)\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

Using partial fractions expansion we get:

$$Y(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{3}{4}e^{-j\omega}\right)} = \frac{A}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} + \frac{B}{\left(1 - \frac{3}{4}e^{-j\omega}\right)} \rightarrow eq1$$

Cross multiplication yields:

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = 2$, gives:

$$1 = A\left(1 - \frac{3}{4}\times(2)\right) + B\left(1 - \frac{1}{2}\times(2)\right)$$

$$1 = A\left(\frac{2-3}{2}\right) + B(0)$$

$$1 = A\left(\frac{-1}{2}\right) \Rightarrow A = -2$$

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = \frac{4}{3}$, gives:

$$1 = A\left(1 - \frac{3}{4}\times\left(\frac{4}{3}\right)\right) + B\left(1 - \frac{1}{2}\times\left(\frac{4}{3}\right)\right)$$

$$1 = A(0) + B\left(\frac{3-2}{3}\right)$$

$$1 = B\left(\frac{1}{3}\right) \Rightarrow B = 3$$

Putting values of A and B in eq(1) gives:

$$Y(e^{j\omega}) = \frac{-2}{\left(1 - \frac{1}{2}e^{-j\omega}\right)} + \frac{3}{\left(1 - \frac{3}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$

Question # 2:

Use the Fourier transform analysis equation to calculate the Fourier transform of:

$$x[n] = 2\left(\frac{3}{4}\right)^n u[n]$$

Solution:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = \sum_{n=-\infty}^{\infty}\left(2\left(\frac{3}{4}\right)^n u[n]\right)e^{-j\omega n}$$

$$X(e^{j\omega}) = 2\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n e^{-j\omega n} = 2\sum_{n=0}^{\infty}\left(\frac{3}{4}e^{-j\omega}\right)^n$$

Applying the transform pair: $(a)^n u[n] \overset{DTFT}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}$

$$X(e^{j\omega}) = 2\left[\frac{1}{1 - \frac{3}{4}e^{-j\omega}}\right] \Rightarrow \frac{2}{1 - \frac{3}{4}e^{-j\omega}}$$

Question # 3:

The following is the Fourier transform of discrete-time signal. Determine the signal x [n] corresponding to the transform:

$$X(e^{j\omega}) = cos^2\,\omega + sin^2\,3\omega$$

Solution:

$$X(e^{j\omega}) = cos^2\,\omega + sin^2\,3\omega$$

$$X(e^{j\omega}) = \left(\frac{1+\cos 2\omega}{2}\right) + \left(\frac{1-\cos 2(3\omega)}{2}\right) = \left(\frac{1+\cos 2\omega}{2}\right) + \left(\frac{1-\cos 6\omega}{2}\right)$$

$$\therefore cos^2 x = \frac{1+\cos 2x}{2}\ \&\ sin^2 x = \frac{1-\cos 2x}{2}$$

$$X(e^{j\omega}) = \frac{1}{2} + \frac{\cos 2\omega}{2} + \frac{1}{2} - \frac{\cos 6\omega}{2}$$

$$= \frac{1}{2} + \left[\frac{e^{j2\omega} + e^{-j2\omega}}{4}\right] + \frac{1}{2} - \left[\frac{e^{j6\omega} + e^{-j6\omega}}{4}\right]$$

$$= 1 + \frac{e^{j2\omega}}{4} + \frac{e^{-j2\omega}}{4} - \frac{e^{j6\omega}}{4} - \frac{e^{-j6\omega}}{4}$$

Inverse Fourier transform x [n] is:

$$x[n] = \delta[n] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n+2] - \frac{1}{4}\delta[n-6] - \frac{1}{4}\delta[n-6]$$

Question # 4:

A particular LTI system is described by the difference equation:

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1]$$

Find the impulse response h [n] of the system.

Solution:

The use of the Fourier transform simplifies the analysis of the difference equation:

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1]$$

$$Y(e^{j\omega}) + \frac{1}{4}e^{-j\omega}Y(e^{j\omega}) - \frac{1}{8}e^{-2j\omega}Y(e^{j\omega}) = X(e^{j\omega}) - e^{-j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1 + \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}\right) = X(e^{j\omega})\left(1 - e^{-j\omega}\right)$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{1 + \frac{1}{4}e^{-j\omega} - \frac{1}{8}e^{-2j\omega}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{\left(1 + \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Using Partial fraction expansion, we see that:

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{\left(1 + \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{1}{4}e^{-j\omega}\right)} = \frac{A}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \frac{B}{\left(1 - \frac{1}{4}e^{-j\omega}\right)} \rightarrow eq1$$

Cross multiplication yields:

$$1 - e^{-j\omega} = A\left(1 - \frac{1}{4}e^{-j\omega}\right) + B\left(1 + \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = 4$, gives:

$$1 - 4 = A\left(1 - \frac{1}{4}\times(4)\right) + B\left(1 + \frac{1}{2}\times(4)\right)$$

$$1 - 4 = A(0) + B(1 + 2)$$

$$-3 = B(3) \Longrightarrow B = -1$$

$$1 = A\left(1 - \frac{3}{4}e^{-j\omega}\right) + B\left(1 - \frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = -2$, gives:

$$1 - (-2) = A\left(1 - \frac{1}{4}\times(-2)\right) + B\left(1 + \frac{1}{2}\times(-2)\right)$$

$$1 + 2 = A\left(\frac{2 + 1}{2}\right) + B(0)$$

$$3 = A\left(\frac{3}{2}\right) \Longrightarrow A = 2$$

Putting values of A and B in eq(1) gives:

$$H\left(e^{j\omega}\right) = \frac{2}{\left(1 + \frac{1}{2}e^{-j\omega}\right)} + \frac{-1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$h[n] = 2\left(-\frac{1}{2}\right)^{n} u[n] - \left(\frac{1}{4}\right)^{n} u[n]$$

Good Luck