**ISRA UNIVERSITY**

Islamabad Campus

**Department of Electrical Engineering**
**Program: B.E. (Electrical)**
**Semester - Fall 2016**

**EL313- Signal & Systems**

**Quiz – 4 Solution**
**Marks: 15** **Handout Date: 23/12/2016**

## Question # 1:

**a)** Consider the linear constant coefficient difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

Which describes a linear, time-invariant system initially at rest. What is the system function $H(e^{j\omega})$ that describes $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$?

**b)** Using Fourier transform, evaluate y [n] if x [n] is:

$$\left(\frac{3}{4}\right)^n u[n]$$

## Solution:

**a)** Consider the linear constant coefficient difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

Which describes a linear, time-invariant system initially at rest. What is the system function $H(e^{j\omega})$ that describes $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$?

The difference equation $y[n] - \frac{1}{2}y[n-1] = x[n]$, which is initially at rest, has a system transfer function that can be obtained by taking the Fourier transform of both sides of the equation.
This yields:

$$Y(e^{j\omega}) - \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$$

$$Y(e^{j\omega})\left(1 - \frac{1}{2}e^{-j\omega}\right) = X(e^{j\omega})$$

So,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1 - \frac{1}{2}e^{-j\omega}}$$

**b)** Using Fourier transform, evaluate y [n] if x [n] is:

$$\left(\frac{3}{4}\right)^n u[n]$$

If $x[n] = \left(\frac{3}{4}\right)^n u[n]$, then:

$$X(e^{j\omega}) = \frac{1}{1-\frac{3}{4}e^{-j\omega}}$$

$$\therefore H(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

Therefore,

$$Y(e^{j\omega}) = \left[\frac{1}{1-\frac{3}{4}e^{-j\omega}}\right]\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right]$$

$$Y(e^{j\omega}) = \frac{1}{\left(1-\frac{3}{4}e^{-j\omega}\right)\left(1-\frac{1}{2}e^{-j\omega}\right)}$$

Using partial fractions expansion we get:

$$Y(e^{j\omega}) = \frac{1}{\left(1-\frac{1}{2}e^{-j\omega}\right)\left(1-\frac{3}{4}e^{-j\omega}\right)} = \frac{A}{\left(1-\frac{1}{2}e^{-j\omega}\right)} + \frac{B}{\left(1-\frac{3}{4}e^{-j\omega}\right)} \rightarrow eq1$$

Cross multiplication yields:

$$1 = A\left(1-\frac{3}{4}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = 2$, gives:

$$1 = A\left(1-\frac{3}{4}\times(2)\right) + B\left(1-\frac{1}{2}\times(2)\right)$$

$$1 = A\left(\frac{2-3}{2}\right) + B(0)$$

$$1 = A\left(\frac{-1}{2}\right) \Rightarrow A = -2$$

$$1 = A\left(1-\frac{3}{4}e^{-j\omega}\right) + B\left(1-\frac{1}{2}e^{-j\omega}\right)$$

Putting $e^{-j\omega} = \frac{4}{3}$, gives:

$$1 = A\left(1-\frac{3}{4}\times\left(\frac{4}{3}\right)\right) + B\left(1-\frac{1}{2}\times\left(\frac{4}{3}\right)\right)$$

$$1 = A(0) + B\left(\frac{3-2}{3}\right)$$

$$1 = B\left(\frac{1}{3}\right) \Rightarrow B = 3$$

Putting values of A and B in eq(1) gives:

$$Y(e^{j\omega}) = \frac{-2}{\left(1-\frac{1}{2}e^{-j\omega}\right)} + \frac{3}{\left(1-\frac{3}{4}e^{-j\omega}\right)}$$

Taking the inverse Fourier transform, we obtain:

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$

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**Question # 2:**

Find the Fourier transform using analysis equation:

$$x[n] = \begin{cases} -1, & n = -3, -1, 1, 3 \\ 1, & n = -2, 0, 2 \\ 0, & otherwise \end{cases}$$

**Solution:**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-3}^{3} x[n]e^{-j\omega n}$$

$$= x[-3]e^{j3\omega} + x[-2]e^{j2\omega} + x[-1]e^{j\omega} + x[0]e^{0} + x[1]e^{-j\omega} + x[2]e^{-j2\omega} + x[3]e^{-j3\omega}$$

$$= (-1)e^{j3\omega} + (1)e^{j2\omega} + (-1)e^{j\omega} + (1)1 + (-1)e^{-j\omega} + (1)e^{-j2\omega} + (-1)e^{-j3\omega}$$

$$= -e^{j3\omega} + e^{j2\omega} - e^{j\omega} + 1 - e^{-j\omega} + e^{-j2\omega} - e^{-j3\omega}$$

$$= 1 - \left(e^{j3\omega} + e^{-j3\omega}\right) + \left(e^{j2\omega} + e^{-j2\omega}\right) - \left(e^{j\omega} + e^{-j\omega}\right)$$

Using Euler's identity: $\cos x = \frac{e^{jx}+e^{-jx}}{2}$ $or\ 2\cos x = e^{jx} + e^{-jx}$

$$X(e^{j\omega}) \Rightarrow 1 - 2\cos(3\omega) + 2\cos(2\omega) - 2\cos(\omega)$$

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**Good Luck**