ISRA UNIVERSITY

Islamabad Campus

Department of Electrical Engineering
Program: B.E. (Electrical)
Semester - Fall 2016

EL313- Signal & Systems

Quiz – 5 Solution
Marks: 20 **Handout Date: 04/01/2017**

Question # 1:

Using partial fraction expansion and the fact that:

$$(a)^n u[n] \leftrightarrow \frac{1}{1-az^{-1}} \ , \ |z| > |a|$$

Find the inverse z-transform of:

$$X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}} \ , \ |z| > \frac{1}{2}$$

Also, determine whether the system is causal or stable or both?

Solution:

Using partial fraction expansion:

$$X(z) = \frac{1-z^{-1}}{1-\frac{1}{4}z^{-2}}$$

$$X(z) = \frac{1-z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{A}{\left(1-\frac{1}{2}z^{-1}\right)} + \frac{B}{\left(1+\frac{1}{2}z^{-1}\right)} \rightarrow eq(1)$$

Cross multiplication gives:

$$1-z^{-1} = A\left(1+\frac{1}{2}z^{-1}\right) + B\left(1-\frac{1}{2}z^{-1}\right)$$

Putting $z^{-1} = -2$, gives:

$$1-(-2) = A\left(1+\frac{1}{2}(-2)\right) + B\left(1-\frac{1}{2}(-2)\right)$$

$$1+2 = A(0) + B(1+1)$$

$$3 = B(2) \Longrightarrow B = \frac{2}{3}$$

Putting $z^{-1} = 2$, gives:

$$1-(2) = A\left(1+\frac{1}{2}(2)\right) + B\left(1-\frac{1}{2}(2)\right)$$

$$-1 = A(1+1) + B(0)$$

$$-1 = A(2) \Rightarrow A = -\frac{1}{2}$$

Putting values of A and B in eq (1) gives:

$$X(z) = \frac{-\frac{1}{2}}{\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{\frac{3}{2}}{\left(1 + \frac{1}{2}z^{-1}\right)}$$

Since the ROC is $|z| > 1/2$,

Taking the inverse z-transform, we obtain:

$$x[n] = -\frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{3}{2}\left(-\frac{1}{2}\right)^n u[n]$$

Since the ROC is $|z| > \frac{1}{2}$, it includes the unit circle as well and both the poles are inside the unit circle so the system is both stable and causal.

Question # 2:

Consider the linear discrete-time, shift invariant system with input x [n] and output y [n] for which:

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n]$$

The system is stable. Determine the unit sample response i.e., h [n]=?

(***Hint: For a stable system ROC must include the unity circle. So, do focus on the poles and ROC for determining the unit sample response.***)

Solution:

Taking the z-transform of both sides of the given difference equation and simplifying, we get:

$$z^{-1}Y(z) - \frac{10}{3}Y(z) + z^{1}Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{z^{-1} - \frac{10}{3} + z} = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}$$

Using partial fraction expansion:

$$H(z) = \frac{z^{-1}}{1 - \frac{10}{3}z^{-1} + z^{-2}}$$

$$H(z) = \frac{z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)(1 - 3z^{-1})} = \frac{A}{\left(1 - \frac{1}{3}z^{-1}\right)} + \frac{B}{(1 - 3z^{-1})} \rightarrow eq(1)$$

Cross multiplication gives:

$$z^{-1} = A(1 - 3z^{-1}) + B\left(1 - \frac{1}{3}z^{-1}\right)$$

Putting $z^{-1} = \frac{1}{3}$, gives:

$$\frac{1}{3} = A\left(1 - 3\left(\frac{1}{3}\right)\right) + B\left(1 - \frac{1}{3}\left(\frac{1}{3}\right)\right)$$

$$\frac{1}{3} = A(0) + B\left(1 - \frac{1}{9}\right)$$
$$\frac{1}{3} = B\left(\frac{9-1}{9}\right)$$
$$\frac{1}{3} = B\left(\frac{8}{9}\right) \Longrightarrow B = \frac{1}{3} \times \frac{9}{8} = \frac{3}{8}$$

Putting $z^{-1} = 3$, gives:

$$3 = A\big(1 - 3(3)\big) + B\left(1 - \frac{1}{3}(3)\right)$$
$$3 = A(1-9) + B(0)$$
$$3 = A(-8) \Longrightarrow A = -\frac{3}{8}$$

Putting values of A and B in eq (1) gives:

$$H(z) = \frac{-\frac{3}{8}}{\left(1 - \frac{1}{3}z^{-1}\right)} + \frac{\frac{3}{8}}{(1 - 3z^{-1})}$$

Since H (z) corresponds to a stable system, the ROC has to be $(1/3) < |z| < 3$. Therefore,

$$h[n] = -\frac{3}{8}\left(\frac{1}{3}\right)^n u[n] - \frac{3}{8}(3)^n u[-n-1]$$

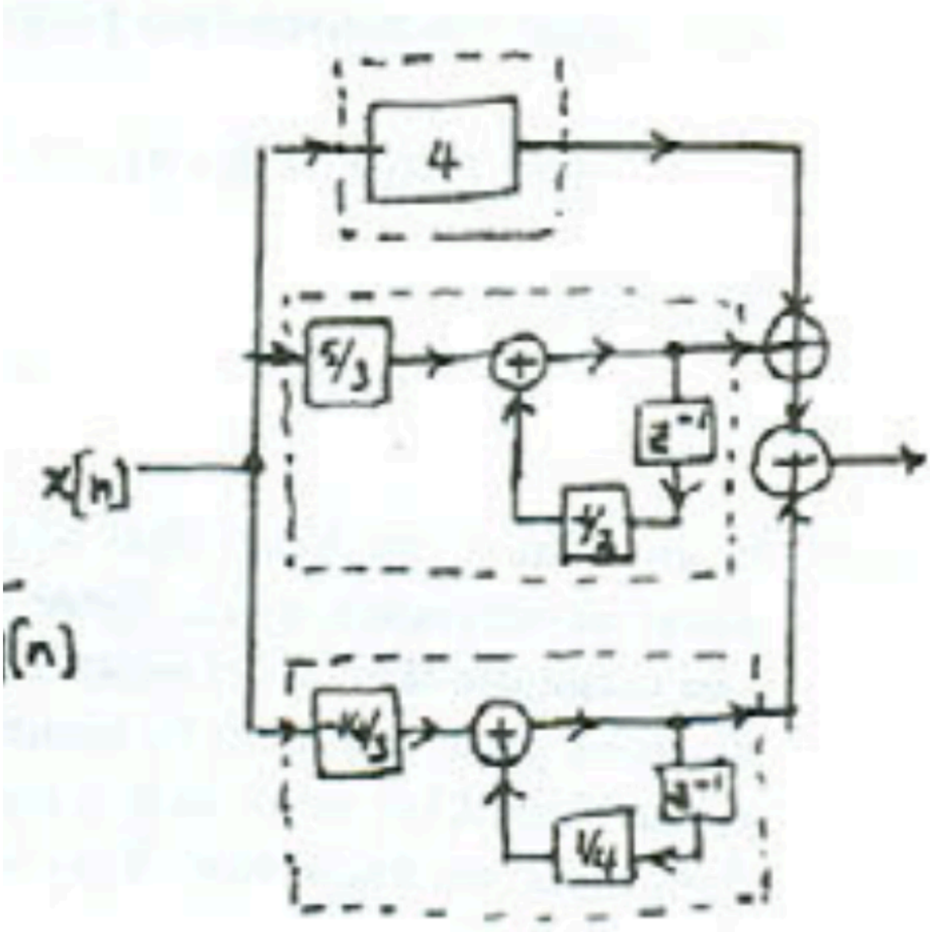
Question # 3:

Draw a parallel-form block diagram representation for S based on the observation that:

$$H(z) = 4 + \frac{\frac{5}{3}}{1 + \frac{1}{2}z^{-1}} - \frac{\frac{14}{3}}{1 - \frac{1}{4}z^{-1}}$$

Solution:

We may draw the block-diagram of $H_1(z) = 4$, $H_2(z) = \frac{\frac{5}{3}}{1+\frac{1}{2}z^{-1}}$ and $H_3(z) = -\frac{\frac{14}{3}}{1-\frac{1}{4}z^{-1}}$ as shown below. H (z) is the parallel combination of $H_1(z), H_2(z)\ and\ H_3(z)$.



Good Luck