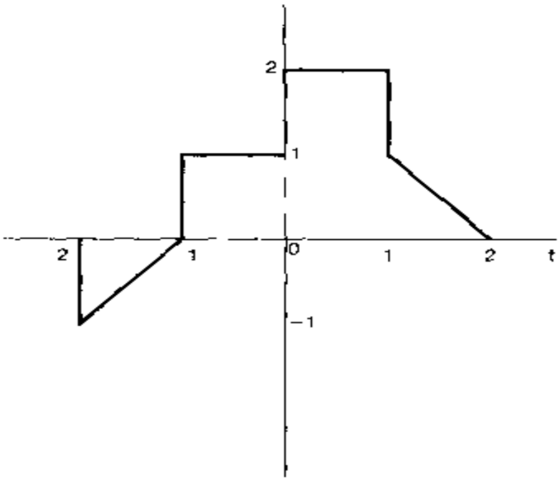
Name: **Regd. No.**

Course Title: Signal & Systems **Course Code:** EL-313

MID SEMESTER EXAMINATION – Fall 2016
Program: B.E. (Electrical)
Solution
SECTION-II: 24 MARKS
Time Allowed: 1hr 10 min

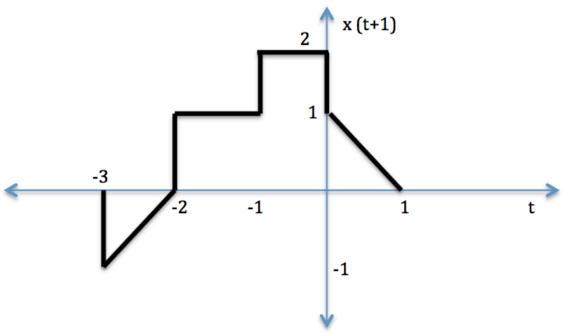
Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

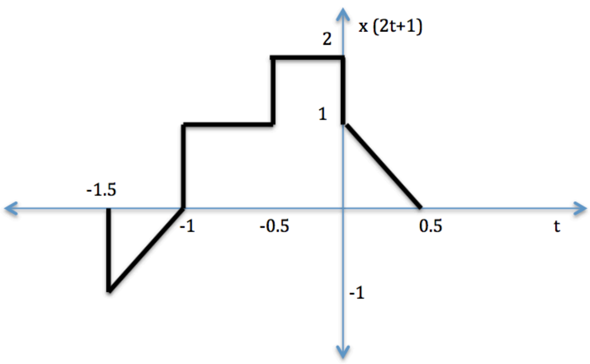
Q1. A continuous-time signal x (t) is shown in figure below. Sketch and label the signal $x(2t + 1)$:



(03 Marks)

Solution:





Q2. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period:

$$x[n] = 1 + e^{\frac{j4\pi n}{7}} - e^{\frac{j2\pi n}{5}}$$

(03 Marks)

Solution:

Step #1: Determine the fundamental period of individual signals.
Period of the first term in the RHS $N_1 = 1$
Period for the second term in the RHS is:

$$\frac{N}{m} = \frac{2\pi}{\omega_0}, where\ \omega_0 = \frac{4\pi}{7}$$

$$\frac{N}{m} = \frac{2\pi}{\frac{4\pi}{7}} \Rightarrow \frac{7}{2}$$

Where $N_2 = 7$ and $m = 2$.
Period for the third term in the RHS is:

$$\frac{N}{m} = \frac{2\pi}{\omega_0}, where\ \omega_0 = \frac{2\pi}{5}$$

$$\frac{N}{m} = \frac{2\pi}{\frac{2\pi}{5}} \Rightarrow 5$$

Where $N_3 = 5$ and $m = 1$.

Step #2: Find the ratio of fundamental period of 1st signal to fundamental period of every other signal.

$$\frac{N_1}{N_2} \Rightarrow \frac{1}{7}, \frac{N_1}{N_3} \Rightarrow \frac{1}{5}$$

Step #3: If the ratios are rational, the composite signal is periodic.
Hence above two ratios are rational so the signal x [n] is periodic.

Step #4: $N_0 = LCM(N_1, N_2, N_3)$

$$LCM = 1 \times 7 \times 5 \Rightarrow 35$$

Therefore, the overall signal x [n] is periodic with a period which is least common multiple of the period of the three terms in x [n]. This is equal to 35.

Q3. Determine whether the following signals are energy signal or power signal:

i. $x(t) = e^{-t}u(t)$
ii. $x(t) = \cos(t) + j\sin(t)$

(03 Marks)

Solution:

i. $x(t) = e^{-t}u(t)$

$$E = \int_{-\infty}^{\infty} [x(t)]^2\, dt$$

$$= \int_0^{\infty} (e^{-t})^2 dt = \int_0^{\infty} e^{-2t} dt = -\frac{e^{-2t}}{2}\bigg|_0^{\infty}$$

$$= -\frac{e^{-2(\infty)}}{2} + \frac{e^{2(0)}}{2} = \frac{1}{2} < \infty$$

Hence, x (t) is an Energy Signal and its Power is P =0.

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ii. $x(t) = \cos(t) + j\sin(t)$

$$|x(t)|^2 = |\cos(t) + j\sin(t)|^2 = \cos^2(t) + \sin^2(t) \Longrightarrow 1$$

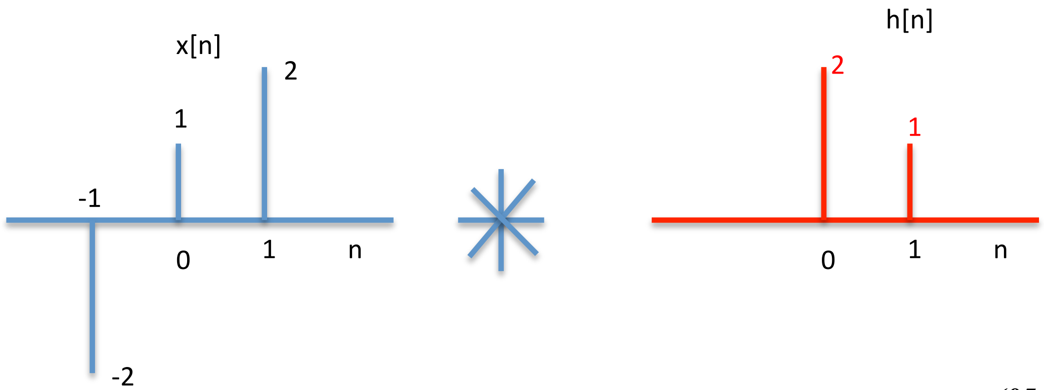
$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} [x(t)]^2\, dt$$

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 1\, dt = \lim_{T\to\infty} \frac{1}{2T}[t]_{-T}^{T} = \lim_{T\to\infty} \frac{1}{2T}[T-(-T)]$$

$$P = \lim_{T\to\infty} \frac{1}{2T}[2T] \Longrightarrow 1 < \infty$$

Since, signal x (t) is a Power signal and its $E = \infty$.

Q4. Determine the discrete-time convolution of x [n] and h [n] for the following case:

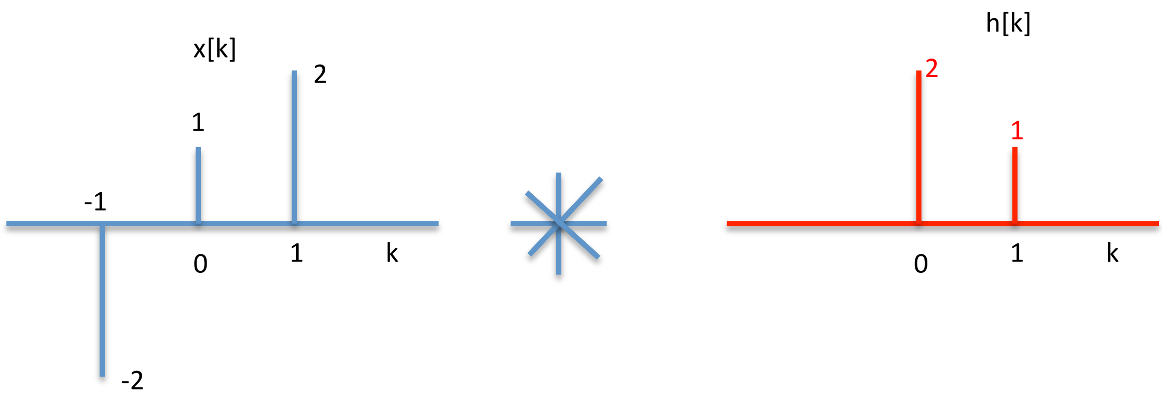


(05 Marks)

<u>Solution:</u>

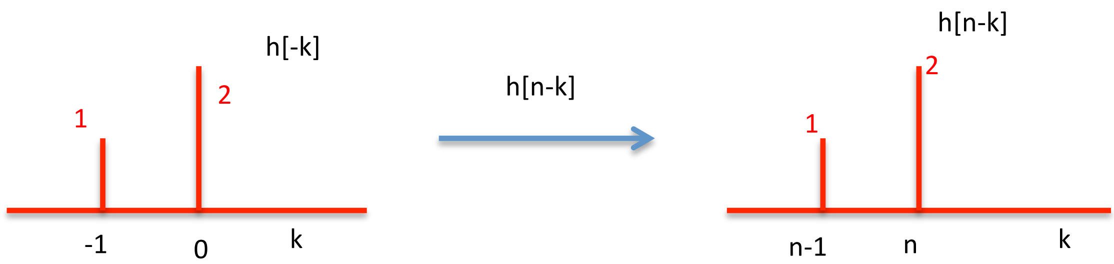
<u>Step#1:</u>

Change the subscript n to k.



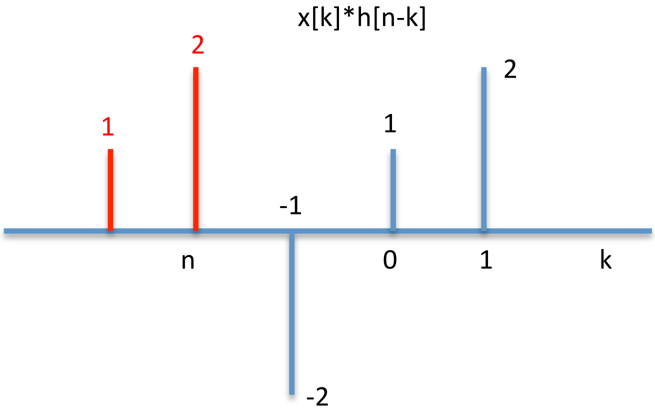
Step#2:

Flip and shift anyone of the signal. Here we are flipping and shifting h[k].



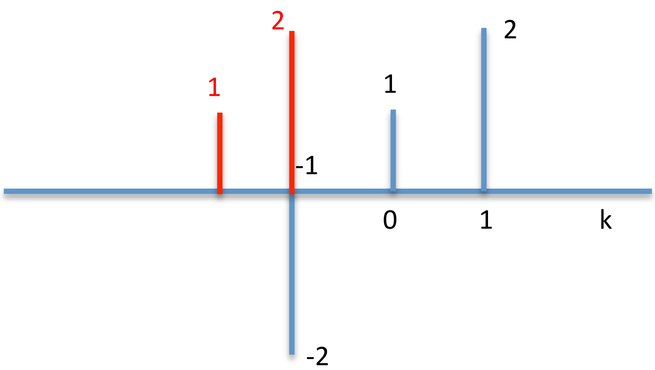
Step#3:

Start sliding h [n-k] over the signal x [k] and convolve.



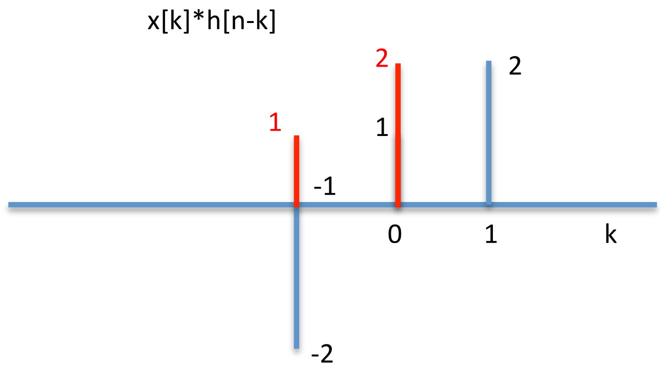
$$when\ n < -1$$

$$\sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0\ As\ there\ is\ no\ overlapping$$



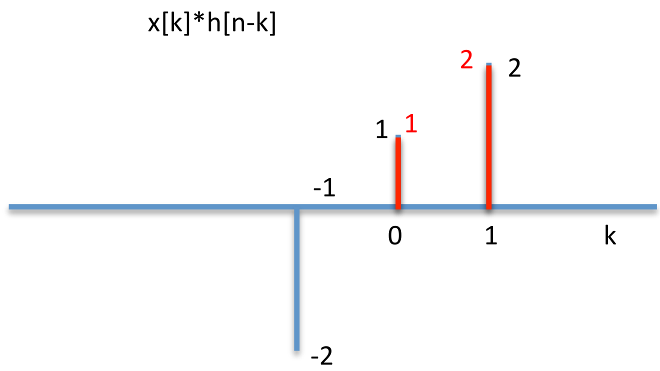
$$when\ n = -1$$

$$y[-1] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 2\times(-2) \Rightarrow -4$$



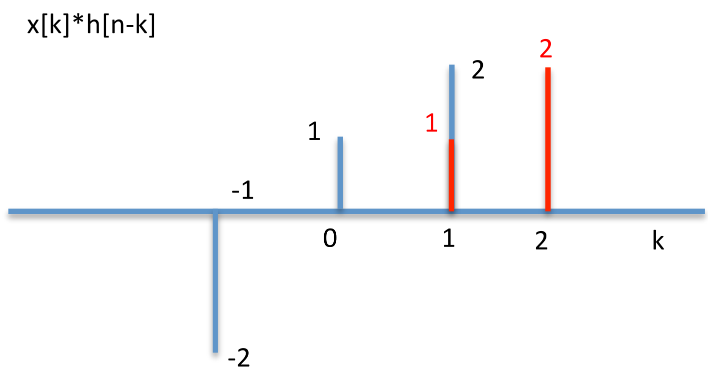
when $n = 0$

$$y[0] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 1\times(-2) + (2\times1) = -2 + 2 \Longrightarrow 0$$



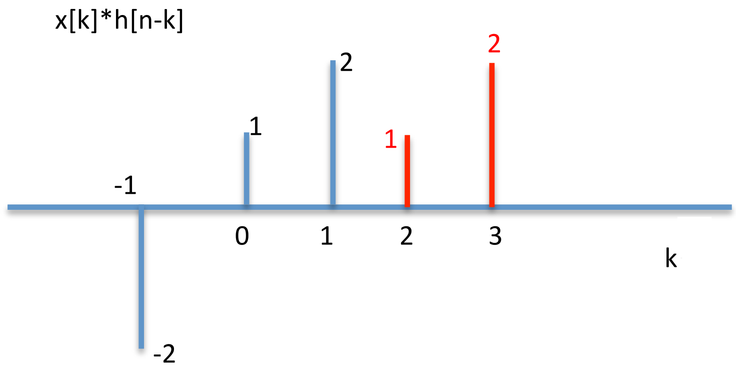
when $n = 1$

$$y[1] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 2\times(2) + (1\times1) = 4 + 1 \Longrightarrow 5$$



when $n = 2$

$$y[2] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 2\times(0) + (1\times2) = 0 + 2 \Longrightarrow 2$$

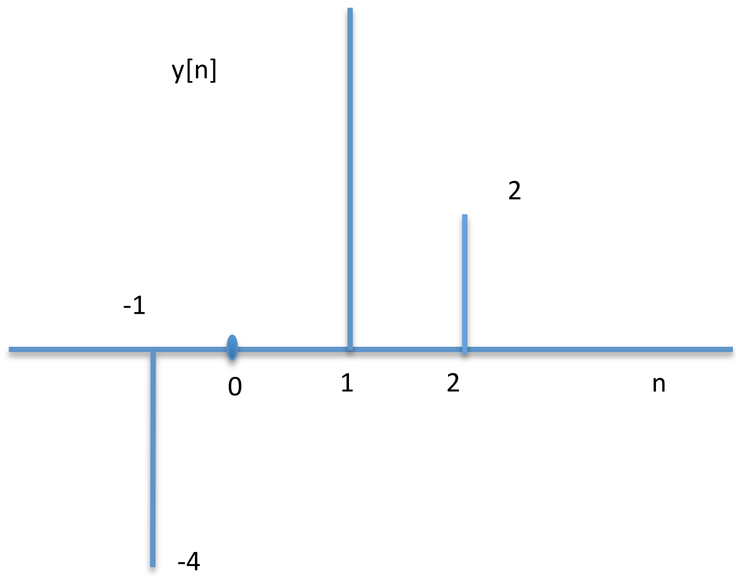


$$when\ n = 3$$

$$y[3] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0\ As\ there\ is\ no\ overlapping$$

Step#4:

Sketch the final signal.



Q5. Find the Fourier series coefficients for each of the following signals:

i. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

ii. $x(t) = 1 + \cos(2\pi t)$

(05 Marks)

Solution:

i. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

$$x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$$

Using Euler's identity:

$$x(t) = \frac{e^{j\pi/6}}{2j} e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j} e^{-j2\pi t5}$$

The fundamental frequency, $\omega_0 = 2\pi$.

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

Where, by inspection:

$$a_5 = \frac{e^{j\pi/6}}{2j} \quad , a_{-5} = \frac{-e^{-j\pi/6}}{2j}$$

Otherwise $a_k = 0$.

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ii. $x(t) = 1 + \cos(2\pi t)$

$$x(t) = 1 + \cos(2\pi t)$$

Using Euler's identity:

$$x(t) = 1 + \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2}$$

The fundamental frequency, $\omega_0 = 2\pi$.

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

Where, by inspection:

$$a_{-1} = a_1 = \frac{1}{2} \ \ \& \ a_0 = 1$$

Otherwise $a_k = 0$.

Q6. Convolve the signal x (t) and h (t) and find out its output y (t). Where:

$$x(t) = e^{-at}\, u(t)$$

$$h(t) = e^{-bt}\, u(t)$$

Where $b \neq a$.

(05 Marks)

Solution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau}u(\tau)\, e^{-b(t-\tau)}u(t-\tau)d\tau = \int_{-\infty}^{\infty} e^{-a\tau}\, e^{-bt}e^{b\tau}u(\tau)u(t-\tau)d\tau$$

$$= e^{-bt}\int_{0}^{t} e^{-a\tau}\, e^{b\tau}d\tau \ , t > 0$$

$$= e^{-bt}\int_{0}^{t} e^{b\tau-a\tau}d\tau = e^{-bt}\int_{0}^{t} e^{(-a+b)\tau}d\tau$$

$$= e^{-bt}\left[\frac{e^{(-a+b)\tau}}{-a+b}\right]_0^t = e^{-bt}\left[\frac{e^{(-a+b)t}}{-a+b} - \frac{e^{(-a+b)0}}{-a+b}\right]$$

$$= e^{-bt}\left[\frac{e^{(-a+b)t}}{-a+b} - \frac{e^{0}}{-a+b}\right] = e^{-bt}\left[\frac{e^{-at}e^{bt}}{-a+b} - \frac{1}{-a+b}\right]$$

$$= \frac{e^{-at}e^{bt}e^{-bt}}{-a+b} - \frac{e^{-bt}}{-a+b} = \frac{e^{-at}e^{bt-bt} - e^{-bt}}{-a+b} = \frac{e^{-at}e^{0} - e^{-bt}}{-a+b}$$

$$y(t) = \frac{e^{-at} - e^{-bt}}{b-a}u(t)$$

When t < 0 then y (t)=0.

Formula Sheet

S. No.	Continuous-Time	Discrete-Time
1.	$Frequency: \ f = \frac{1}{T}$	$Angular\ Frequency: \ \omega = \frac{2\pi k}{N}$
	$Angular\ Frequency: \ \omega = 2\pi f = \frac{2\pi}{T}$	$Fundamental\ Period: \frac{N}{k} = \frac{2\pi}{\omega}$
	$Fundamental\ Period: \ T = \frac{2\pi}{\omega}$	
2.	$Energy: \ E = \int_{-\infty}^{\infty} [x(t)]^2 dt$	$Energy: \ E = \sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2$
	$Power: \ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [x(t)]^2 dt$ If x (t) is periodic, then its average power becomes: $P = \frac{1}{T}\int_0^T [x(t)]^2 dt$	$Power: \ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x[n] \rvert^2$
3.	Convolution Integral $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$	Convolution Sum $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$
4.	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \ , Synthesis\ Equation$	Fourier Series $x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} \ , Synthesis\ Equation$
	$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt \ , Analysis\ Equation$	$a_k = \frac{1}{N}\sum_{n=\langle N \rangle} x[n]e^{-jk\omega_0 n} \ , Analysis\ Equation$