Name: **Regd. No.**

Course Title: Signal & Systems **Course Code:** EL-313

SOLUTION

MID SEMESTER EXAMINATION – Fall 2016

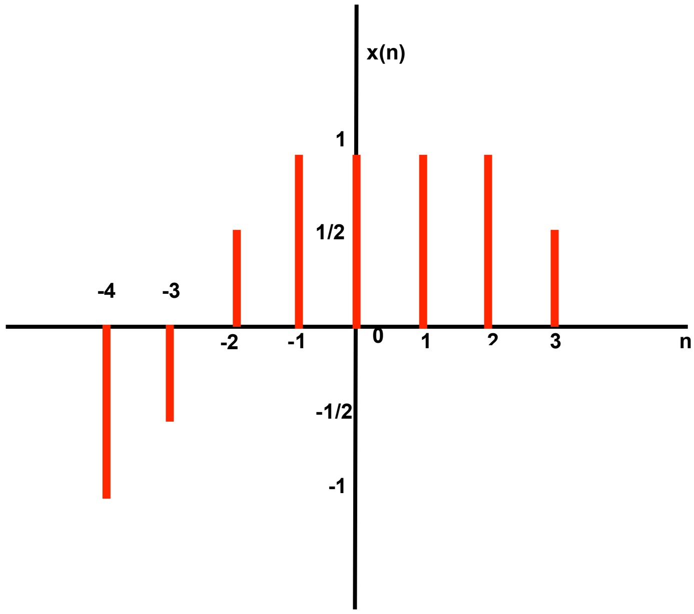
Program: B.E. (Electrical)

SECTION-II: 24 MARKS

Time Allowed: 1hr 10 min

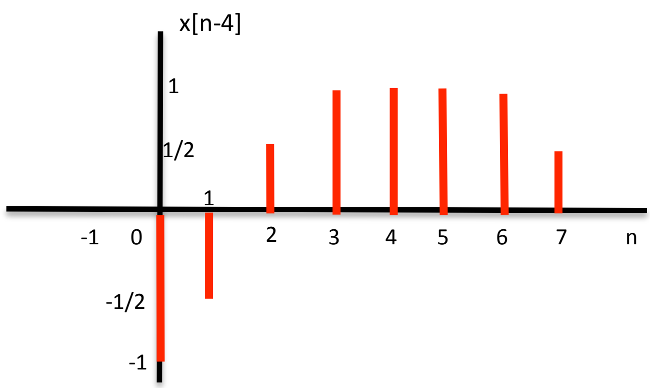
Attempt all questions. Marks are mentioned against the questions. **Note: Please attach the question paper at the end of the answer sheet.**

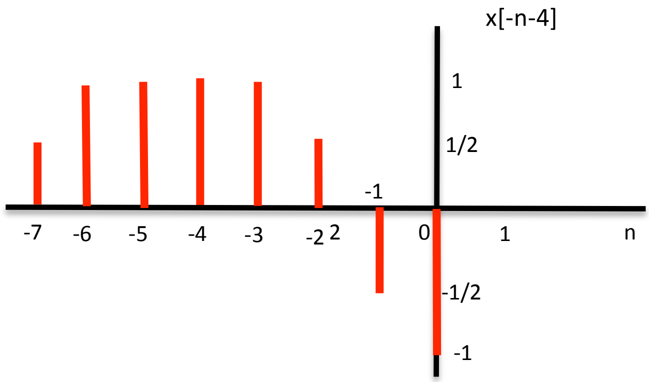
Q1. A discrete time signal is shown below. Sketch and label the signal $x\,[-n-4]$:



(03 Marks)

Solution:





Q2. Determine whether or not each of the following signals is periodic. If a signal is periodic, specify its fundamental period:

i. $x[n] = e^{j\frac{3\pi n}{5}}$

ii. $x(t) = 5\cos \pi t \sin 3\pi\, t$

(03 Marks)

<u>**Solution:**</u>

i. $x[n] = e^{j\frac{3\pi n}{5}}$

$$x[n] = e^{j\frac{3\pi n}{5}}$$

$$N = ?$$

$$\Omega = \frac{3\pi}{5}$$

$$N = \frac{2\pi}{\Omega} = \frac{2\pi}{\frac{3\pi}{5}} = \frac{2\pi}{3\pi}\times 5$$

$$N = \frac{2\times 5}{3} \Rightarrow \frac{10}{3}$$

$$N \Rightarrow 10$$

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ii. $x(t) = 5\cos \pi t \sin 3\pi\, t$

Step #1: Determine fundamental period of individual signals:

$$T_1 = \frac{2\pi}{\omega_1}, \omega_1 = \pi \quad ; T_2 = \frac{2\pi}{\omega_2}, \omega_2 = 3\pi$$

$$T_1 = \frac{2\pi}{\pi} \Rightarrow 2 \quad ; T_2 = \frac{2\pi}{3\pi} \Rightarrow \frac{2}{3}$$

Step #2: Find the ratio of fundamental period of 1st signal to fundamental period of every other signal.

$$\frac{T_1}{T_2} = \frac{2}{\frac{2}{3}} = \frac{2}{2}\times 3 \Rightarrow 3$$

Step #3: If the ratios are Rational, the composite signal is periodic.
Number 3 is rational so x (t) is periodic.

Step #4: $T_0 = \text{LCM}(T_1, T_2, \ldots\ldots)$

$$T_0 = LCM\left(2, \frac{2}{3}\right)$$

$$LCM\ of\ fraction = \frac{LCM\ of\ numerators}{HCF\ of\ denominators}$$

$$T_0 = \frac{LCM(2,,2\ )}{HCF(1,3)} = \frac{2}{1} \Rightarrow 2$$

Hence the fundamental time period of x (t) is 2.

Q3. Determine whether the following signals are energy signal or power signal:

i. $x(t) = e^{-5t}u(t)$
ii. $x[n] = (-0.3)^n\, u[n]$

(03 Marks)

<u>**Solution:**</u>

i. $x(t) = e^{-5t}u(t)$

$$E = \int_{-\infty}^{\infty} [x(t)]^2\, dt$$

$$= \int_0^{\infty} (e^{-5t})^2 dt = \int_0^{\infty} e^{-10t} dt = -\frac{e^{-10t}}{10}\Bigg|_0^{\infty}$$

$$= -\frac{e^{-10(\infty)}}{10} + \frac{e^{10(0)}}{10} = \frac{1}{10} < \infty$$

Hence, x (t) is an Energy Signal and its Power is P =0.

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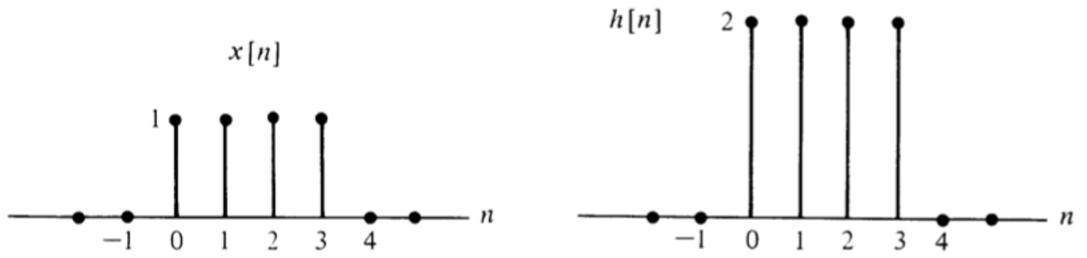
ii. $x[n] = (-0.3)^n\, u[n]$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=0}^{\infty} |(-0.3)^n|^2 = \sum_{n=0}^{\infty} (0.09)^n = \frac{1}{1-0.09} \Rightarrow \frac{1}{0.91} < \infty$$

Hence, x [n] is an Energy Signal and its Power is P =0.

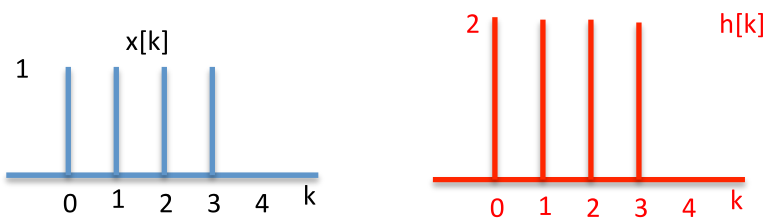
Q4. Determine the discrete-time convolution of x [n] and h [n] for the following case:



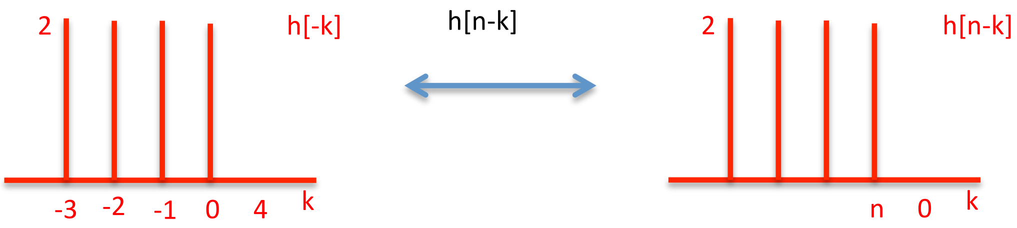
(05 Marks)

<u>Solution:</u>

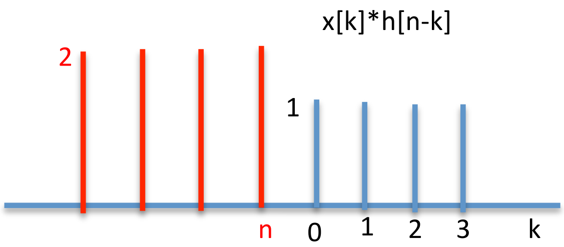
<u>Step#1:</u> Change the subscript n to k.



<u>Step#2:</u> Flip and shift anyone of the signal. Here we are flipping and shifting h[k].

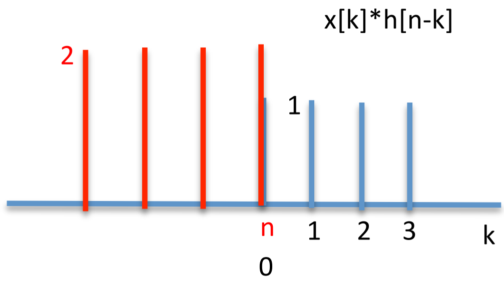


<u>Step#3:</u> Start sliding h [n-k] over the signal x [k] and convolve.



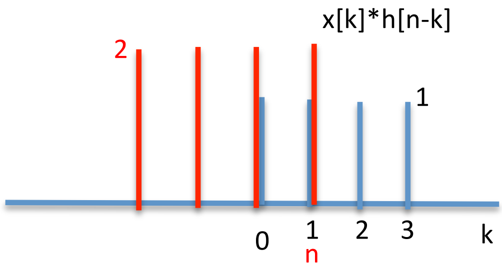
$$when\ n < 0$$

$$\sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0\ As\ there\ is\ no\ overlapping$$



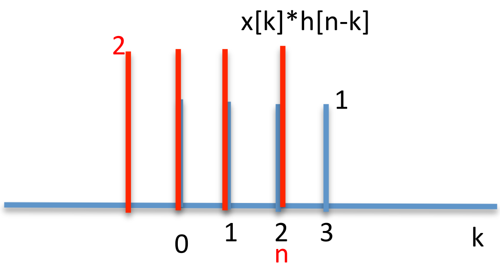
$$when\ n = 0$$

$$y[0] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 2\times(1) \Longrightarrow 2$$



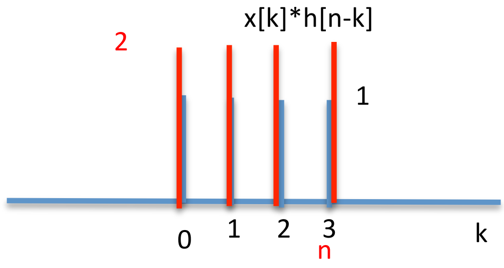
$$when\ n = 1$$

$$y[1] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) \Longrightarrow 4$$



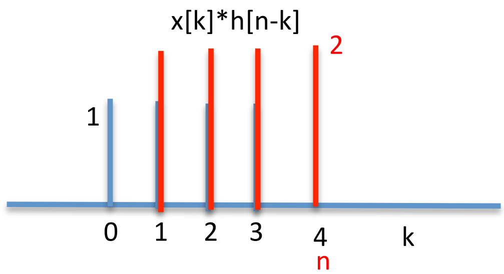
$$when\ n = 2$$

$$y[2] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) + (2\times1) = 2+2+2 \Longrightarrow 6$$



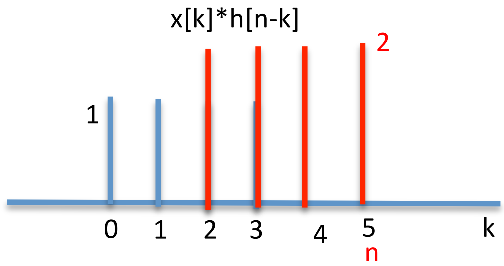
$$when\; n = 3$$

$$y[3] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times1) + (2\times1) + (2\times1) + (2\times1) = 2 + 2 + 2 + 2 \Longrightarrow 8$$



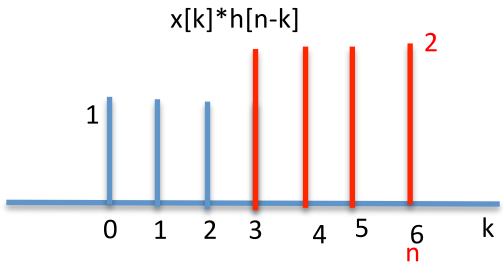
$$when\; n = 4$$

$$y[4] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times1) + (2\times1) + (2\times1) = 0 + 2 + 2 + 2 \Longrightarrow 6$$



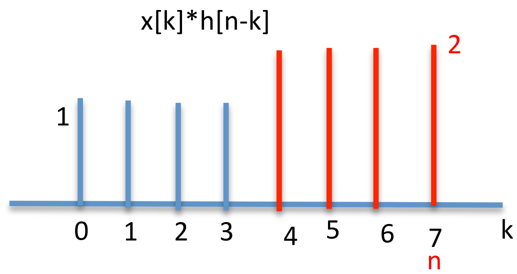
$$when\; n = 5$$

$$y[5] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times0) + (2\times1) + (2\times1) = 0 + 0 + 2 + 2 \Longrightarrow 4$$



$$when\ n = 6$$

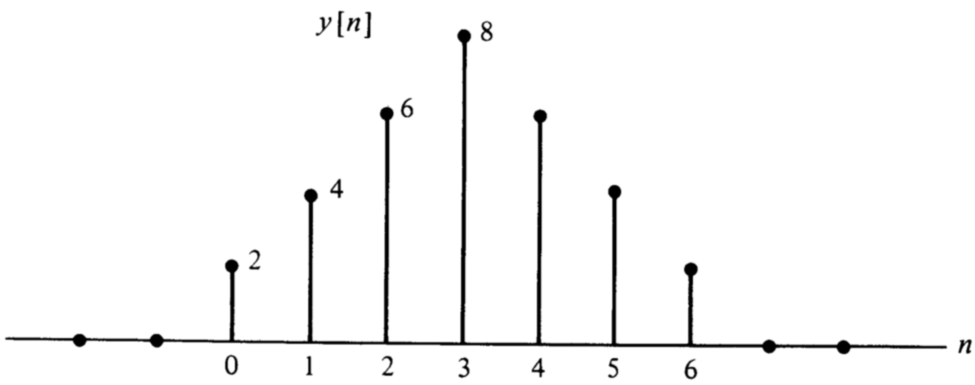
$$y[6] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = (2\times0) + (2\times0) + (2\times0) + (2\times1) = 0 + 0 + 0 + 2 \Longrightarrow 2$$



$$when\ n = 7$$

$$y[7] = \sum_{n=-\infty}^{\infty} x[k]h[n-k] = 0\ As\ there\ is\ no\ overlapping$$

Step#4: Sketch the final signal.



Q5. Find the Fourier series coefficients for each of the following signals:

i. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

ii. $x(t) = 1 + \cos(2\pi t)$

(05 Marks)

Solution:

i. $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

$$x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$$

Using Euler's identity:

$$x(t) = \frac{e^{j\pi/6}}{2j} e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j} e^{-j2\pi t5}$$

The fundamental frequency, $\omega_0 = 2\pi$.

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

Where, by inspection:

$$a_5 = \frac{e^{j\pi/6}}{2j} \;, a_{-5} = \frac{-e^{-j\pi/6}}{2j}$$

Otherwise $a_k = 0$.

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ii. $x(t) = 1 + \cos(2\pi t)$

$$x(t) = 1 + \cos(2\pi t)$$

Using Euler's identity:

$$x(t) = 1 + \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2}$$

The fundamental frequency, $\omega_0 = 2\pi$.

$$x(t) = \sum_k a_k e^{jk\omega_0 t}$$

Where, by inspection:

$$a_{-1} = a_1 = \frac{1}{2} \;\&\; a_0 = 1$$

Otherwise $a_k = 0$.

Q6. Convolve the signal x (t) and h (t) and find out its output y (t). Where:

$$x(t) = e^{-at}\, u(t)$$

$$h(t) = e^{-bt}\, u(t)$$

Where $b \neq a$.

(05 Marks)

Solution:

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau)\, e^{-b(t-\tau)} u(t-\tau) d\tau = \int_{-\infty}^{\infty} e^{-a\tau}\, e^{-bt} e^{b\tau} u(\tau) u(t-\tau) d\tau$$

$$= e^{-bt} \int_0^t e^{-a\tau}\, e^{b\tau} d\tau \;, t > 0$$

$$= e^{-bt}\int_0^t e^{b\tau - a\tau}d\tau = e^{-bt}\int_0^t e^{(-a+b)\tau}d\tau$$

$$= e^{-bt}\left[\frac{e^{(-a+b)\tau}}{-a+b}\right]_0^t = e^{-bt}\left[\frac{e^{(-a+b)t}}{-a+b} - \frac{e^{(-a+b)0}}{-a+b}\right]$$

$$= e^{-bt}\left[\frac{e^{(-a+b)t}}{-a+b} - \frac{e^{0}}{-a+b}\right] = e^{-bt}\left[\frac{e^{-at}e^{bt}}{-a+b} - \frac{1}{-a+b}\right]$$

$$= \frac{e^{-at}e^{bt}e^{-bt}}{-a+b} - \frac{e^{-bt}}{-a+b} = \frac{e^{-at}e^{bt-bt} - e^{-bt}}{-a+b} = \frac{e^{-at}e^{0} - e^{-bt}}{-a+b}$$

$$y(t) = \frac{e^{-at} - e^{-bt}}{b-a}u(t)$$

When t < 0 then y (t)=0.
