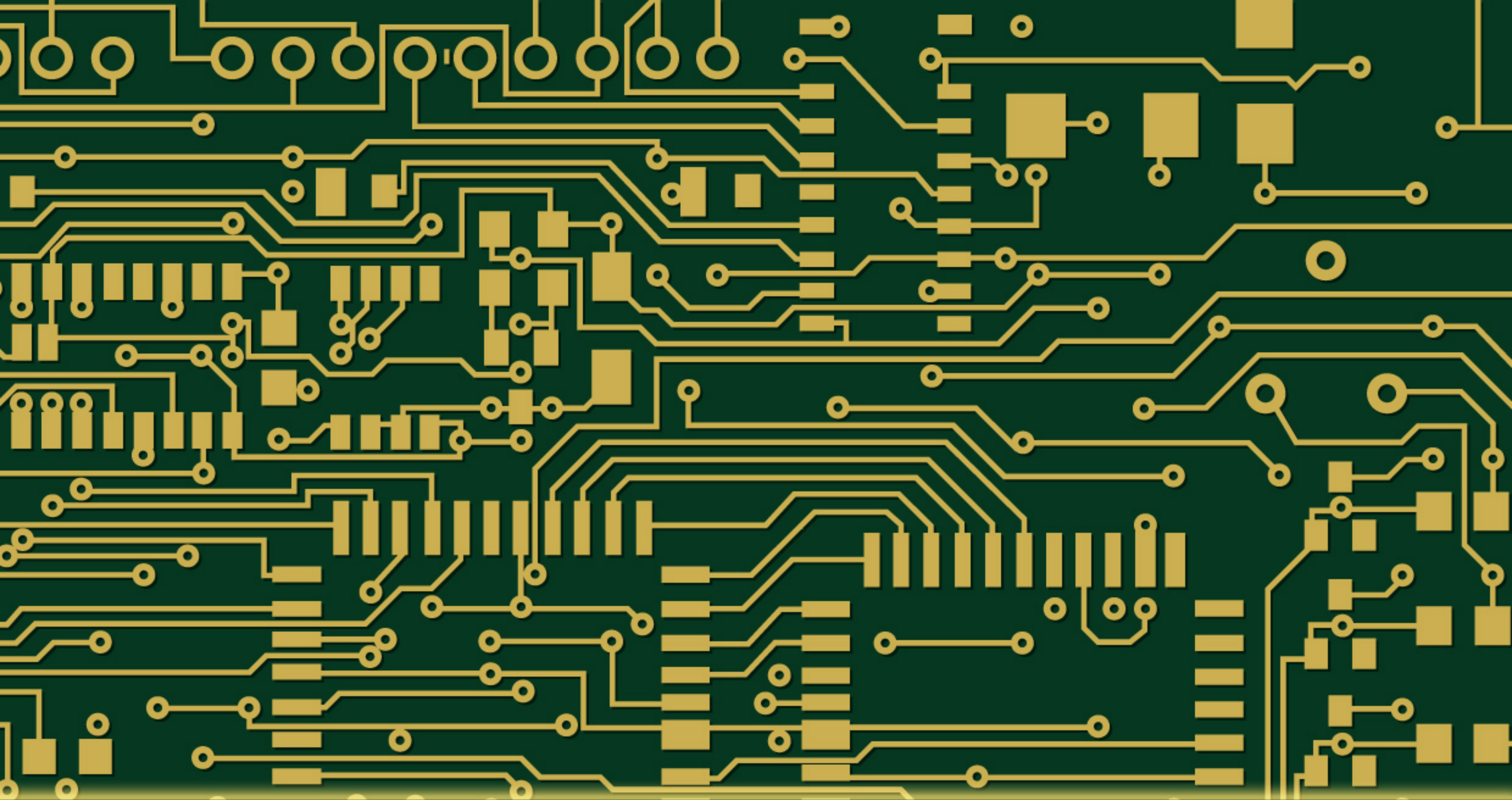
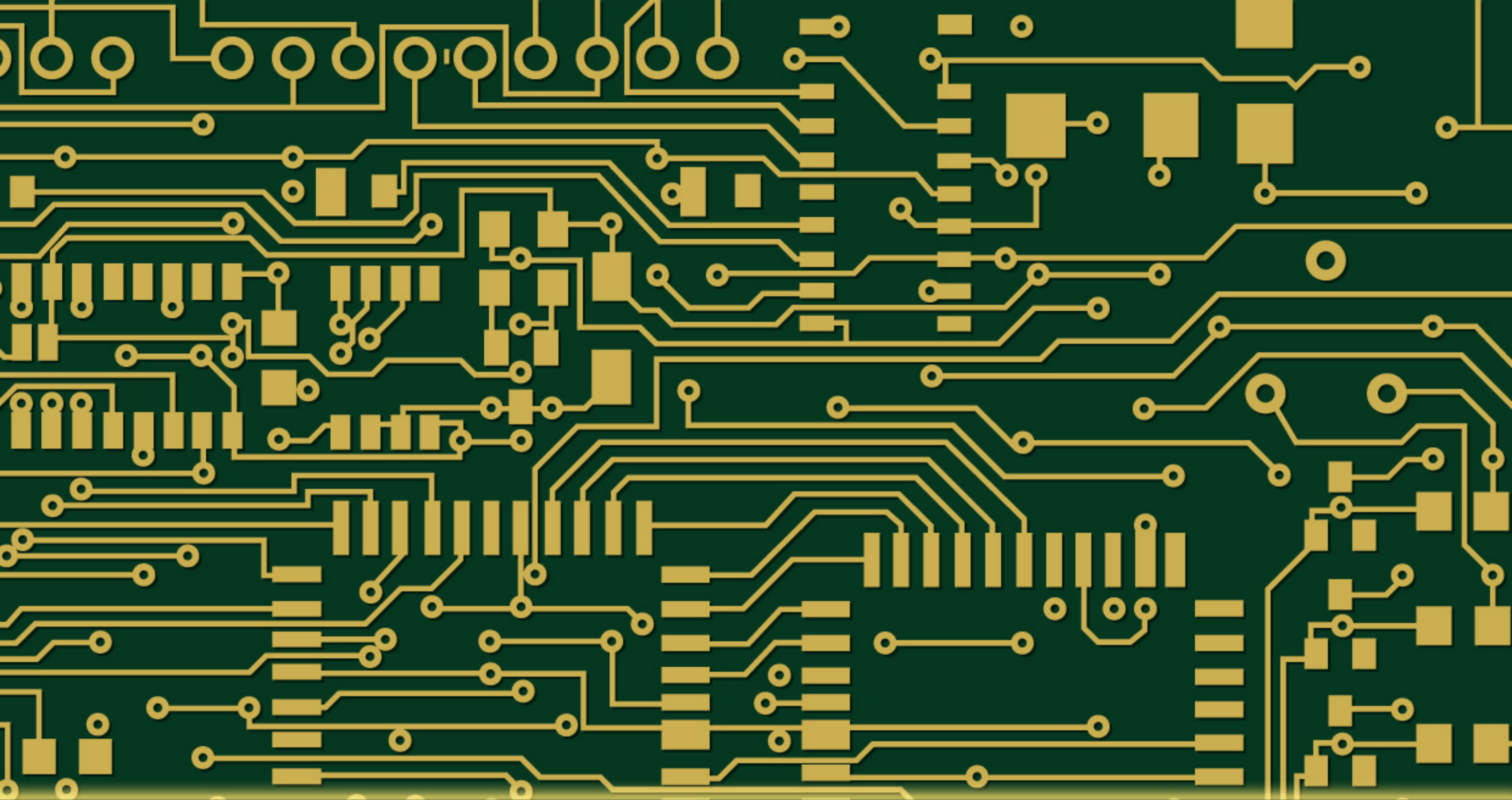


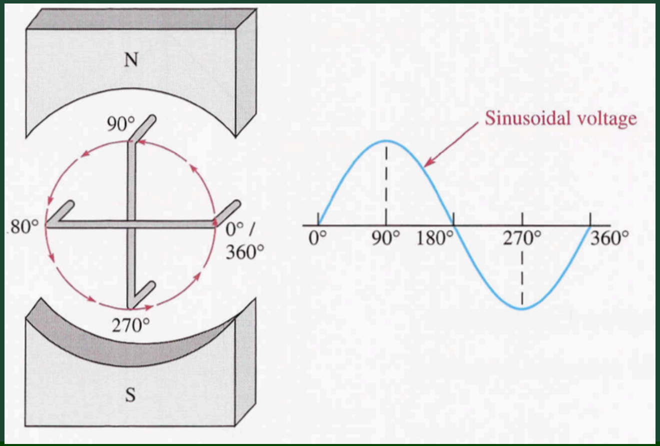
Circuit Analysis-II



Angular Measurement

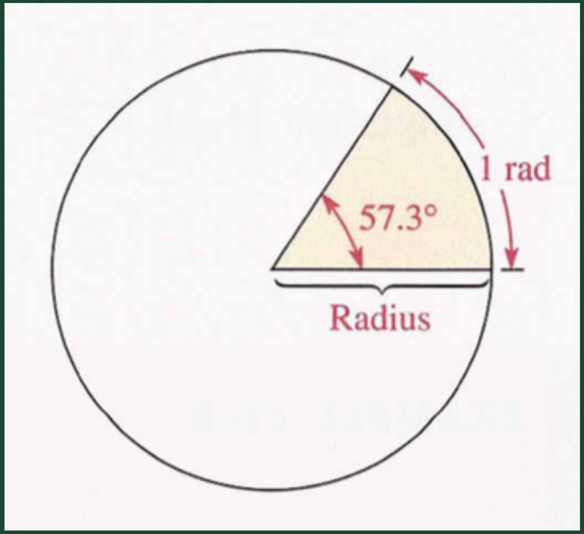
Angular Measurement of a Sine Wave

- ✓ As we already know that a sinusoidal voltage can be produced by an ac generator.
- ✓ As the windings on the rotor of the ac generator go through a full 360° of rotation, the resulting voltage is one full cycle of a sine wave.
- ✓ Thus the angular measurement of a sine wave can be related to the angular rotation of a generator as shown below:



Angular Measurement of a Sine Wave (cont.)

- ✓ A degree is an angular measurement corresponding to 1/360 of a circle or a complete revolution.
- ✓ A radian is the angular measurement along the circumference of a circle that is equal to the radius of the circle.
- ✓ One radian (rad) is equivalent to 57.3°, as shown below:



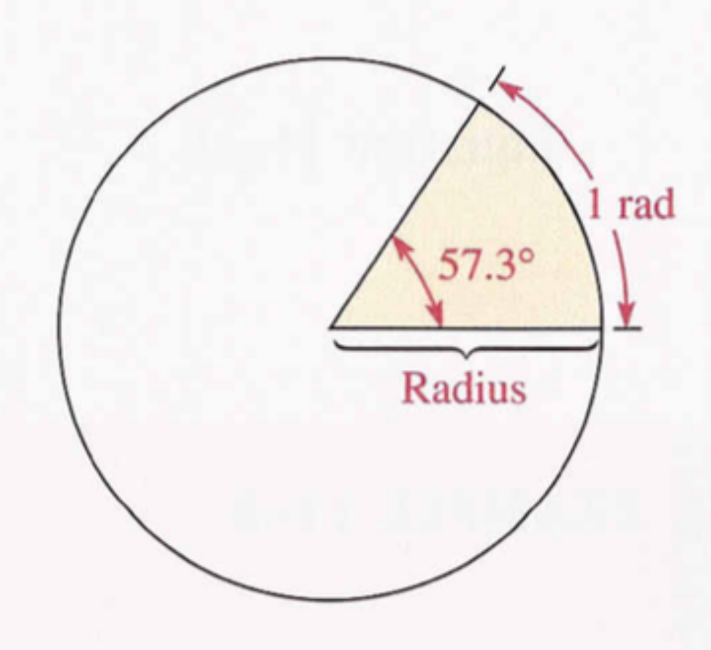
Angular Measurement of a Sine Wave (cont.)

- ✓ In a 360° revolution, there are 2π radians.
- ✓ The Greek letter π (pi) represents the ratio of circumference of any circle to its diameter and has a constant value of approximately 3.1416.

Degree & Radian

✓ Below is the list of several values of degree and the corresponding radian values and there angular measurements:

DEGREES (°)	RADIANS (RAD)
0	0
45	$\pi/4$
90	$\pi/2$
135	$3\pi/4$
180	π
225	$5\pi/4$
270	$3\pi/2$
315	$7\pi/4$
360	2π



Radian / Degree Conversion

- ✓ Degrees can be converted to radians:

$$rad = \left(\frac{\pi\, rad}{180^\circ}\right) \times \deg rees$$

- ✓ Similarly, radians can be converted to degree:

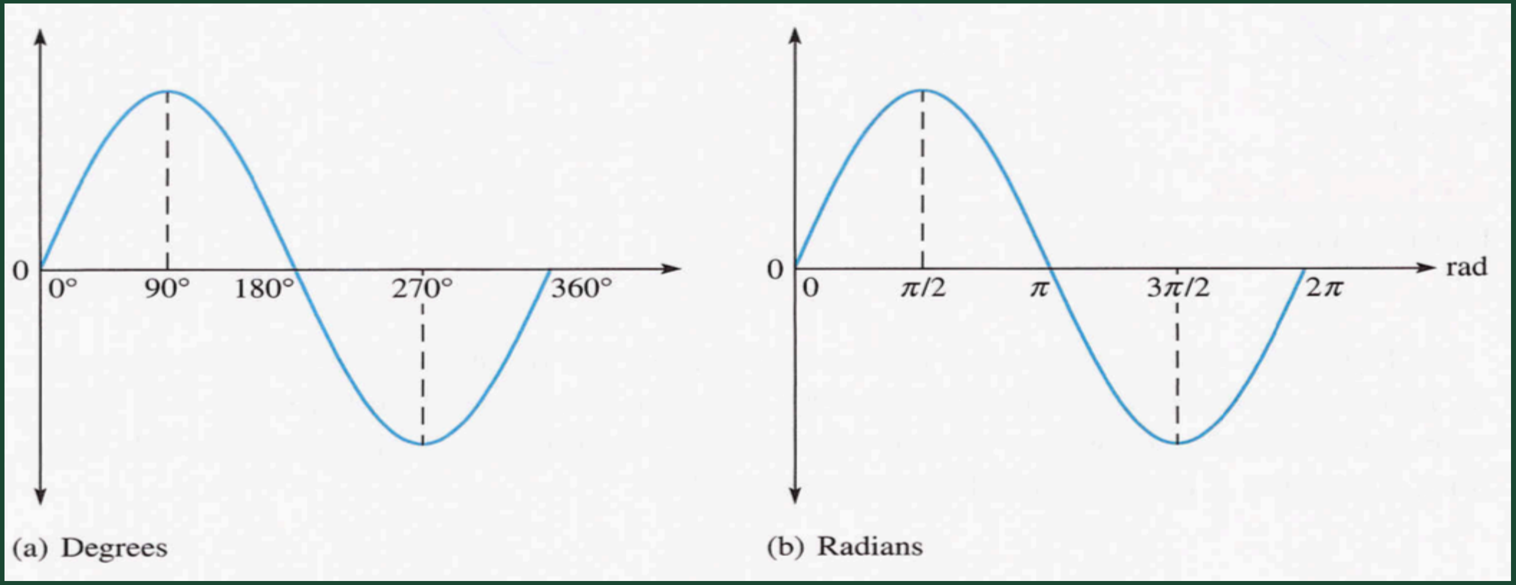
$$\deg rees = \left(\frac{180^\circ}{\pi\, rad}\right) \times rad$$

Example #1

- ✓ (a): Convert 60° to radians.
- ✓ (b): Convert $\pi/6$ rad to degrees.

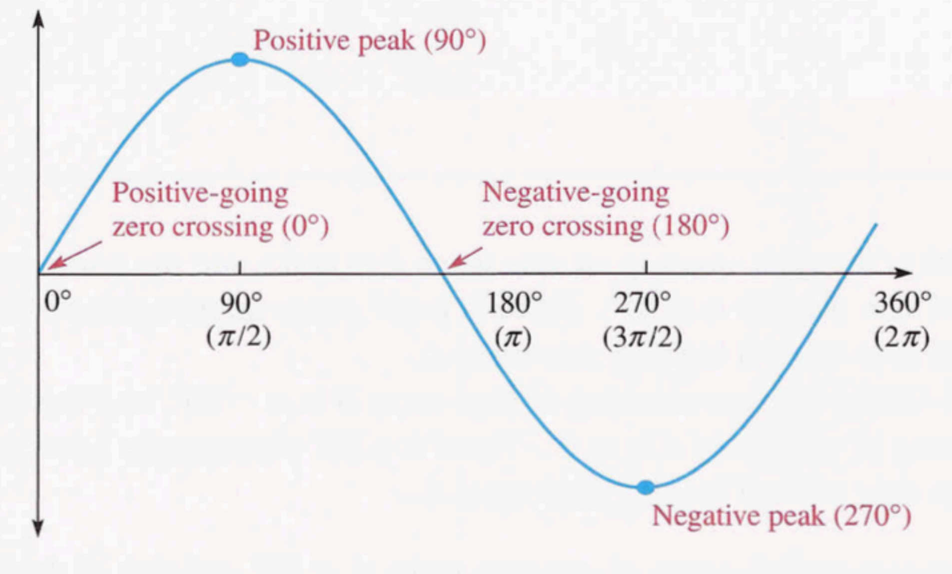
Sine Wave Angles

- ✓ The angular measurement of a sine wave is based on 360° or 2π for a complete cycle.
- ✓ A half cycle is 180° or π rad; a quarter cycle is 90° or π/2 rad and so on.



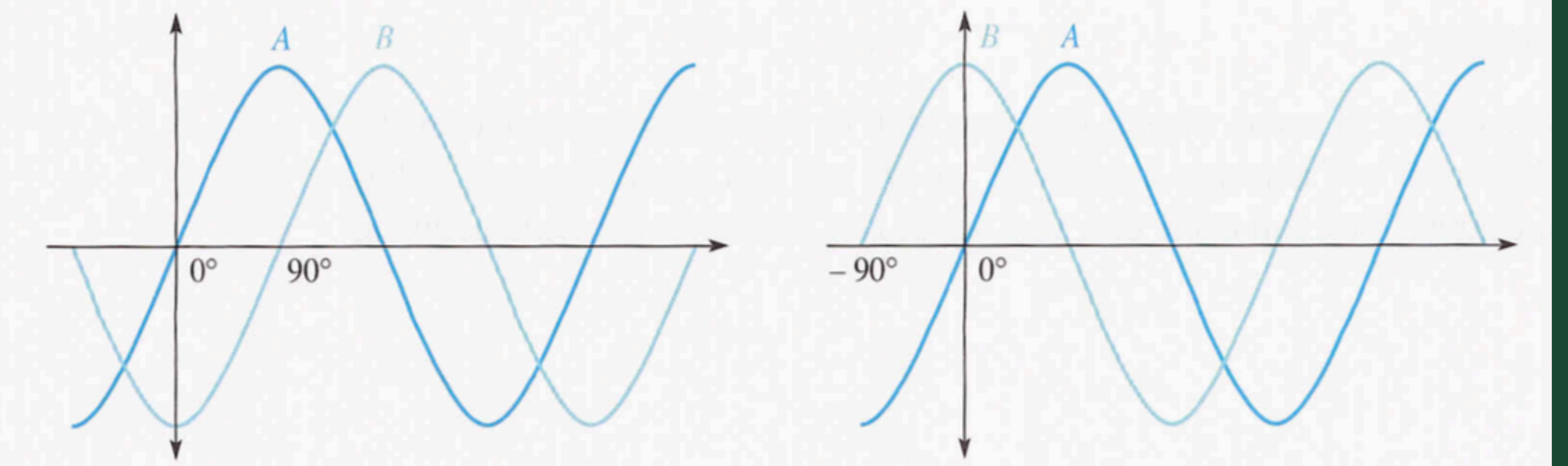
Phase of a Sine Wave

- ✓ The phase of a sine wave is an angular measurement that specifies the position of that sine wave relative to a reference.
- ✓ When the sine wave is shifted left or right with respect to this reference, there is a phase shift.



Phase Shift

✓ Phase shift of a sine wave is shown below:

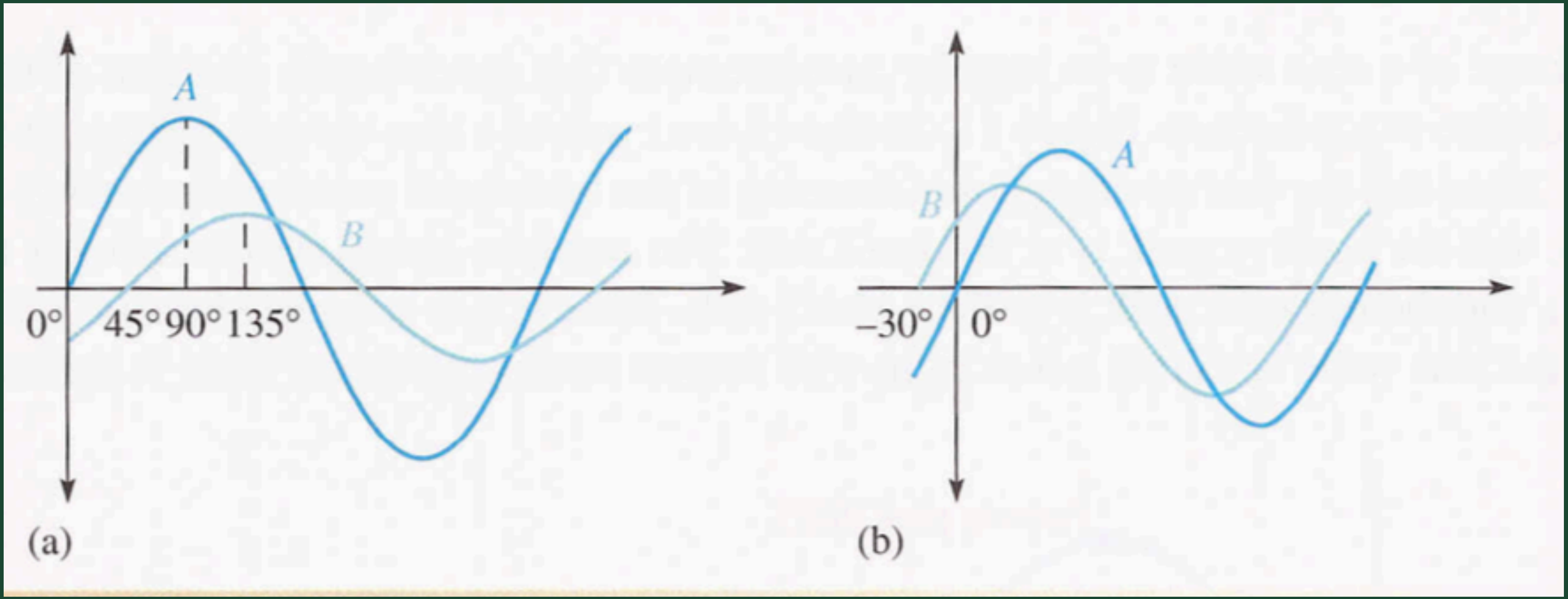


a) A leads B by 90°, or B lags A by 90°.

(b) B leads A by 90°, or A lags B by 90°.

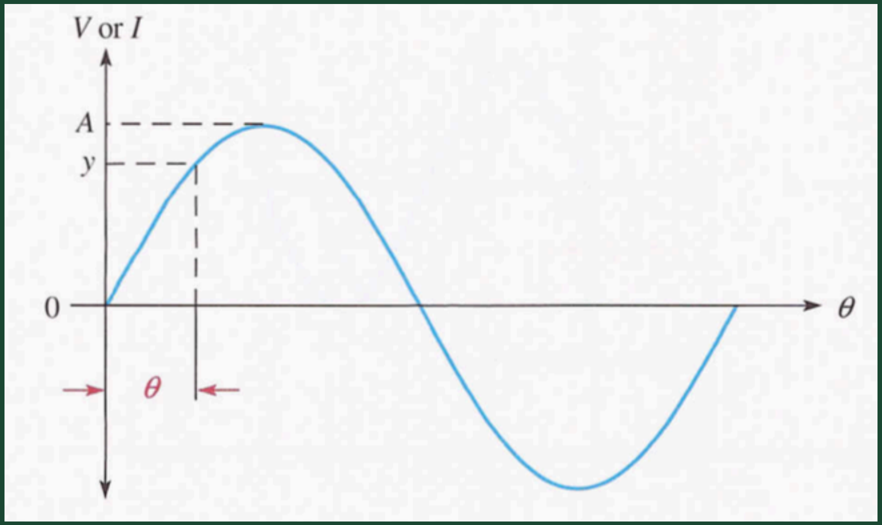
Example #2

- ✓ What are the phase angles between the two sine waves in parts (a) and (b) in the figure below:



Sine Wave Formula

Formula



- ✓ The sine wave amplitude (A) is the maximum value of the voltage or current on the vertical axis, angular values run along the horizontal axis.
- ✓ The variable y is an instantaneous value that represents either voltage or current at a given angle, θ.

Formula (cont.)

- ✓ The general expression for the sine wave is:

$$y = A\sin\theta$$

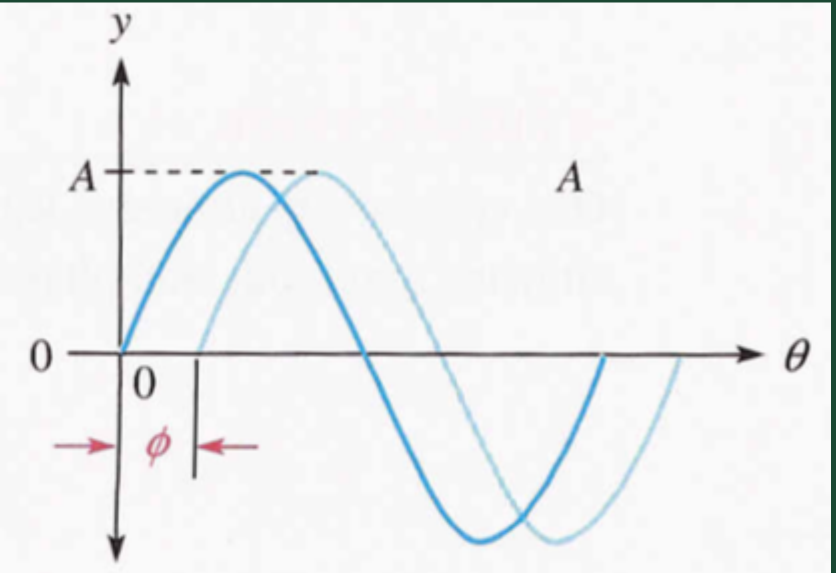
- ✓ This formula states that at any point on the sine wave, represented by an instantaneous value (y) , is equal to the maximum value A time the sine (sin) of the angle θ at that point.
- ✓ For example:
  - ✓ A certain voltage sine wave has a peak value of 10V. The instantaneous voltage at a point 60° along the horizontal axis is as follows, where y=v and $A=V_p$:

$$v = V_p\sin\theta = (10V)\sin 60^\circ = (10V)(0.866) = 8.66V$$

Expressions for Phase-Shifted Sine Waves

- ✓ When sine wave is shifted to the right of the reference (lagging) by a certain angle Φ, the general expression is :

$$y = A\sin\left(\theta - \phi\right)$$

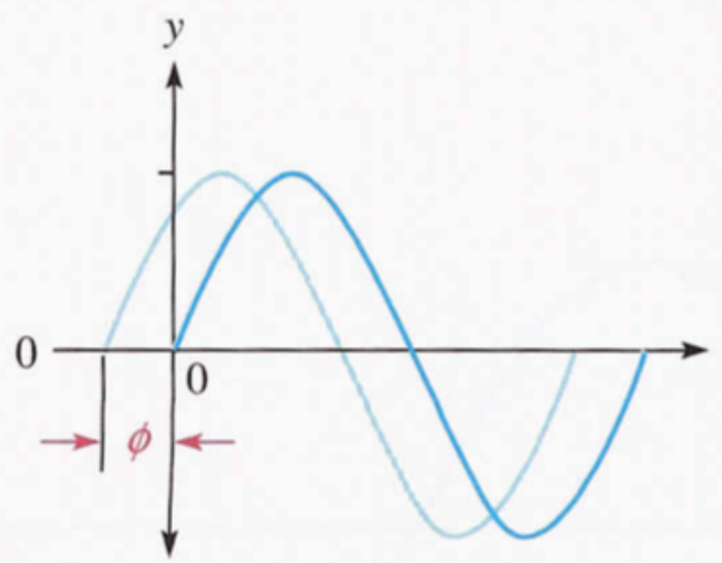


(a) $y = A \sin(\theta - \phi)$

Expressions for Phase-Shifted Sine Waves (cont.)

- ✓ When a sine wave is shifted to the left of the reference (leading) by a certain angle, Φ, the genera expression is:

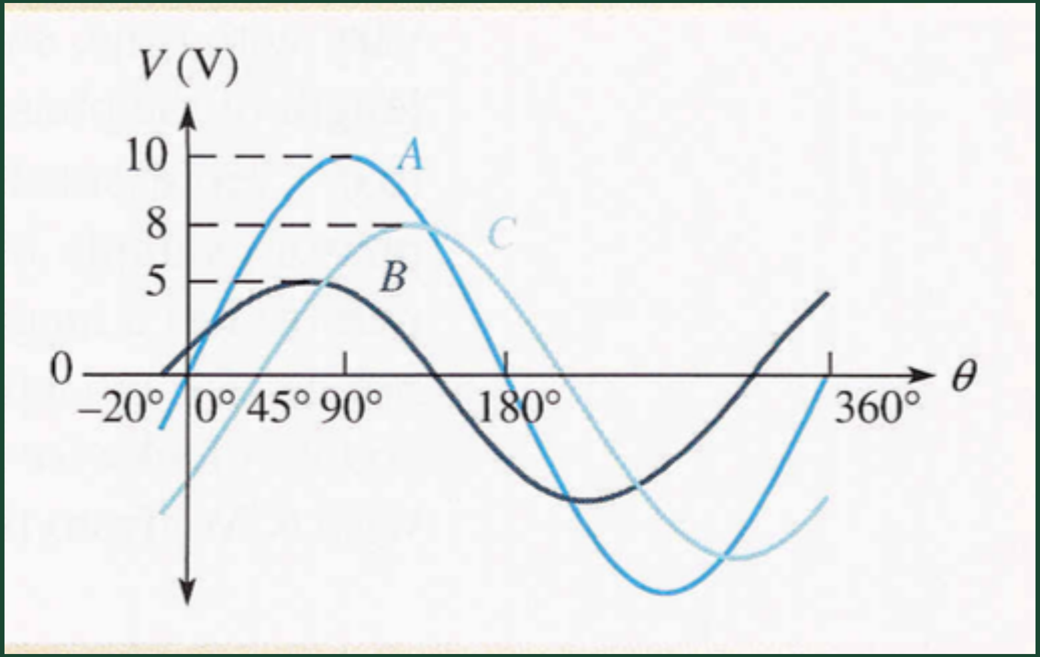
$$y = A\sin\left(\theta + \phi\right)$$

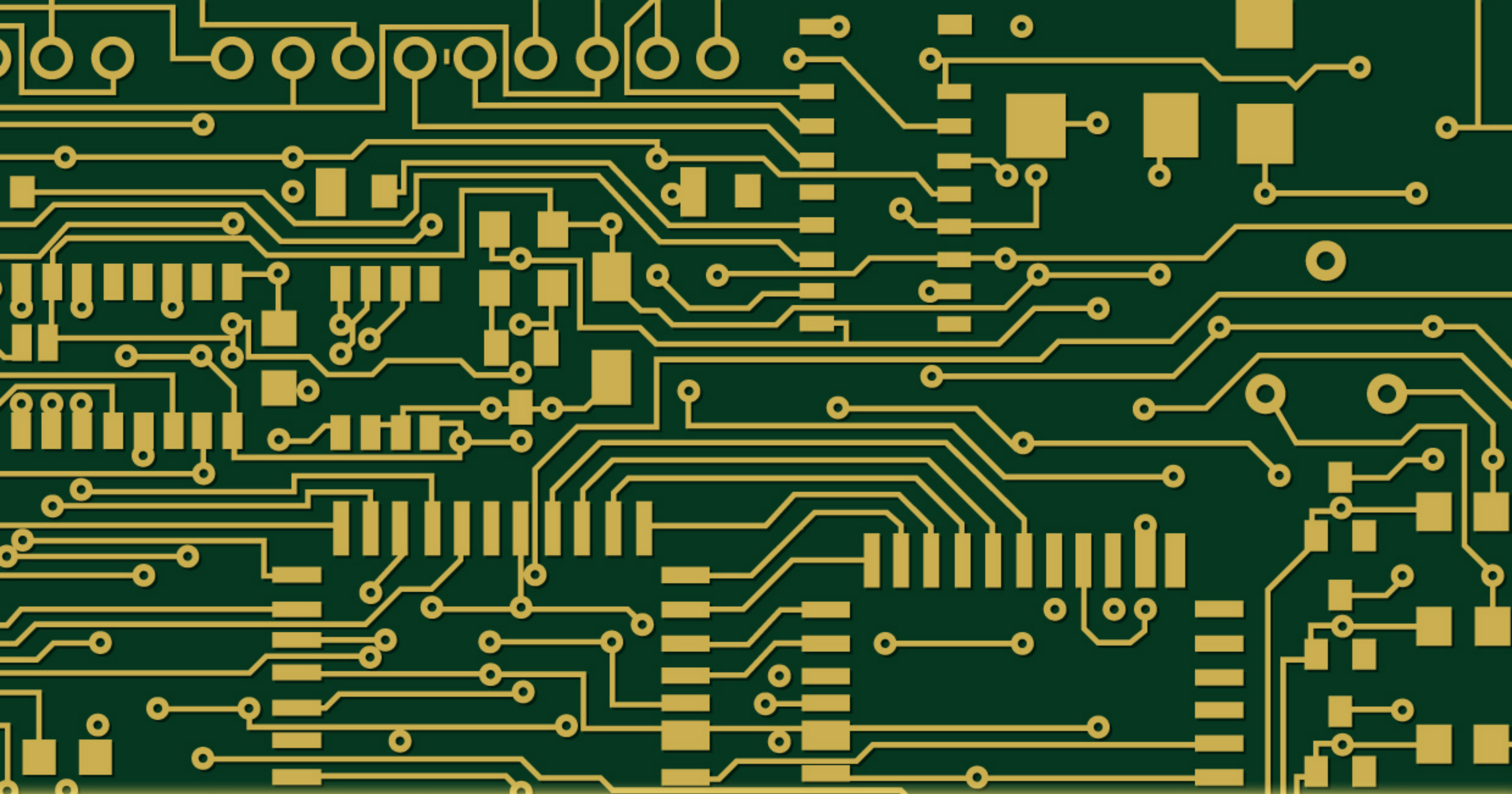


(b) $y = A \sin(\theta + \phi)$

Example #3

✓ Determine the instantaneous value at the 90° reference point on the horizontal axis for each voltage sine wave shown below:

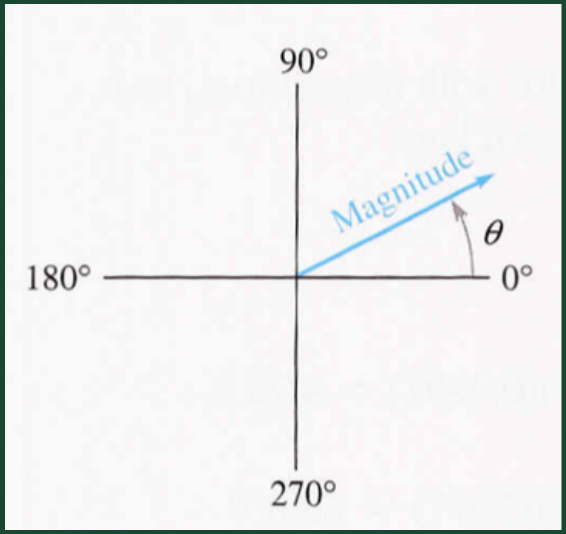




Introduction to Phasors

Phasors

- ✓ A phasor is a type of vector but the term generally refers to quantities that vary with time, such as sine waves.



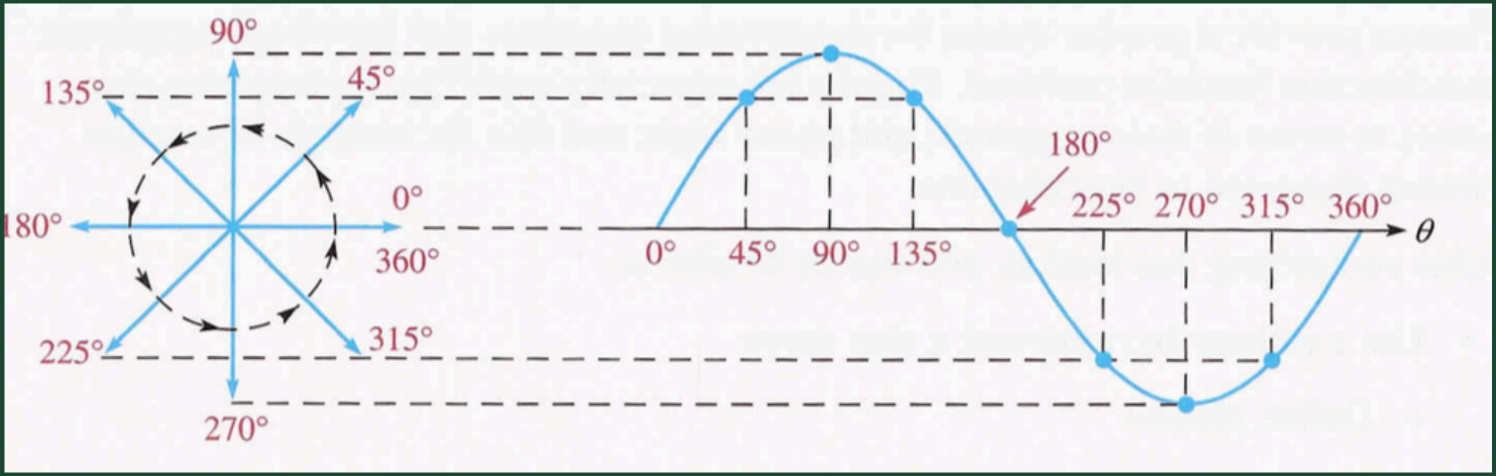
- ✓ The length of the phasor “arrow” represents the magnitude of a quantity.

Phasors (cont.)

- ✓ The angle, θ represents the angular position, for a positive angle.

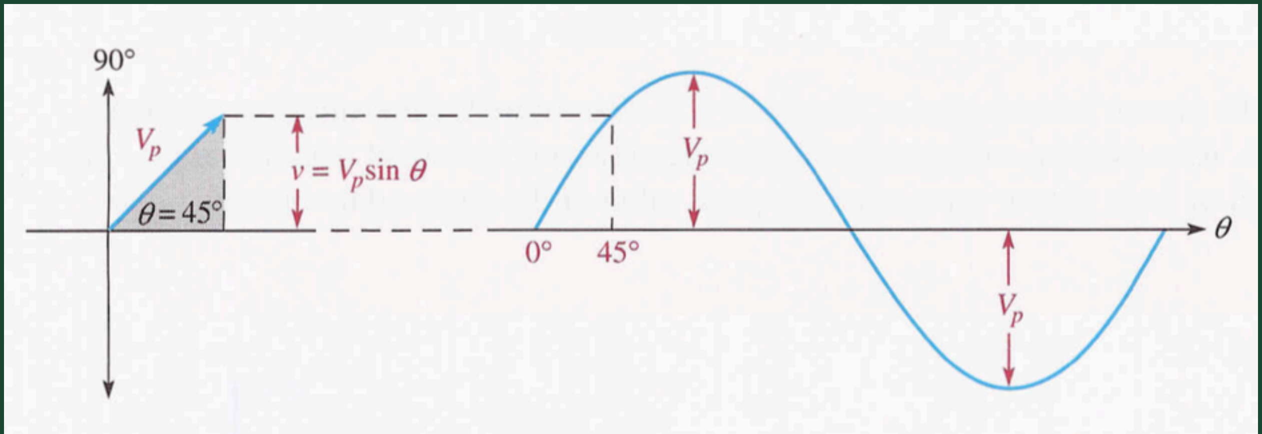
Phasor Representation of a Sine Wave

- ✓ A full cycle of a sine wave can be represented by rotation of a phasor through 360 degrees.
- ✓ The instantaneous value of the sine wave at any point is equal to the vertical distance from the tip of the phasor to the horizontal axis.



Phasors & the Sine Wave Formula

- ✓ The figure below shows a voltage phasor at an angular position of 45° and the corresponding point on the sine wave:



- ✓ The vertical distance from the phasor tip down to the horizontal axis represents the instantaneous value of the sine wave at that point.

Phasors & the Sine Wave Formula (cont.)

- ✓ When a vertical line is drawn from the phasor tip down to the horizontal axis, a right triangle is formed.
- ✓ The length of the phasor is the hypotenuse of the triangle, and the vertical projection is the opposite side.
- ✓ The opposite side of a right triangle is equal to the hypotenuse times the sine of the angle θ.
- ✓ The length of the phasor is the peak value of the sinusoidal voltage, V_p. Thus the opposite side of the triangle, which is the instantaneous value, can be expressed as:

$$v = V_p \sin\theta$$

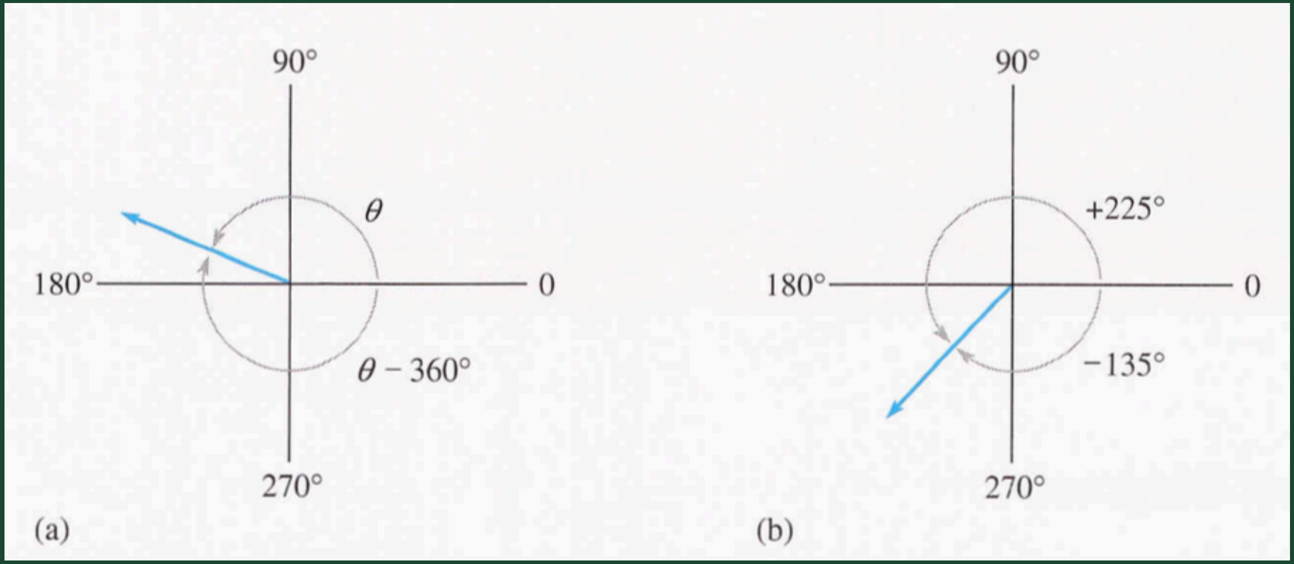
Phasors & the Sine Wave Formula (cont.)

- ✓ A similar formula applies to a sinusoidal current:

$$i = I_p \sin\theta$$

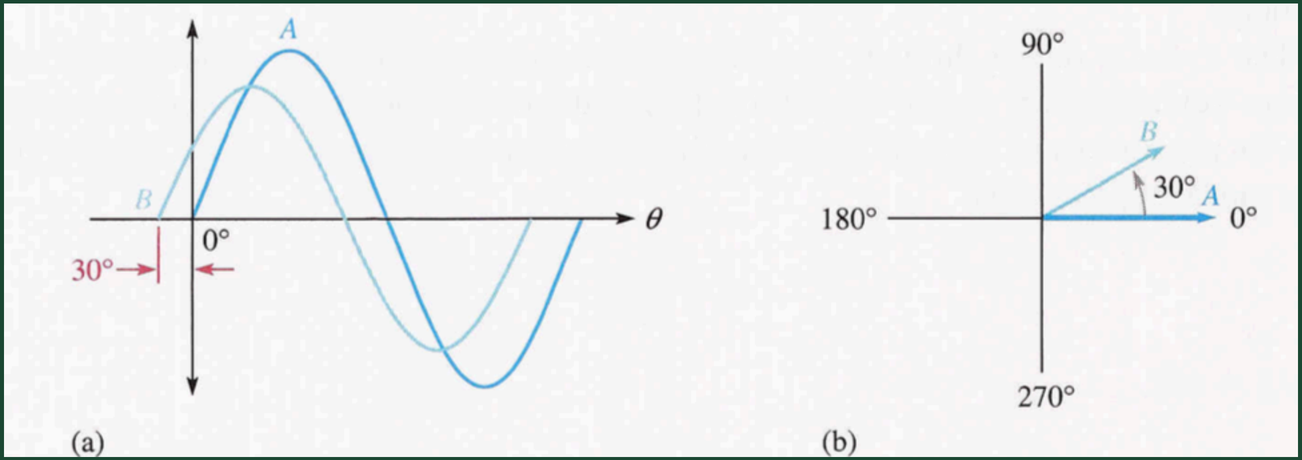
Positive & Negative Phasor Angles

- ✓ The position of a phasor at any instant can be expressed as a positive angle or as an equivalent negative angle.
- ✓ Positive angles are measured counterclockwise from 0° and negative angles are measured clockwise from 0°.



Phasor Diagrams

- ✓ A phasor diagram can be used to show the relative relationship of two or more sine waves of the same frequency.
- ✓ A phasor in a fixed position is used to represent a complete sine wave because once the phase angle b/w two or more sine waves of the same frequency or b/w the sine wave and a reference is established the phase angle remains constant throughout the cycles.



Angular Velocity of a Phasor

- ✓ One cycle of a sine wave is traced out when a phasor is rotated through 360 degrees or 2π radians.
- ✓ The faster it is rotated, the faster the sine wave cycle is traced out. Thus the period and frequency are related to the velocity of rotation of the phasor.
- ✓ The velocity of rotation is called the angular velocity and is designated ω.
- ✓ The angular velocity can be expressed as:

$$\omega = \frac{2\pi}{T}$$

- ✓ Since f=1/T,

$$\omega = 2\pi f$$

Angular Velocity of a Phasor (cont.)

- ✓ When a phasor is rotated at an angular velocity ω, then ωt is the angle through which the phasor has passed at any instant.
- ✓ Therefore, the following relationship can be stated:

$$\theta = \omega t$$

- ✓ Substituting 2πf for ω results in θ = 2πft.
- ✓ The equation for the instantaneous value of a sinusoidal voltage, $v=V_p\sin\theta$ can be written as:

$$v = V_p \sin 2\pi ft$$

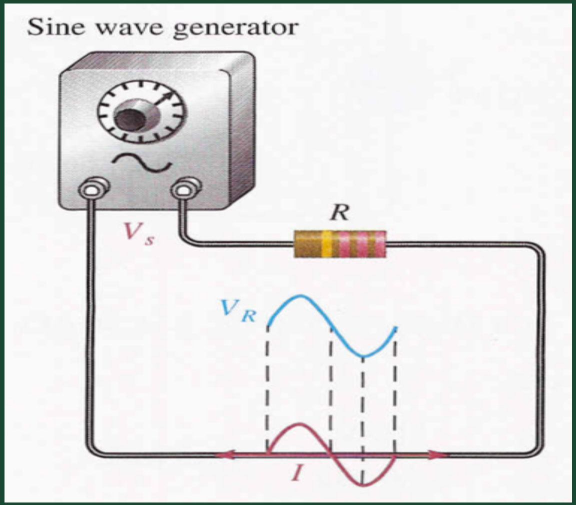
Example #4

- ✓ What is the value of a sinusoidal voltage at 3μs from the positive-going zero crossing when V_p = 10V and f = 50kHz?

AC Circuit Analysis

A.C. Analysis

- ✓ If a sinusoidal voltage is across a resistor there is a sinusoidal current.
- ✓ The current is zero when the voltage is zero and is maximum when the voltage is maximum.
- ✓ When the voltage changes polarity, the current reverses direction.
- ✓ As a result, the voltage and current are said to be in phase with each other.

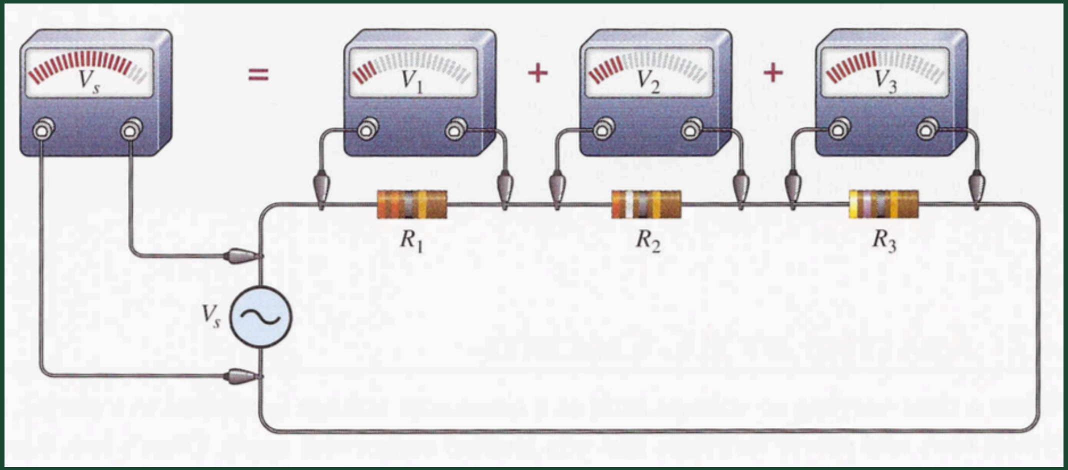


Ohm’s Law

- ✓ When ohm’s law is used in A.C. circuits, both the voltage and the current must be expressed consistently, i.e., both as peak values, both as rms values, both as average values and so on.

Kirchhoff's Voltage

- ✓ Kirchhoff's voltage law in a resistive circuit that has a sinusoidal voltage source is shown below:



- ✓ The source voltage is the sum of all the voltage drops across the resistors, just as in a dc circuit.

Power

- ✓ Power in resistive A.C. circuits is determined the same as for dc circuits except that you must use rms values of current and voltage.
- ✓ The general power formulas are restated for a resistive A.C. circuits as:

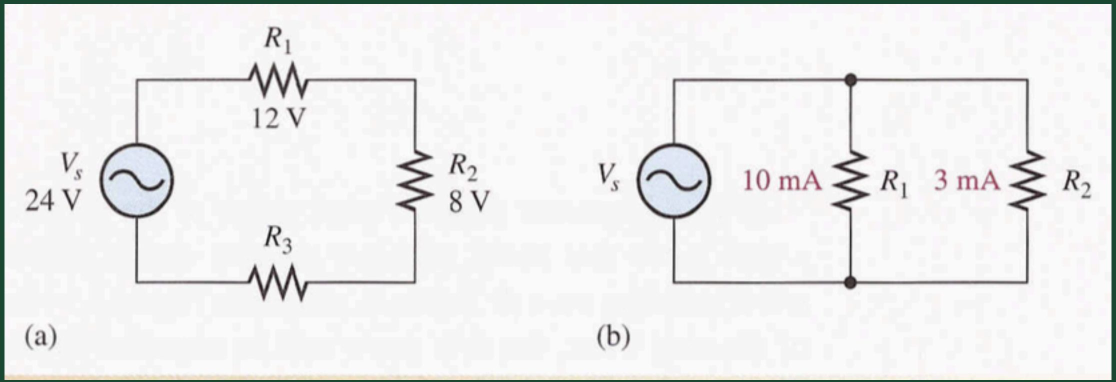
$$P = V_{rms} I_{rms}$$

$$P = \frac{V_{rms}^2}{R}$$

$$P = I_{rms}^2 R$$

Example #5

- ✓ Consider the figure below and calculate the following:



  - ✓ (a): Find the unknown peak voltage drop in fig: a.
  - ✓ (b): Find the total rms current in fig: b.
  - ✓ (c): Find the total power in fig: b if V_{rms}=24V.
- ✓ Note: All values in the circuits are given in rms.

Thank You