Signal & Systems
Lecture #3

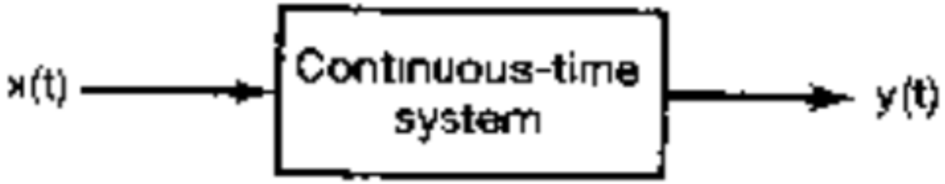
20th March 18

Continuous & Discrete Systems

Fundamentals of Systems

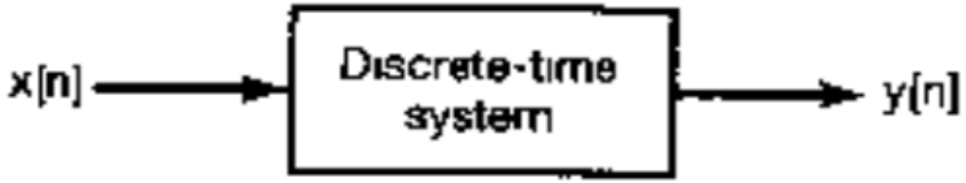
Systems

- A system in the broadcast sense are an interconnection of components, devices or subsystems.
- A system can be viewed as a process in which input signals are transformed by the system or cause the system to respond in some way resulting in other signals as output.
- A continuous time system is a system in which continuous time input signals are applied and result in continuous time output signals. The input-output relation is represented by the following notation: $x(t) \rightarrow y(t)$.



Systems (cont.)

- Similarly a discrete time system is a system that transforms discrete time inputs into discrete time outputs and represented symbolically as: $x[n] \rightarrow y[n]$.



System Properties

Memory

- A system is said to be memory-less if its output for each value of the independent variable is dependent only on the input signal at that same time.

- For example:
  - $y(t) = 5x(t)$, y(t) is the output signal corresponding to the input signal x(t) is memory-less.
  - The system specified by the relationship below is memory-less as the value of y[n] at any particular time n_o depends only on the value of x[n] at that time. $$y[n] = \left(2x[n] - x^2[n]\right)^2$$

- The concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time.

Memory (cont.)

- Examples:
  - The identity system $y(t) = x(t)$ is ofcourse Memory-less.
  - System with description $y[n] = x[n-5]$ has memory. The input at any "instant" depends on the input 5 "instants" earlier.
- Memory-less system is also known as Static system.
- With Memory system is also known as Dynamic system.

Memory (cont.)

- Note:
  - The system will be dynamic (with memory) when there is time scaling.
  - The system will be dynamic (with memory) when there is time shifting.
  - Integration based systems will be dynamic as well.

Example #1

- $y(t) = x(t)$

Causality

- A system is causal if the output at any time depends only on values of the input at the present time and in the past.
- Such a system is often referred to as being non-anticipative, as the system output does not anticipate future values of the input.
- Examples:
  - System with description $y[n] = x[n-1] + x[n]$ is clearly causal, as output "at" n depends on only values of the input "at instants" less than or equal to n.
  - $y[n] = x[n+1] + x[n]$ is not causal, because $x[n+1]$ is a future sample.
- Note: All memory-less systems are causal, since the output responds only to the current value of the input.

Causality (cont.)

- There is an Anti-Causal system which depends only on the future values of input.
- All anti-causal systems are always non-causal but the reverse of this statement is not true.

Linearity

- A system is said to be linear if, for any two input signals, their linear combination yields as output the same linear combination of the corresponding output signals.

- A linear system in continuous time or discrete time is a system that possess the important property of superposition.

- A system is linear if it is additive and scalable. That is:

$$ax_1(t)+bx_2(t)\rightarrow ay_1(t)+by_2(t)$$

$$for \quad all \quad a,b\in C$$

$$ax_1[n]+bx_2[n]\rightarrow ay_1[n]+by_2[n]$$

Linearity (cont.)

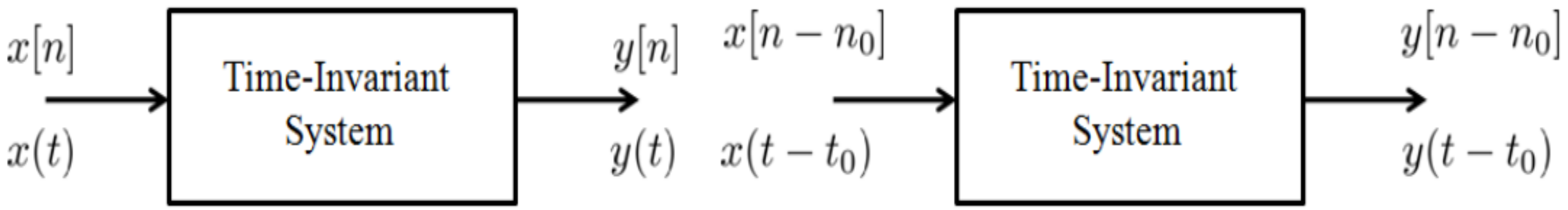
- Examples:
  - System $y(t) = t \cdot x(t)$ is linear.
  - The system $y[n] = 2x[n] + 3$ is not linear.

Example #2

- $y(t) = x(t^2)$

Shift Invariance

- A system is time invariant if the behaviour and characteristics of the system are fixed over time.
- A system is time-invariant if a time-shift of the input signal results in the same time-shift of the output signal. That is if: x (t) ➔ y(t), then the system is time invariant if: x $(t-t_o)$ ➔ $y(t-t_o)$ for any t_o belonging to R.
- Illustration of a time-invariant system:



Shift Invariance (cont.)

- Examples:
  - The system $y(t) = \sin[x(t)]$ is time invariant.
  - The system $y[n] = n\,x[n]$ is not time-invariant.
- Note:
  - When system is performing time scaling the system will be time variant (shift variant).
  - When there is amplitude shifting the system will be time invariant (shift invariant).
- For Time Invariant system:
  - No time scaling.
  - Co-efficient should be constant.
  - Any added / subtracted term in the system relationship (except i/p and o/p) must be constant or zero.

Example #3

- $y(t) = x(\cos t)$

Stability

- The word stability means resistance to change or displacement.
- A stable system is a one in which small inputs lead to predictable responses that do not diverge i.e., are bounded.
- For example: Consider an ideal mechanical spring. If we consider tension in the spring as a function of time as the input signal and elongation as a function of time to be the output signal, it would appear intuitively that the system is stable. A small tension leads only to a finite elongation.
- To describe a stable system, we first need to define the notion of BIBO stability, i.e., Bounded Input-Bounded Output Stability.

Stability (cont.)

- A system is said to be BIBO stable if for any bounded input signal, the output signal is bounded.
  - If $|x(t)| \leq B$ for some $B < \infty$, then $|y(t)| < \infty$.
- It is not necessary for the input and output signal to have the same independent variable for this property to make sense.
- It is valid for continuous time, discrete time and hybrid systems.
- For Example:
  - The system $y(t) = 2x^2(t-1) + x(3t)$ is stable.
  - The system $y[n] = \sum_{k=-\infty}^{\infty} x[k]$ is not stable.

Stability (cont.)

- Bounded signal examples:
  - D.C. value: we will have finite amplitude from -∞ to ∞, i.e., y(t)=6.
  - sin(t) as amplitude varies from -1 to 1.
  - cos(t) as amplitude varies from -1 to 1.
  - u(t) as it's amplitude will be 0 or 1.

Example #4

- $y(t) = \sin[x(t)]$

Invertible System

- A system is invertible if the input signal can be uniquely determined from knowledge of the output signal.
- In other words, a system is invertible if there exists an one-to-oone mapping from the set of input signals to the set of output signals.
- For example in systems for encoding used in a wide variety of communications applications. In such a system a signal that we wish to transmit is first applied as the input to a system known as an encoder.
- There are many reasons to do this, ranging from the desire to encrypt the original message for secure or private communication to the objective of providing some redundancy in the signal.

Invertible System (cont.)

- So that if any error occur in transmission can be detected and possibly corrected. For lossless coding the input to the encoder must be exactly recoverable from the output, i.e., the encoder must be invertible.
- One-to-one Mapping:
  - For each and every input value the output value will be unique.
- Many-to-one Mapping:
  - Many inputs have same outputs.
- For an Invertible system, there should be on to one mapping b/w input and output at each and every distinct of time. Otherewise the system will be non-invertible.

Example #5

- $y(t) = |x(t)|$

Thank You!