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CONTINUOUS & DISCRETE-TIME SYSTEMS:-

$$x(t) \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y(t)$$

i/p   o/p

→ $y(t) = x(t)$ if $t=0$ $y(0) = x(0)$ → present time o/p dependent on present i/p
→ $y(t) = x(t-1)$ if $t=0$ $y(0) = x(-1)$ → past time
→ $y(t) = x(t+1)$ if $t=0$ $y(0) = x(+1)$ → future time

→ $t=0$ (present) can have different past and future values
$t=-1$ (past)
$t=1$ (future)

→ consider any value of 't' and put in the relation given then you will have proper understanding of relation.

MEMORY & MEMORYLESS SYSTEMS:-

→ let's consider 3 cases.

1. $y(t) = x(t-1)$ ← past i/p
2. $y(t) = x(t)$ ← present i/p
3. $y(t) = x(t+1)$ ← future i/p

→ All happening due to system.
→ For example $x(-2) = 1.5$
$x(-1) = 2.0$
$x(0) = 2.5$
$x(1) = 3.0$

considering case #1: $y(t) \overset{t=0}{=} y(0) = x(-1) \Rightarrow 2.0$
depending on past input as system is producing delay. because of that delay we are getting 2.0 instead of 2.5.

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→ memory-less ⇒ o/p depends only on present value. also known as static system. (no storage)
→ with memory ⇒ o/p depends on past and future values of i/p at any instant of time. also known as dynamic system. (only if we have memory element).

EXAMPLE # 1:-

$$y(t) = x(2t)$$

Sol:-

∴ $t = 0$

$y(0) = x(2(0)) \Rightarrow x(0)$ → static present value of i/p

But if $t \neq 0$ the system can be dynamic instead of static

∴ $t = -1$

$y(-1) = x(-2)$ → past value of i/p

if $t = 1$

$y(1) = x(2)$ → future value of i/p

Hence the system is with memory. So the given system is dynamic system.

⇒ $y(t-2) = x(t-2)$

→ $y(t+1) = x(t+1)$

→ When their is time scaling then the system will be dynamic in nature.

→ When ever their is Time shifting the system will be dynamic.

→ Integration based systems will be dynamic as well.

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~~EXAMPLE # 1 (b):~~

CAUSALITY:-

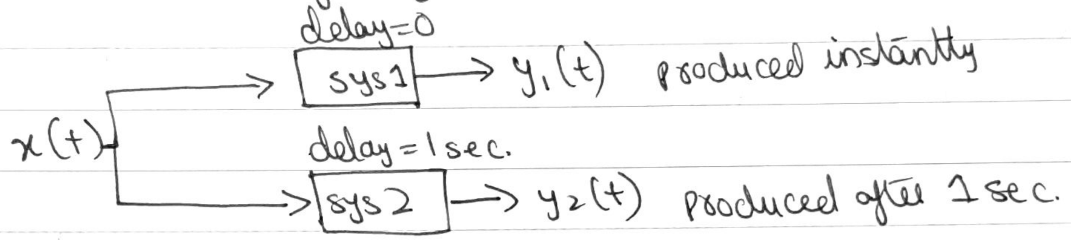
→ independent of future values of i/p is causal system.

→ Ex:- 1) $y(t) = x(t)$

2) $y[n] = x[n-1] + x[n]$

→ All pratical and real life systems are causal.

→



when $t=0$

$y_1(0) \rightarrow x(0)$

$y_2(0) \rightarrow x(-1)$ as there is delay of 1 sec.

→ dependent on future values of i/p is non-causal system.

→ Ex: 1) $y(t) = x(\underbrace{t+2}_{\text{Future value}})$

2) $y(t) = x(t) + x(t-1) + x(t+1)$ non causal.

→ Anti Causal system:-

should depend only on future values of i/p.

→ ~~All non causal are anti~~

→ All anti causal systems are always non causal but the reverse of this statement is not true.

LINEAR & NON-LINEAR SYSTEMS:-

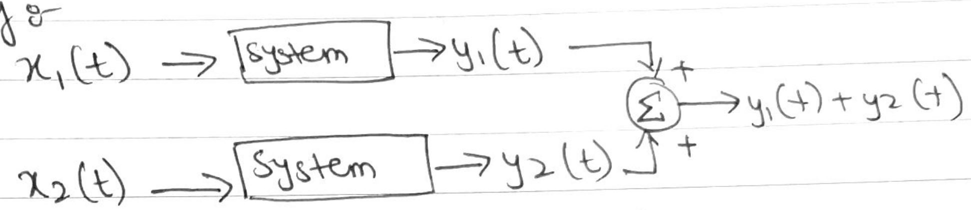
→ The system which follows the principle of superposition is known as linear system.

→ Otherwise non-linear system.

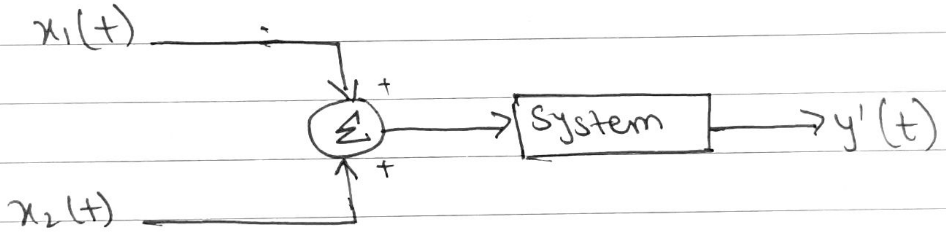
→ Combination of law of additivity and law of homogenity is the law of superposition.

→ 1) law of aditivity :-

First step ⇒



2nd step ⇒ add $x_1(t)$ and $x_2(t)$ and will be inserted in the system.



⇒ $y'(t) = y_1(t) + y_2(t)$ or $\neq y_1(t) + y_2(t)$

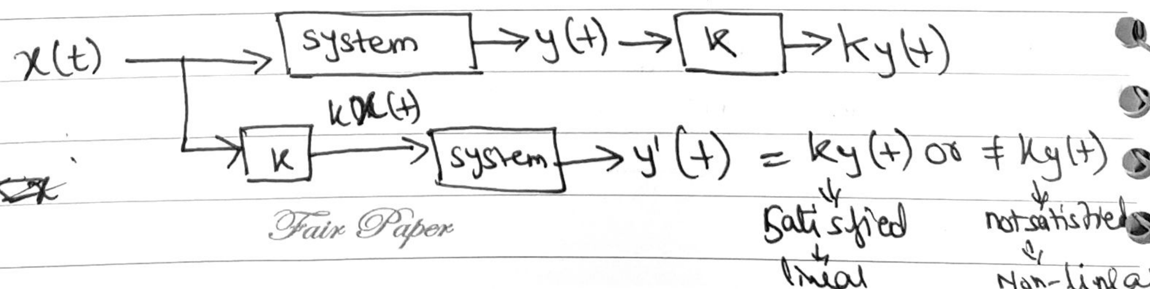
↓ law satisfied ↓ Check homogenity.

↓ not satisfied (Non-linear)

→ 2) law of homogenity :-

step 1:

Step2: ~~kx~~



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$\Rightarrow$ For example :

$\Rightarrow y(t) = t \cdot x(t)$

Check law of additivity

$y_1(t) = t \cdot x_1(t)$

$y_2(t) = t \cdot x_2(t)$

$y_3(t) = t \cdot x_1(t) + t \cdot x_2(t) \Rightarrow t(x_1(t) + x_2(t))$

$y'(t) = t\, x_1(t) + t\, x_2(t) \Rightarrow t(x_1(t) + x_2(t))$

$y_3(t) = y'(t)$ law of additivity satisfied

Check law of homogenity.

$k \cdot y(t) = k t \cdot x(t)$

$t \cdot x(t) \Rightarrow k\, t \cdot x(t) \Rightarrow y'(t)$

$y'(t) = k \cdot y(t)$ law satisfied

The system is linear.

$\Rightarrow y(t) = x(\sin t)$

1- LOA :

$y_1(t) = x_1(\sin t)$

$y_2(t) = x_2(\sin t)$

Step 1 :- $y_1(t) + y_2(t) = x_1(\sin t) + x_2(\sin t) \Rightarrow y_3(t)$

Step 2 :- $x_1(t) + x_2(t) \rightarrow sys \rightarrow x_1(\sin t) + x_2(\sin t) \Rightarrow y'(t)$

$y'(t) = y_3(t)$ satisfied.

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2) LOH:-

Step 1:- $k\,y(t) = k\,x(\sin(t))$

Step 2: $k\,x(t) \rightarrow sys \rightarrow k\,x(\sin t)$

Both are same and law is satisfied.

Hence the system is linear.

EXAMPLE #2:-

$$y(t) = x(t^2)$$

Sol:-

i) LOA:-

Step 1: $y_1(t) = x_1(t^2)$

$y_2(t) = x_2(t^2)$

$y_1(t) + y_2(t) = x_1(t^2) + x_2(t^2) \Rightarrow y_3(t)$

Step 2:- $x_1(t) + x_2(t) \rightarrow sys \rightarrow x_1(t^2) + x_2(t^2) \Rightarrow y'(t)$

$y'(t) = y_3(t)$ law satisfied.

2) LOH:-

Step 1:- $x(t) \rightarrow sys \rightarrow y(t) \rightarrow 'k' \rightarrow k\,y(t) \Rightarrow k\,x(t^2)$

Step 2:- $\cancel{ka}$ $x(t) \Rightarrow 'k' \rightarrow k\,x(t) \rightarrow sys \rightarrow y'(t) \Rightarrow k\,x(t^2)$

Both are equal, thus the law satisfied.

The principle of superposition satisfied, the system is linear.

$\Rightarrow$ when there is time scaling the system is linear.

$\Rightarrow$ system linearity is independent of time scaling.

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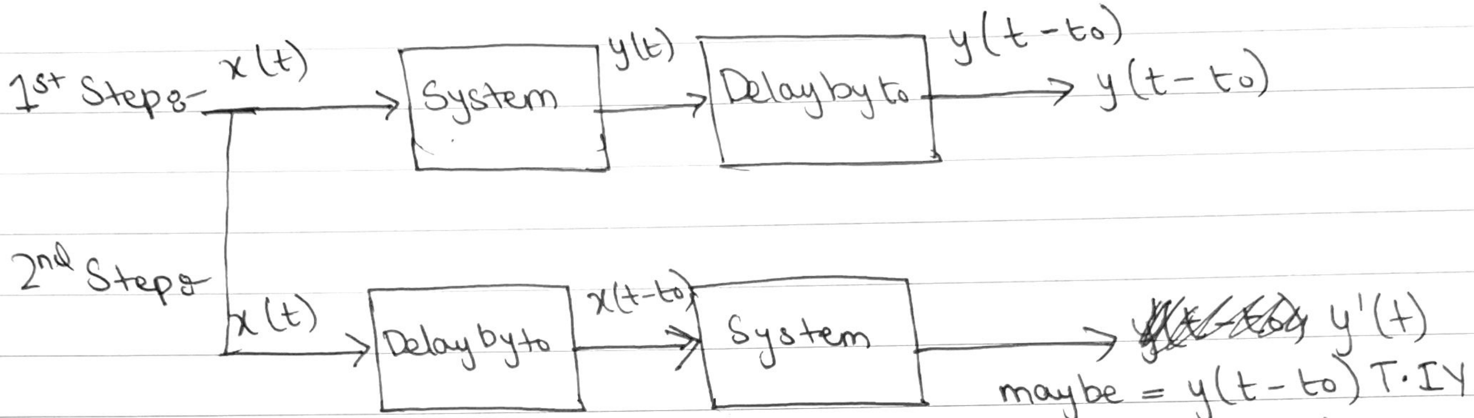
~~FIV~~ ~~TV~~

TIME INVARIANT & TIME VARIANT SYSTEMS :-

⇒ Also known as shift invariant or variant.

⇒ Important property and will be used in L.T.I systems.



maybe $= y(t-t_0)$ T.IV

or $\neq y(t-t_0)$ T.V

⇒ For Example:-

⇒ $y(t) = \sin[x(t)]$.

~~1st step:-~~

$$x(t) \rightarrow sys \rightarrow \sin[x(t)] = y(t)$$

1st Step:-

$$y(t) \xrightarrow{t_0} y(t-t_0) \Rightarrow \sin[x(t-t_0)] = y_1(t)$$

2nd Step:-

$$x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow sys \rightarrow \sin[x(t-t_0)] = y'(t)$$

∴ $y_1(t) = y'(t)$ The system is T.IV.

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⇒ Ex 2:- $y(t) = x(2t)$ ↗ time scaling

$x(t) \rightarrow sys \rightarrow x(2t) = y(t)$

S-1 : $y(t) \xrightarrow{t_0} y(t-t_0) = x[2(t-t_0)] \Rightarrow x(2t-2t_0) \Rightarrow y_1(t)$

S-2:- $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow sys \rightarrow x(2t-t_0) \Rightarrow y'(t)$

As time scaling and $(t-t_0)$ will not be multiplied by 2 as t_0 is not variable

$\therefore$ $y_1(t) \neq y'(t)$ T.V system

⇒ Ex: 3:- $y(t) = 2 + x(t)$

$x(t) \rightarrow sys \rightarrow 2 + x(t) = y(t)$

S-1:- $y(t) \xrightarrow{t_0} y(t-t_0) = 2 + x(t-t_0) \Rightarrow y_1(t)$

S-2:- $x(t) \xrightarrow{t_0} x(t-t_0) \Rightarrow sys \rightarrow 2 + x(t-t_0) \Rightarrow y'(t)$

$\therefore$ $y_1(t) = y'(t)$. T.IV system

~~EXAMPLE #3:~~

~~$y(t) = x$~~

⇒ when system is performing time scaling the system will be T.V.

⇒ when there is amplitude shifting the system will be T.I.V.

Properties continued:-

EXAMPLE # 3:-

$$y(t) = x(\cos t).$$

Sol:-

$$x(t) \rightarrow sys \rightarrow x(\cos t) = y(t)$$

S-1:- $y(t) \xrightarrow{t_0} y(t-t_0) = x[\cos(t-t_0)] \Rightarrow y_1(t)$

S-2:- $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow sys \Rightarrow y'(t) = \cancel{x[\cos(t-t_0)]}\; x[\cos t - t_0]$

as t_0 is constant and operation is on variable t so we have $\cos t$.

$y_1(t) \neq y'(t)$ - The system is time variant.

FOR TIV SYSTEMS:-

1) No time scaling.
2) Coeff. should be constant.
3) Any added / subtracted term in the system relationship (except i/p and o/p) must be constant or zero.

STABLE & UNSTABLE SYSTEMS:-

→ Bounded i/p - Bounded o/p Criteria:- For a stable system o/p should be bounded for bounded i/p at each and every instant of time.
→ i-e BIBO criteria.
→ Bounded means restricting ~~or tend~~ something in defined range.
→ We are limiting the amplitude of signal in $-\infty$ to ∞.
→ Amplitude of i/p signal is finite also for o/p signal. from $-\infty$ to ∞.
→ Unstable system → when bounded i/p is provided the o/p is unbounded.

⇒ **Bounded signals:**

Examples:

1) dc value. we have finite amplitude from $-\infty$ to ∞,
   i-e $y(t) = 6$.
2) $\sin(t)$ as amplitude varies from -1 to $+1$.
3) $\cos(t)$ " " " -1 to $+1$
4) $u(t)$ 0 or 1

⇒ For example:

$$y(t) = t \cdot x(t)$$

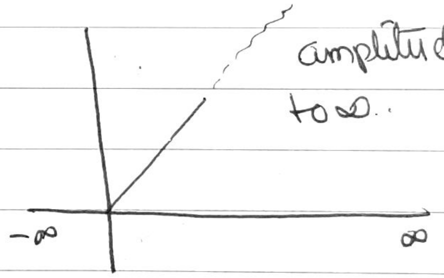
$$x(t) \Rightarrow sys \Rightarrow y(t) = t \cdot x(t)$$

let's select $u(t)$ as bounded i/p

$$u(t) \Rightarrow sys \Rightarrow y'(t) = t \cdot u(t) \Rightarrow \text{unit ram signal} \Rightarrow \text{unbounded}$$

↑
bounded

amplitude will increase to ∞..



∴ hence the o/p is not bounded. So the system is ~~not~~ unstable.

⇒ For example:

$$y(t) = 2x^2(t-1) + x(3t)$$

$$x(t) \rightarrow sys \Rightarrow y(t) = 2x^2(t-1) + x(3t)$$

let's select $\sin(t)$ as bounded i/p

$$\sin(t) \rightarrow sys \Rightarrow y'(t) = 2\sin^2(t-1) + \sin(3t)$$

amplitude is 0 to 1   amplitude is -1 to 2

Hence the o/p is bounded, So the system is stable.

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EXAMPLE # 4:

$$y(t) = \sin[x(t)]$$

Sol:-

$$x(t) \rightarrow \text{sys} \rightarrow y(t) = \sin[x(t)]$$

Case 1: Feeding $x(t) \rightarrow$ as bounded.

$y(t) = \sin[\text{finite value}]$

$-1 \leq \sin(\underset{\downarrow}{\ }) \leq 1$

anything

The property will remain same so output $y(t)$ will also have $-1 \leq y(t) \leq 1$ hence it will be bounded. System is stable.

Case 2:- $x(t) \leftarrow$ unbounded

$y(t) = \sin(\infty)$

$-1 \leq \sin(\infty) \leq 1$

again same result the output is bounded.

$$\Rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

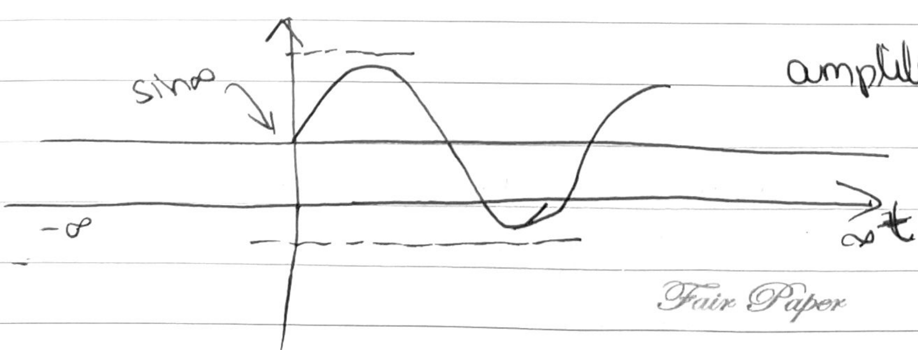
$$x(t) = \cos(t)$$

$$y(t) = \int_{-\infty}^{t} \cos(\tau)\, d\tau \Rightarrow [\sin(\tau)]_{-\infty}^{t}$$

$$\Rightarrow \sin t - \sin(-\infty)$$

$$= \sin t + \sin \infty$$

also $-1 \leq \sin \infty \leq 1$



amplitude is finite so $y(t)$ is bounded.

Hence system is bounded in case of cost is stable.

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but for $x(t) = u(t)$

$$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \text{ramp signal.}$$

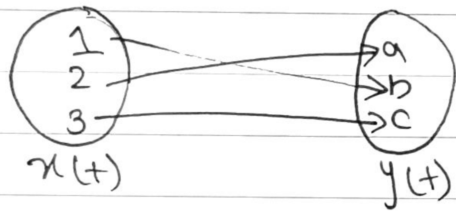
ramp signal is unbounded so the o/p is unbounded hence system is unstable.

The system is unstable.

INVERTIBLE & NON-INVERTIBLE SYSTEMS

⇒ One to one mapping:- For each and any input value the o/p value will be unique.

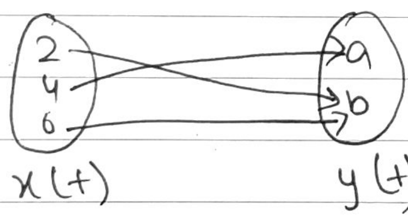
set →



3 dinstinct value

all the three values are producing distinct o/p values.

⇒ Many to one mapping:-



~~many~~ Two dinstinct i/p values the output is b.

many iputs have same output

⇒ For an invertible system, ~~the~~ there should be one to one mapping b/w i/p and o/p at each and every distinct of time.

⇒ Otherwise non-invertible. (many to one mapping.

⇒ For example: $y(t) = x^2(t)$

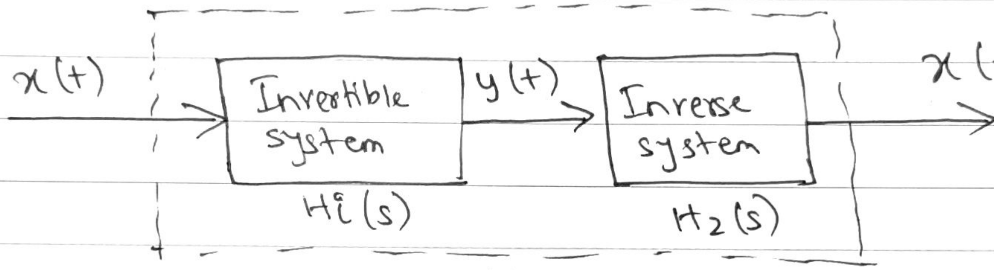
$x(t)$	$y(t)$
-2	4
2	4

many to one mapping so the system is non invertible

⇒ Ex 2:- $y(t) = x(t) + 2$

$x(t)$	$y(t)$
0	2
1	3
-1	1

One to one mapping
Hence the system is invertible



composite system
overall gain will be unity as o/p is same as i/p

$$H(s) = 1 \Rightarrow H_i(s) \times H_2(s)$$

$$H_2(s) = \frac{1}{H_i(s)}$$

EXAMPLE # 5:-

$y(t) = |x(t)|$

Sol:-

$x(t)$	$y(t)$
-2	2
2	
2j	
-2j	

many to one mapping so the system is Non-invertible.

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EXERCISE PROBLEMS:-

Q1:- In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be:
1) Memoryless
2) Time Invariant
3) Linear
4) Causal
5) Stable

Determine which of these properties hold and which do not hold for each of the following systems. Justify your answers:-

a) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

Sol:

1) Memoryless:-
This system clearly is not memoryless as it is dependent on past and present values of the input. So, it is with memory.

2) Time Invariant:-

ST-1 $\quad y(t) \xrightarrow{t_0} y(t-t_0) = x(t-t_0) + x(t-t_0-2) \Rightarrow y_1(t)$

ST-2 $\quad x(t) \xrightarrow{t_0} x(t-t_0) \Rightarrow sys \rightarrow x(t-t_0) + x[(t-t_0)-2]$

$x(t-t_0) + x[t-2-t_0] \Rightarrow y'(t)$

$y_1(t) = y'(t)$ hence the system is Time invariant

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3) Linear :-

a) law of addition :-

$x_1(t) \rightarrow sys \rightarrow y_1(t) = x_1(t) + x_1(t-2)$

$x_2(t) \rightarrow sys \rightarrow y_2(t) = x_2(t) + x_2(t-2)$

$y_1(t) + y_2(t) = x_1(t) + x_1(t-2) + x_2(t) + x_2(t-2) \Rightarrow y_1(t)$

$x_1(t) + x_2(t) \rightarrow sys \rightarrow y'(t) = x_1(t) + x_1(t-2) + x_2(t) + x_2(t-2)$

$y_1(t) = y'(t)$ law satisfied.

b) law of homogenity :-

S-1 $x(t) \Rightarrow sys \Rightarrow y(t) \Rightarrow 'k' \Rightarrow ky(t) \Rightarrow k\,x(t) + k\,x(t-2)$

S-2 $x(t) \rightarrow 'k' \rightarrow kx(t) \rightarrow sys \rightarrow y'(t) = k[x(t)] + k[x(t-2)]$

as both are equal so law is satisfied.

Since law of addition and law of homogenity is satisfied, hence the system is linear.

4) Causal :-

Since the given output is dependent on present and past i/p hence the system is Causal.

5) Stable :-

Since for the given function the i/p amplitude is between $-\infty$ to ∞ the system is stable. ~~the~~

b) $y[n] = nx[n]$.

Sol:-

1) Memoryless:-

Since $y[n]$ only depends on the present value of $x[n]$, hence it is memoryless.

2) Time Invariant:-

$$x[n] \rightarrow sys \rightarrow nx[n] = y[n]$$

S-1:-

$$y[n] \xrightarrow{n_0} y[n-n_0] = (n-n_0)\, x[n-n_0] = y_1[n]$$

S-2:-

$$x[n] \xrightarrow{n_0} x[n-n_0] \rightarrow sys \rightarrow nx[n-n_0] = y'[n]$$

since, $y_1[n] \neq y'[n]$ the system is time variant.

3) Linearity:-

a) law of addition:-

$$x_1[n] \rightarrow sys \rightarrow y_1[n] = n\,x_1[n]$$
$$x_2[n] \rightarrow sys \rightarrow y_2[n] = nx_2[n]$$

$$y_1[n] + y_2[n] = nx_1[n] + nx_2[n] \Rightarrow n\left(x_1[n] + x_2[n]\right) = y_3[n]$$

$$x_1[n] + x_2[n] \rightarrow sys \rightarrow y'[n] = n\left(x_1[n] + x_2[n]\right)$$

$\therefore$ $y_3[n] = y'[n]$ law is satisfied.

b) law of homogenity :

S-1 : $x[n] \rightarrow sys \rightarrow y[n] \rightarrow 'k' \rightarrow ky[n] = knx[n]$

S-2 :- $x[n] \rightarrow 'k' \rightarrow kx[n] \rightarrow sys \rightarrow y'[n] = knx[n]$

$\therefore ky[n] = y'[n]$ law is satisfied

As law of superposition is satisfied the system is linear.

4) Causality :-

Since the o/p $y[n]$ is independent of future values of i/p the system is causal.

5) Stability :-

$$y[n] = nx[n]$$

$$x[n] \rightarrow sys \rightarrow y[n] = n \cdot x[n]$$

let's select $u[n]$ as bounded i/p

$u[n] \rightarrow sys \rightarrow y'[n] = nu[n] \rightarrow$ unit ramp

as $y'[n]$ grows without bound despite a bounded input. The system is therefore not stable.

→ The signal energy in the signal $x(t)$ is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

For discrete $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

→ The signal power in the signal $x(t)$ is

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

For discrete $P = \lim_{N \to \infty} \left[\frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 \right]$

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Q2:- Compute the signal energy and signal power for the discrete time signal:

$$x[n] = \left[\frac{1}{4}\right]^n u(n)$$

Sol:-

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} \left|\left(\frac{1}{4}\right)^n\right|^2$$

$$= \sum_{n=0}^{\infty} \left[\frac{1}{16}\right]^n$$

The expression on the right hand side is a geometric series; hence, we have

$$\therefore S_\infty = \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

$$E = \frac{1}{1 - 1/16} = \frac{16}{15}$$

Since $0 < E < \infty$, signal $x(n)$ is an energy signal, consequently $P=0$.

Q3:- Compute the signal power and signal energy for the discrete-time signal:

$$x[n] = e^{10jn} u(n)$$

Sol:-

$$P = \lim_{N\to\infty} \left[\frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2\right]$$

$$= \lim_{N\to\infty} \left[\frac{1}{2N+1} \sum_{n=0}^{N} 1_n^2\right] = \lim_{N\to\infty} \frac{N+1}{2N+1} \Rightarrow \frac{1}{2}$$

since $0 < P < \infty$, the signal $x[n]$ is a power signal and its $E = \infty$