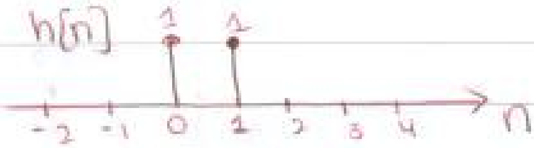
Day/Date Tuesday / 27 - March - 18

Lecture # 4.

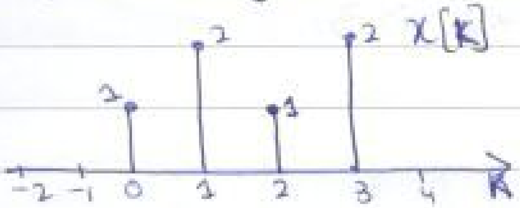
Example # 1:-

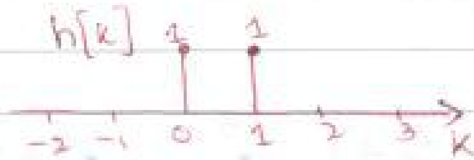
Consider the signal $x[n]$ and the impulse response $h[n]$ shown below:



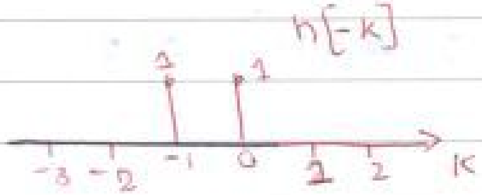
Sol:

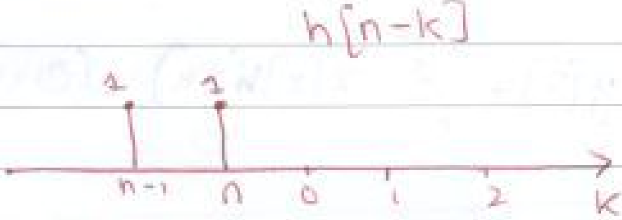
Step 1:- Change n with k.





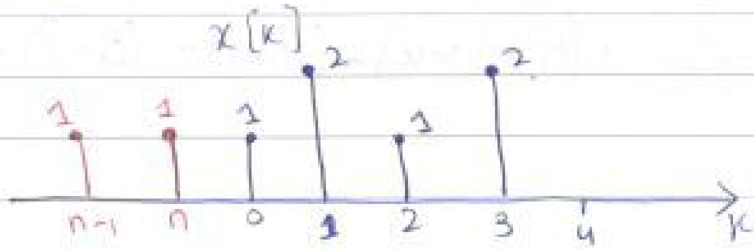
Step 2:- Reverse $h[k]$ and then shift it





Step 3:- Now slide $h[n-k]$ on $x[k]$ and multiply each overlapping point. and ~~sum~~ apply summation.

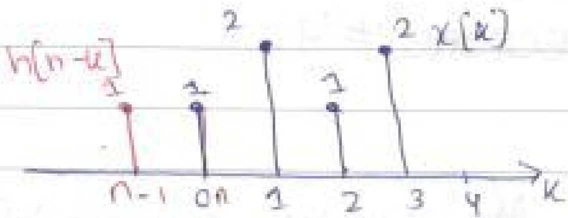
when $n<0$



There is no overlapping. Hence $x[k] \cdot h[n-k] = 0$ $\quad y[n] = 0$

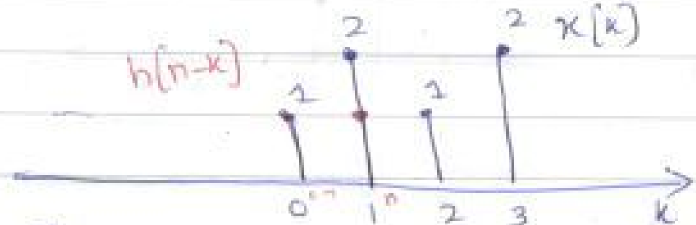
Day/Date

$\Rightarrow n=0$



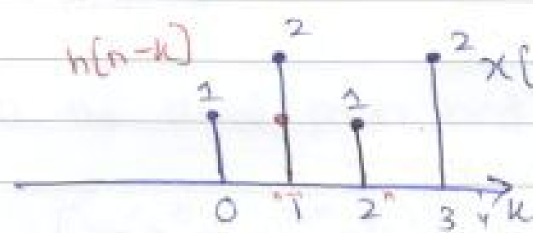
$$y[0]=\sum_{k=-\infty}^{\infty} x[k]h[0-k]=(1\times 0)+(1\times 1)\Rightarrow 1$$

$\Rightarrow n=1$



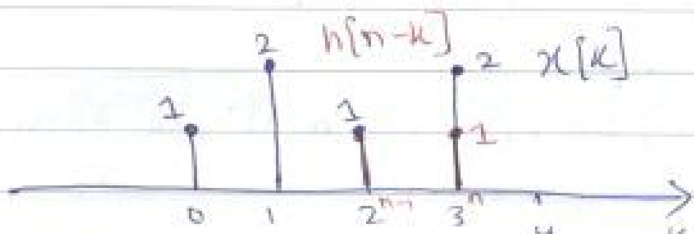
$$y[1]=\sum_{k=-\infty}^{\infty} x[k]h[1-k]\Rightarrow (1\times 1)+(1\times 2)\Rightarrow 3$$

$\Rightarrow n=2$



$$y[2]=\sum_{k=-\infty}^{\infty} x[k]h[2-k]=(0\times 1)+(1\times 2)+(1\times 1)\Rightarrow 3$$

$\Rightarrow n=3$

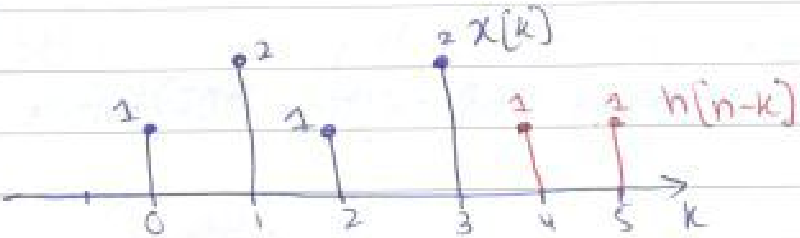


$$y[3]=\sum_{k=-\infty}^{\infty} x[k]h[3-k]=(0\times 1)+(0\times 2)+(1\times 1)+(1\times 2)\Rightarrow 3$$

$\Rightarrow n=4$

$$y[4]=\sum_{k=-\infty}^{\infty} x[k]\,h[4-k] = (1\times 2)+(1\times 0) \Rightarrow 2$$

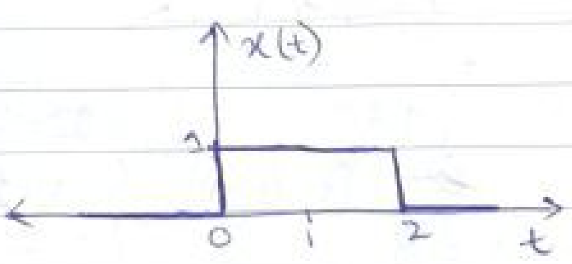
$\Rightarrow n>4$

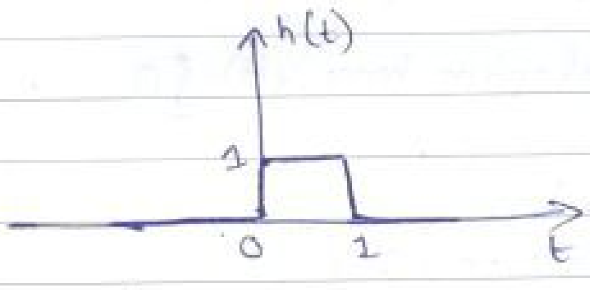


hence there is no overlapping b/w $x[k]$ and $h[n-k]$ so, $y[n]=0$.

$$y[n]=\begin{cases} 0 & , n<0 \\ 1 & , n=0 \\ 3 & , n=1 \\ 3 & , n=2 \\ 3 & , n=3 \\ 2 & , n=4 \\ 0 & , n>4 \end{cases}$$

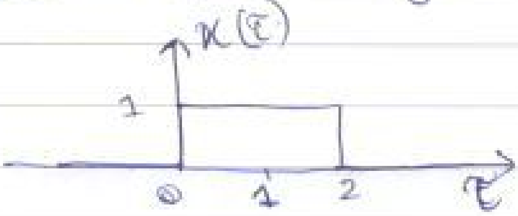
EXAMPLE #2:

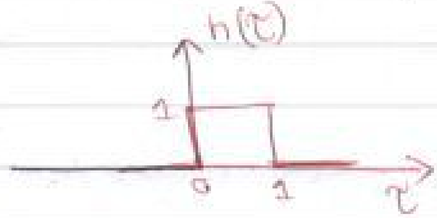




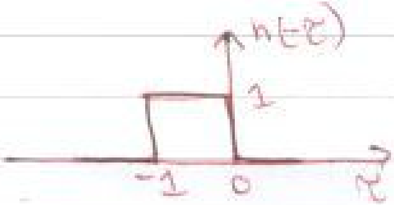
Sol:-

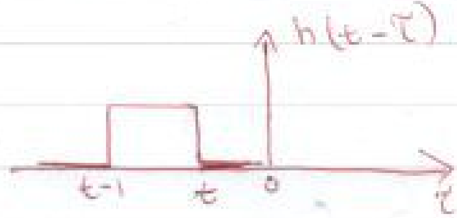
STEP#1:- Replace t by τ.





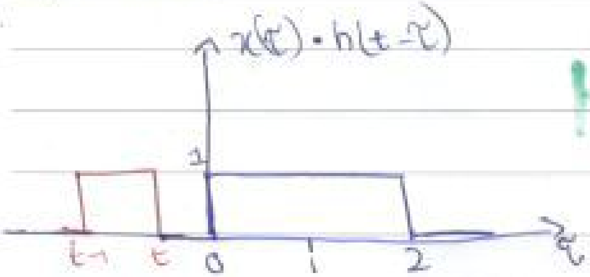
STEP 2:- Perform timereversal and shifting on one of the signal. here we are reversing and shifting $h(\tau)$ while keeping $x(\tau)$ fixed.





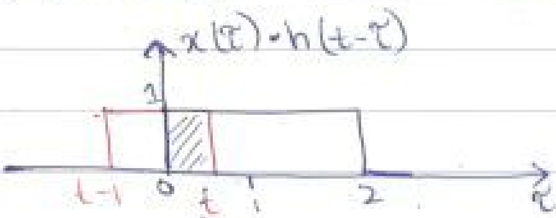
STEP 3:- Now slide $h(t-\tau)$ over $x(\tau)$ and multiply the overlapping portion and integrate.

$\Rightarrow t < 0$



no overlapping hence, $y(t) = 0$

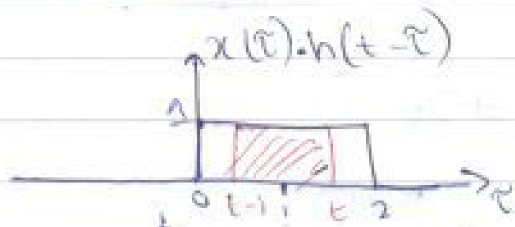
$\Rightarrow 0 < t < 1$



$$y(t) = \int_0^t (1 \times 1) d\tau$$

$$y(t) = \tau\Big|_0^t \Rightarrow [t-0] \Rightarrow t$$

$\Rightarrow 1 < t < 2$

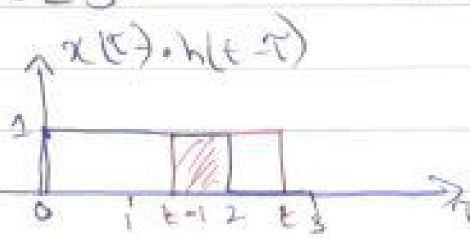


$$y(t) = \int_{t-1}^{t} 1\, d\tau \Rightarrow \tau\Big|_{t-1}^{t}$$

$$= [t-(t-1)] \Rightarrow (t - t + 1)$$

$$\Rightarrow 1$$

$\Rightarrow 2 < t < 3$

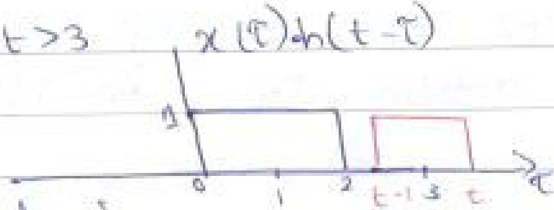


$$y(t) = \int_{t-1}^{2} 1\, d\tau \Rightarrow \tau \Big|_{t-1}^{2}$$

$$= [2 - (t-1)] = 2 - t + 1$$

$$= 3 - t$$

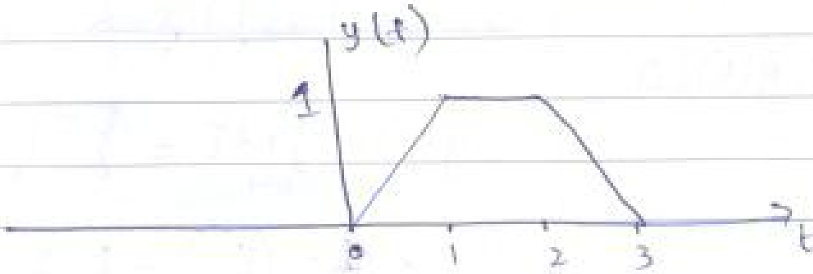
$\Rightarrow$ when $t > 3$



no overlapping

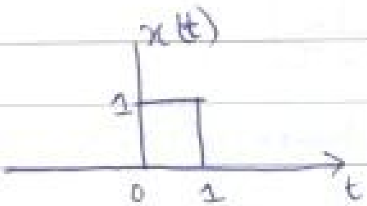
$y(t) = 0$

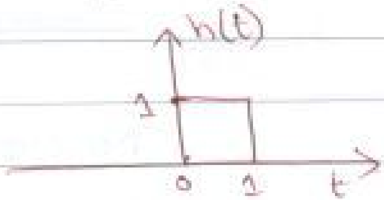
$$y(t) = \begin{cases} 0 & , t < 0 \\ t & , 0 < t < 1 \\ 1 & , 1 < t < 2 \\ 3-t & , 2 < t < 3 \\ 0 & , t > 3 \end{cases}$$



EXAMPLE # 3:-

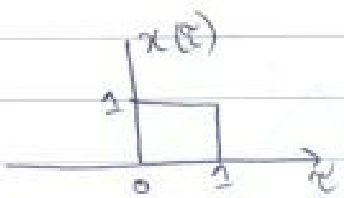
Convolve the two continuous time signals.

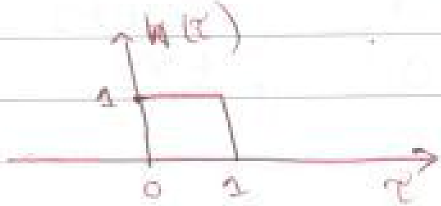




Sol:-

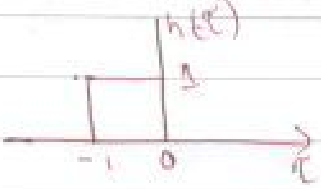
Step 1:- Replace t by τ

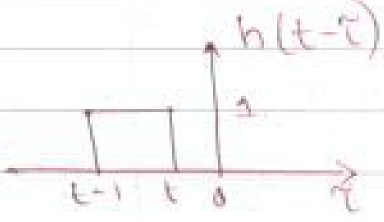




Step 2:- Reverse and shift one of the signal.

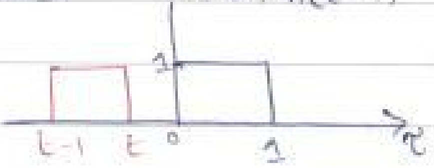
applying step 2 on $h(t)$





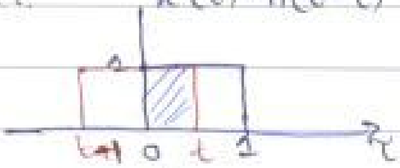
Step 3:- Perform multiplication and integration.

$\Rightarrow t < 0$



no overlapping hence, $y(t) = 0$

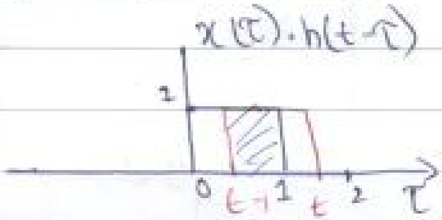
$\Rightarrow$ ~~t~~ $0 < t < 1$



$$y(t) = \int_0^t 1\, d\tau = \tau\Big|_0^t$$

$$\Rightarrow t - 0 \Rightarrow t$$

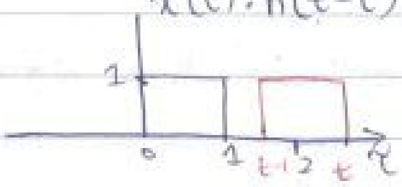
$\Rightarrow$ ~~t~~ $1 < t < 2$



$$y(t) = \int_{t-1}^{1} 1\, d\tau = \tau\Big|_{t-1}^{1}$$

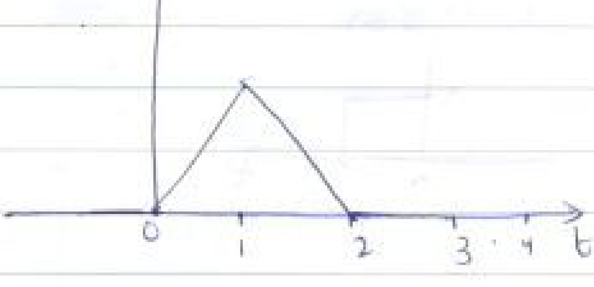
$$= 1 - (t-1) = 1 - t + 1 \Rightarrow 2 - t$$

$\Rightarrow$ when $t > 2$



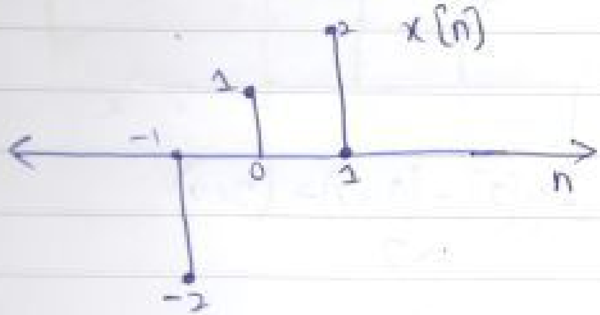
no overlapping hence, $y(t) = 0$

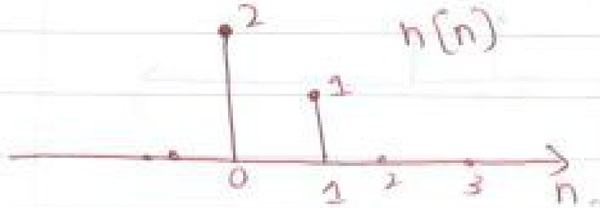
$$y(t) = \begin{cases} 0 & , t < 0 \\ t & , 0 < t < 1 \\ 2 - t & , 1 < t < 2 \\ 0 & , t > 2 \end{cases}$$



Example #4:

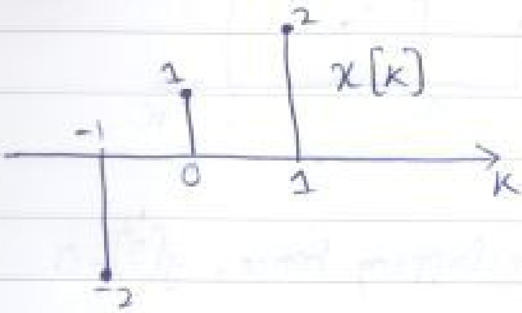
Convolve the two discrete time signals:

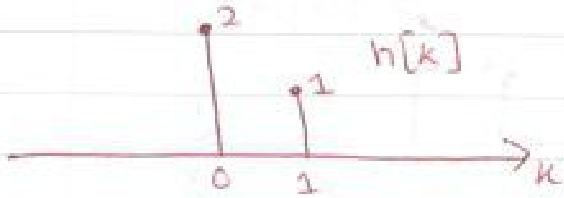




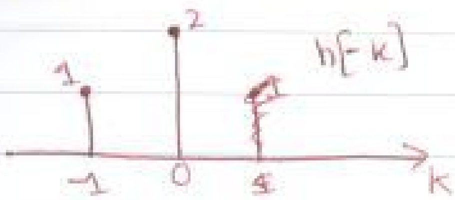
Sol:

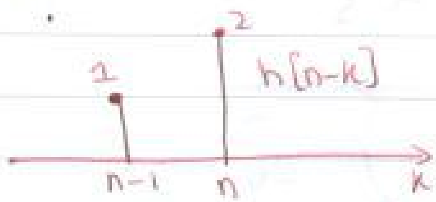
Step 1: Replace n by k.





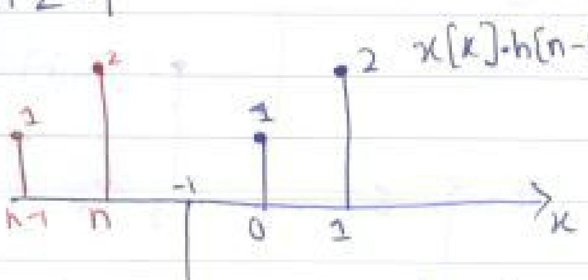
Step 2: Reverse and shift h[k].





Step 3: Multiply two signals and add up the results.

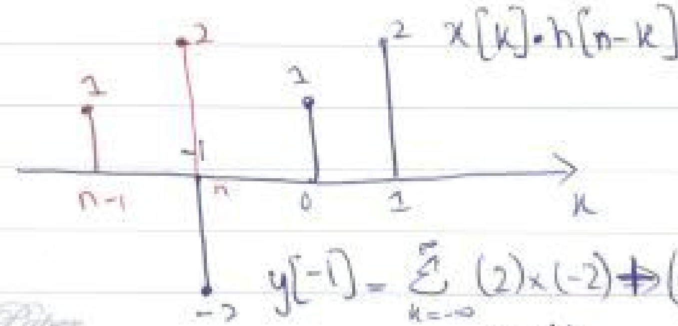
⇒ $n < -1$



no overlapping

$y[n] = 0$

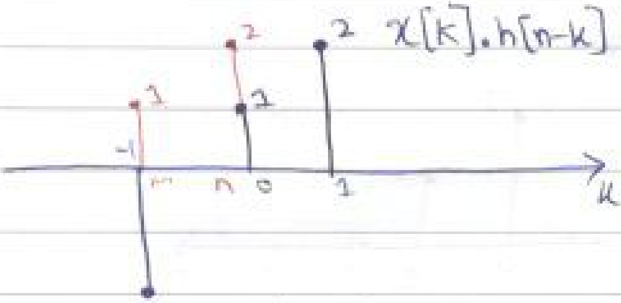
⇒ $n = -1$



$y[-1] = \sum_{k=-\infty}^{\infty} (2)\times(-2) + (0\times 1)$

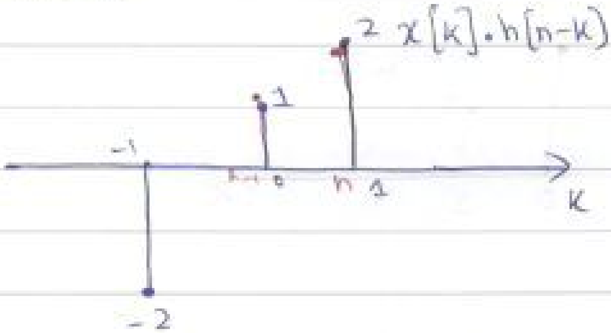
$\Rightarrow -4.$

$\Rightarrow n=0$



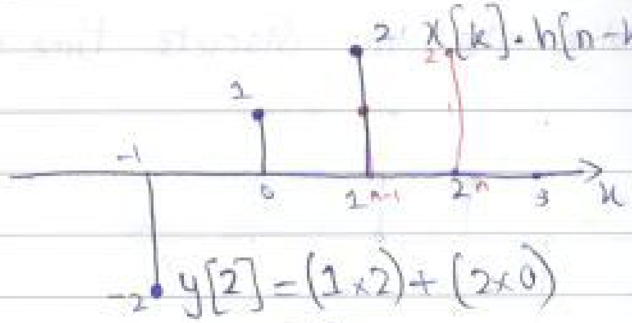
$y[0] = (2\times 1) + (1\times -2)$
$= 2 + (-2) \Rightarrow 0$

$\Rightarrow n=1$



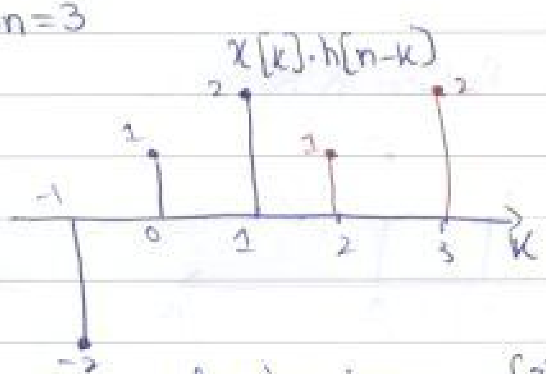
$y[1] = (1\times 1) + (2\times 2)$
$= 1 + 4 \Rightarrow 5$

$\Rightarrow n=2$



$y[2] = (1\times 2) + (2\times 0)$
$\Rightarrow 2$

$\Rightarrow n=3$



no overlapping hence, $y[3]=0$

$$y[n] = \begin{cases} 0 & , n < -1 \\ -4 & , n = -1 \\ 0 & , n = 0 \\ 5 & , n = 1 \\ 2 & , n = 2 \\ 0 & , n = 3 \end{cases}$$