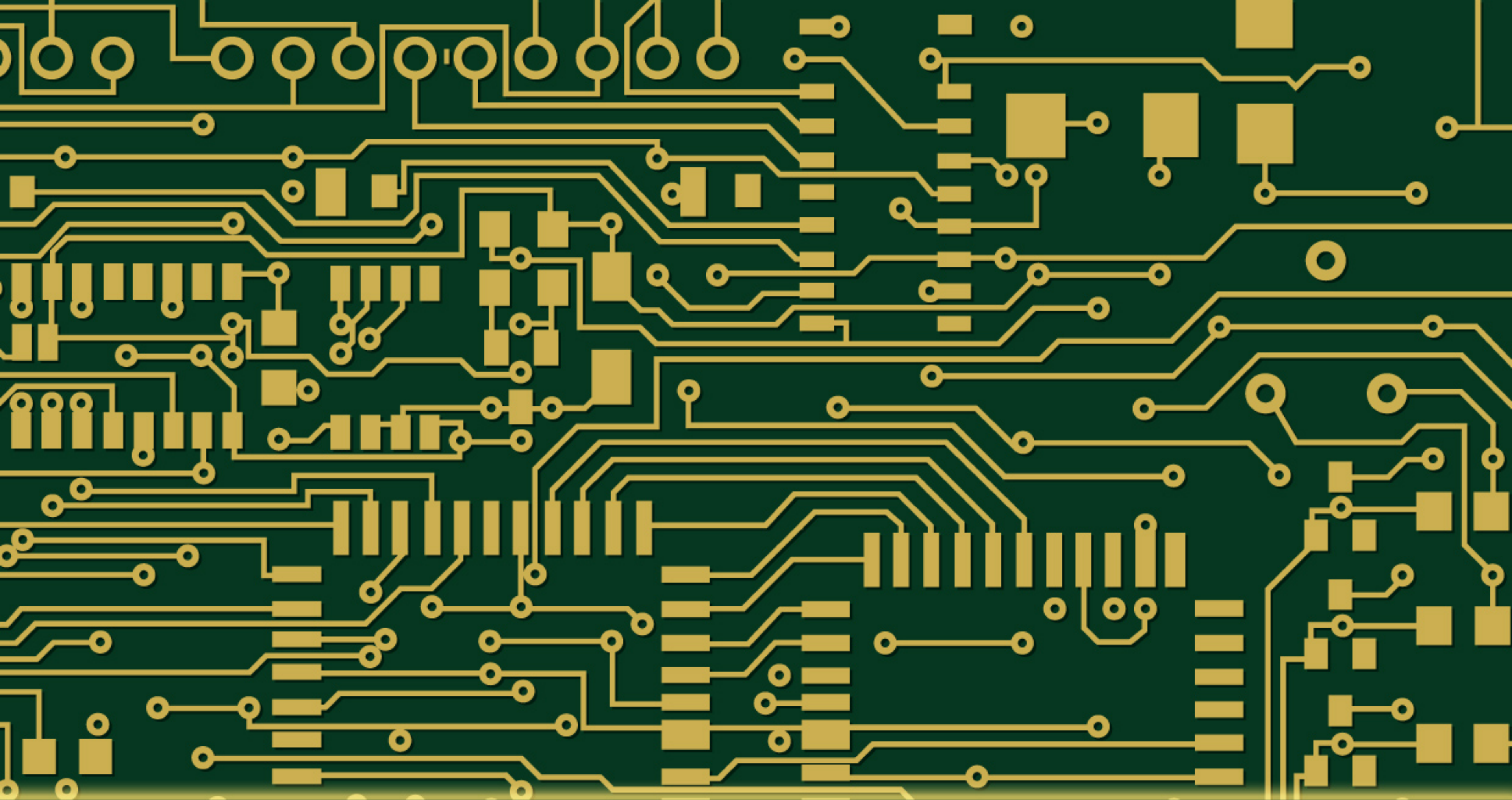
Circuit Analysis-III



Sinusoids

Example #1

- ✓ Find the amplitude, phase, period and frequency of the sinusoid:

$$v(t) = 12\cos(50t + 10°)$$

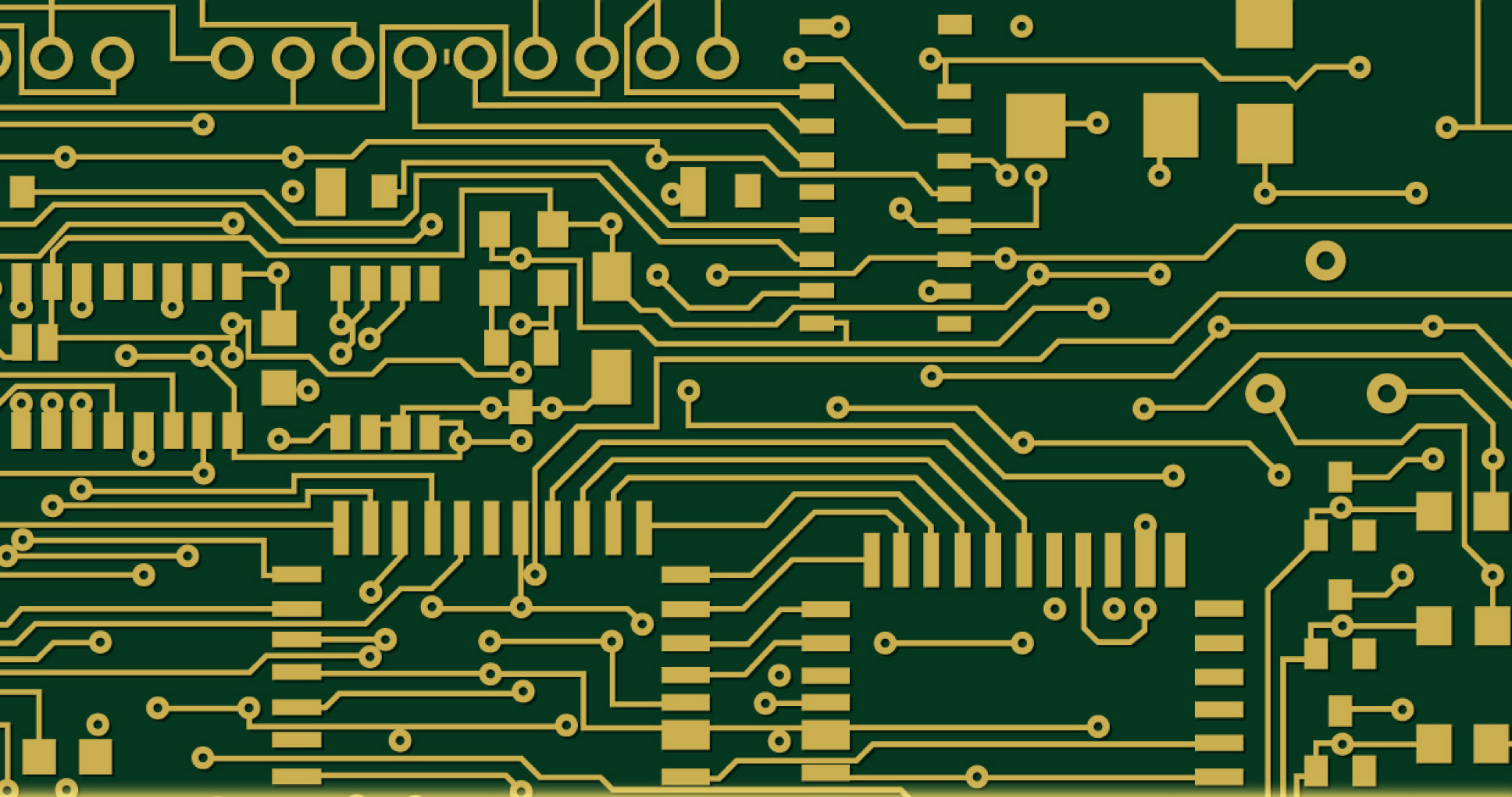
Signal Conversion

- ✓ From sine to cosine and vice versa.
  - ✓ $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
  - ✓ $\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$
- ✓ With trigonometric identities:
  - ✓ $\cos(\omega t) = \sin(\omega t + 90°)$
  - ✓ $-\cos(\omega t) = \sin(\omega t - 90°)$
  - ✓ $\sin(\omega t) = \cos(\omega t - 90°)$
  - ✓ $-\sin(\omega t) = \cos(\omega t + 90°)$

Example #2

✓ Convert the following into the cosine form:

$$v(t) = 5\sin(4\pi t - 60°)$$



Phasors

Definition

- ✓ A phasor is a complex number that represents the amplitude and phase of a sinusoid.
- ✓ It is a vector that represents a sinusoidal varying quantity by means of a line rotating about a point in a plane.
- ✓ The length of the line being proportional to the magnitude of the quantity.
- ✓ The angle between the line and a reference line being equal to the phase of the quantity.

Phasors Representation

- ✓ A complex number z can be written in rectangular form as:

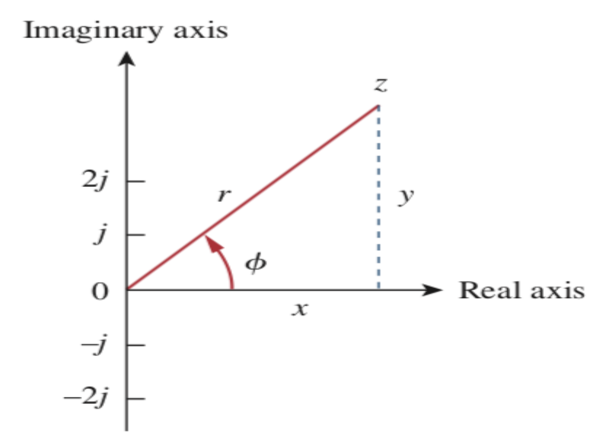
$$z = x + jy \quad where \quad j = \sqrt{-1}$$

- ✓ Where x is the real part of z and y is the imaginary part of z.
- ✓ In polar and exponential form complex number z can be written as:

$$z = r\angle\phi = re^{j\phi}$$

- ✓ Where r is the magnitude of z and Φ is the phase of z.

Relationship b/w Rectangular & Polar Form



- ✓ Where x represents the real part and the y axis represents the imaginary part of a complex number.
- ✓ Given x and y, we can get r and Φ as:

$$r = \sqrt{x^2 + y^2} \quad , \quad \phi = \tan^{-1}\frac{y}{x}$$

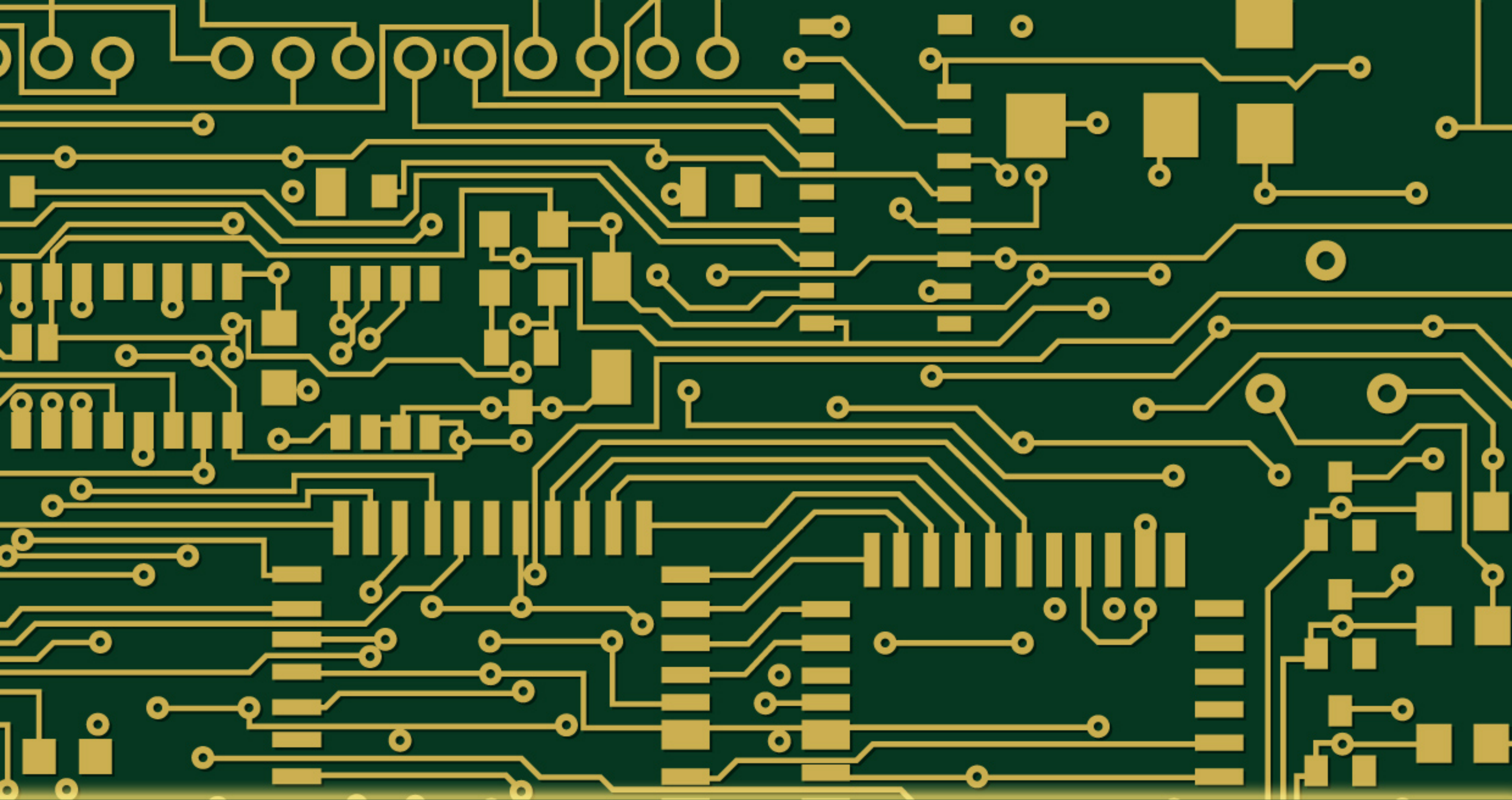
- ✓ On the other hand, if we know r and Φ, we can obtain x and y as:

Relationship b/w Rectangular & Polar Form (cont.)

$$r = \cos\phi \quad , \quad y = r\sin\phi$$

- ✓ Thus, z may be written as:

$$z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)$$



Operations on Phasors

Complex Number Operations

- ✓ Addition and subtraction of complex numbers are better performed in rectangular form; multiplication and division are better done in polar form.
- ✓ Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

- ✓ Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

- ✓ Multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

- ✓ Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

Complex Number Operations (cont.)

- ✓ Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle -\phi$$

- ✓ Square root:

$$\sqrt{z} = \sqrt{r}\angle \phi / 2$$

- ✓ Complex conjugate:

$$z^* = x - jy = r\angle -\phi = re^{-j\phi}$$

Representing A Phasor

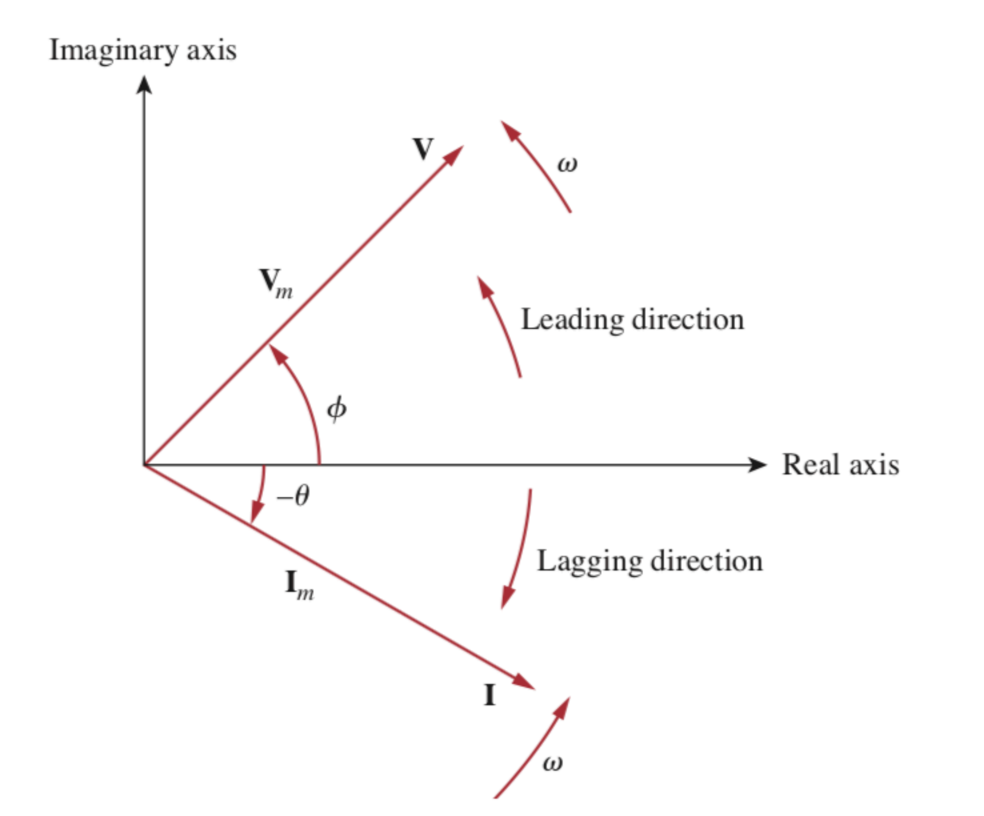
- ✓ Time domain representation:

$$v(t) = V_m \cos(\omega t + \phi)$$

- ✓ Phasor domain representation:

$$V = V_m \angle \phi$$

Representing A Phasor (cont.)



Sinusoid-Phasor Transformation

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

Example #3

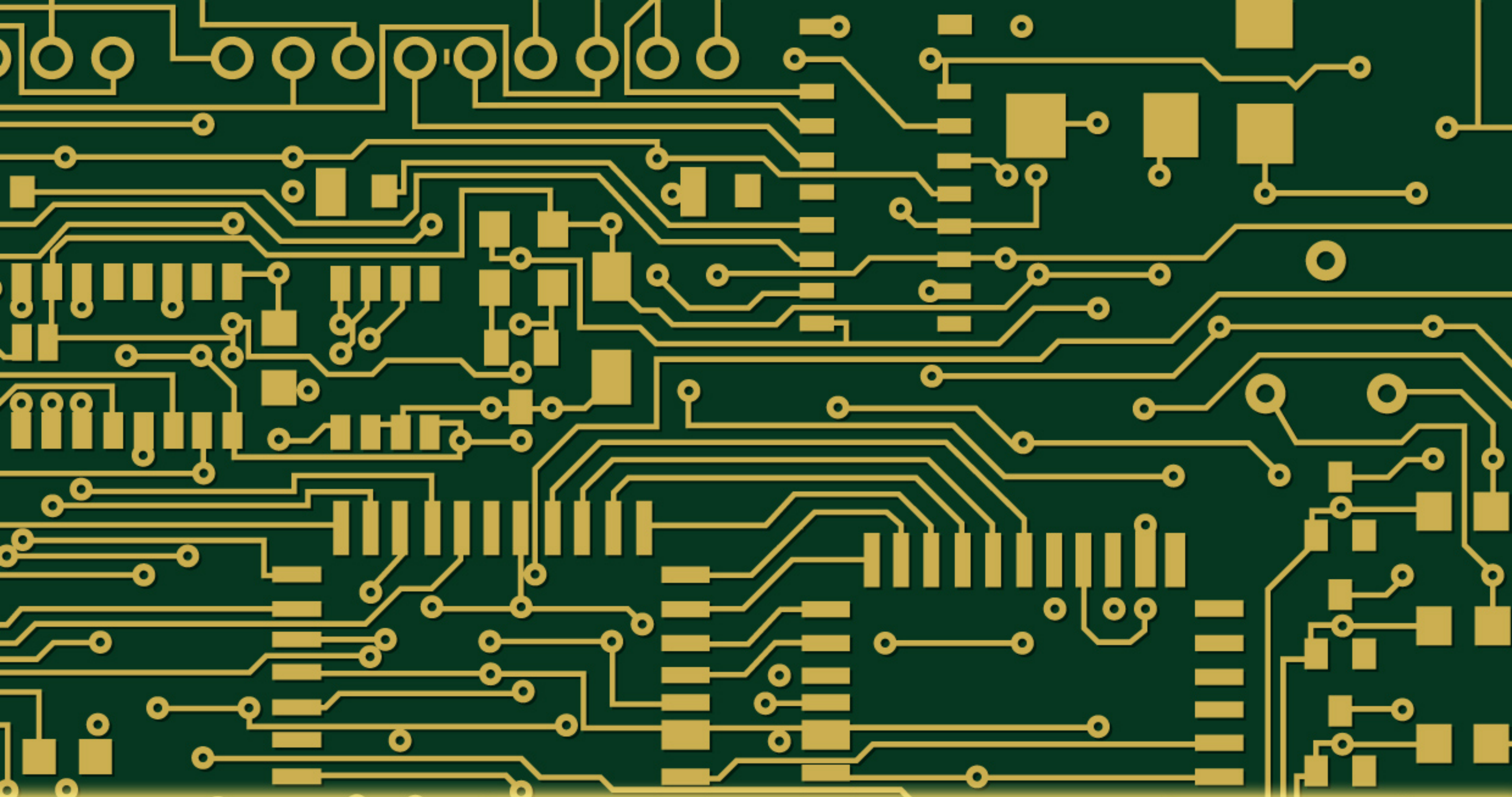
✓ Transform the sinusoid to phasors:

$$i = 6\cos\left(50t - 40^\circ\right) A$$

Example #4

- ✓ Find the sinusoid representation by the phasor:

$$\bar{I} = -3 + j4A$$



Phasor Relationships for Circuit Elements

V-I Relationships For Resistors

- ✓ Note: while writing phasors , we can write them with a bar over the letter.
- ✓ If the current through a resistor R is: $i = I_m \cos(\omega t + \phi)$
- ✓ The voltage across it is given by Ohm's law as:

$$v = iR = Ri = RI_m \cos(\omega t + \phi)$$

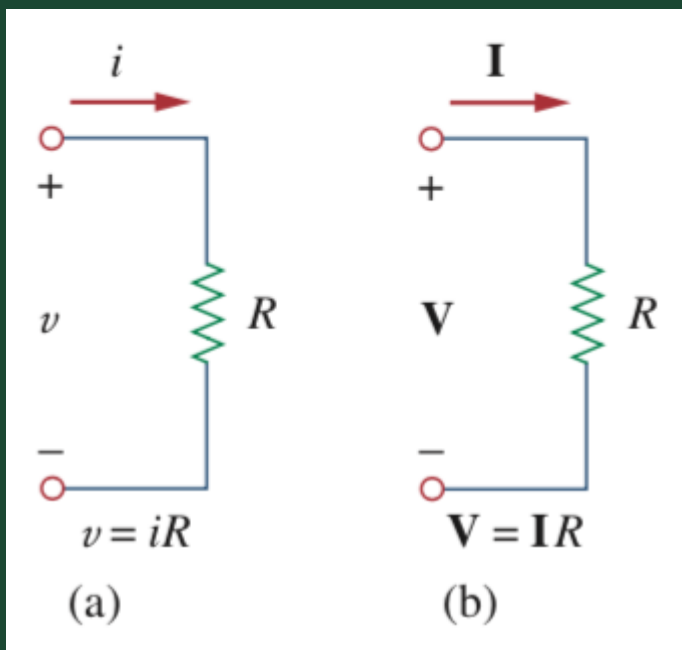
- ✓ The phasor form of this voltage is:

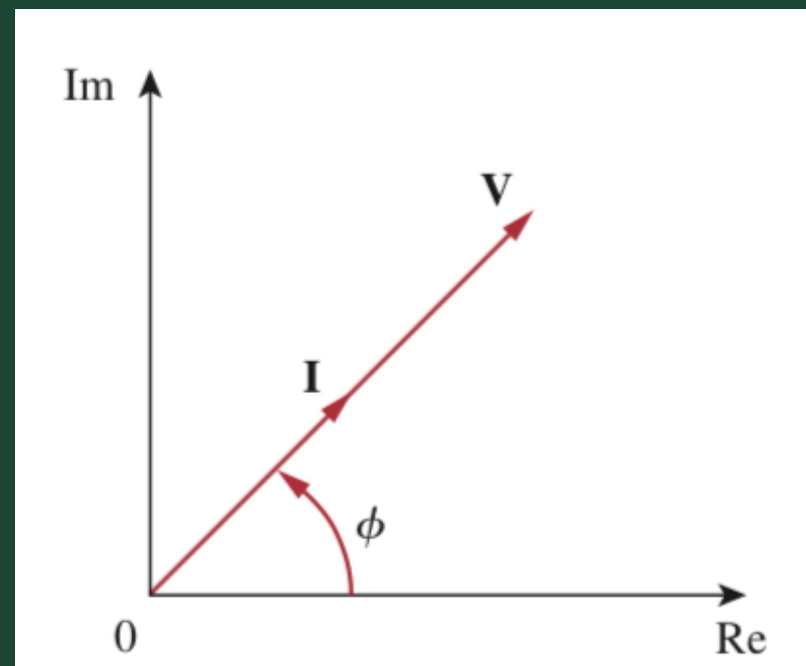
$$\bar{V} = R\,\mathrm{I}_m \angle \phi$$

- ✓ But the phasor representation of the current is $\mathrm{I}_m \angle \phi$. Hence:

$$\bar{V} = R\bar{I}$$

V-I Relationships For Resistors (cont.)





V-I Relationships For Inductors

- For the inductor L, assume the current through it is:

$$i = I_m \cos(\omega t + \phi)$$

- The voltage across the inductor is:

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

- As –sin A=cos (A+90°), we can write:

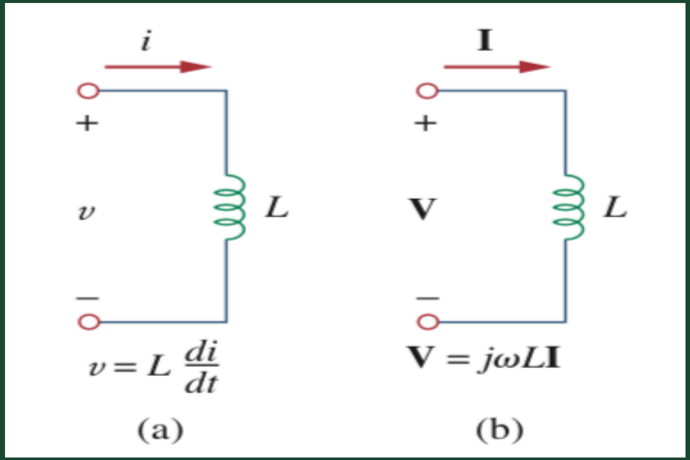
$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

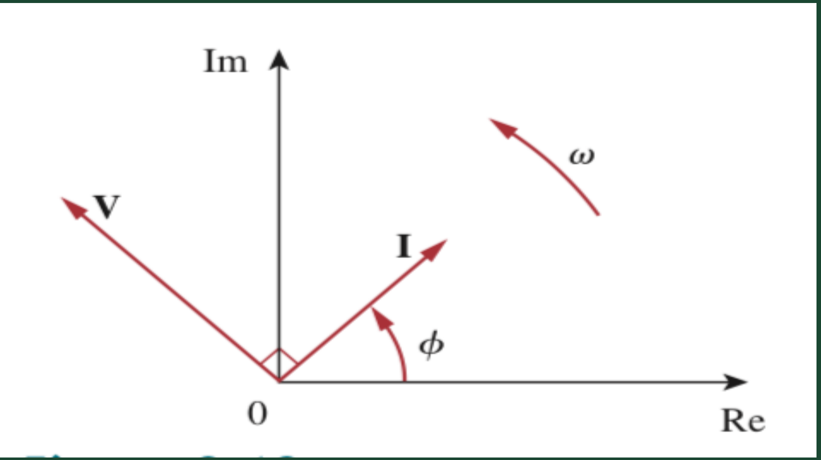
- Which transforms to the phasor:

$$\bar{V} = \omega L I_m e^{j(\phi + 90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \angle \phi + 90^\circ$$

V-I Relationships For Inductors (cont.)

- ✓ But $$I_m \angle \phi = \bar{I}, \quad e^{j90^\circ} = j$$
- ✓ Thus, $$\bar{V} = j\omega L \bar{I}$$
- ✓ The voltage and current are 90° out of phase.





V-I Relationships For Capacitors

- For the capacitor C, the voltage across it is:

$$v = V_m \cos(\omega t + \phi)$$

- The current through the capacitor is:

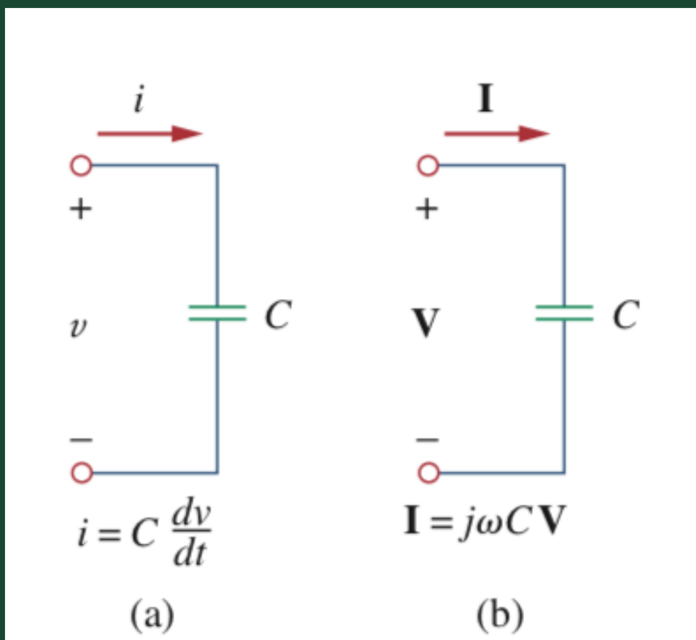
$$i = C\frac{dv}{dt}$$

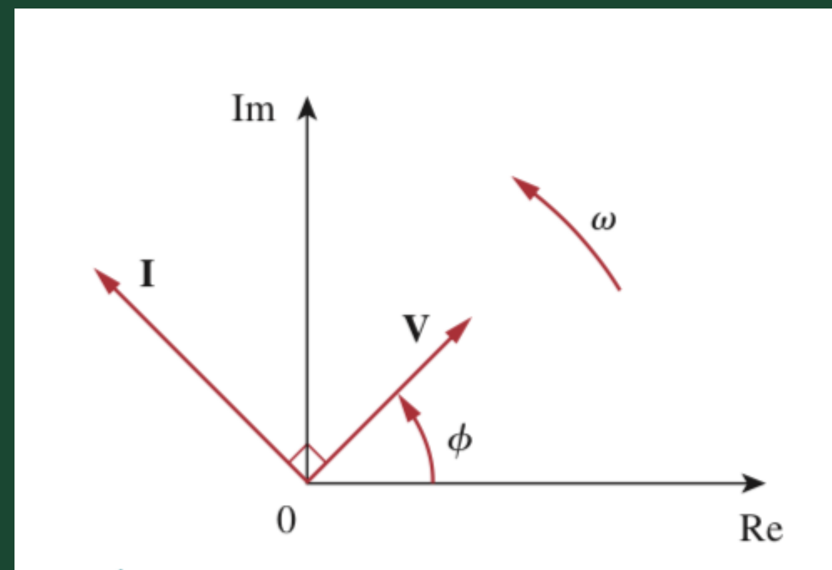
- By following the same steps we took for the inductor, we obtain:

$$\bar{I} = j\omega C\bar{V} \Rightarrow \bar{V} = \frac{\bar{I}}{j\omega C}$$

- The current leads the voltage by 90°.

V-I Relationships For Capacitors (cont.)



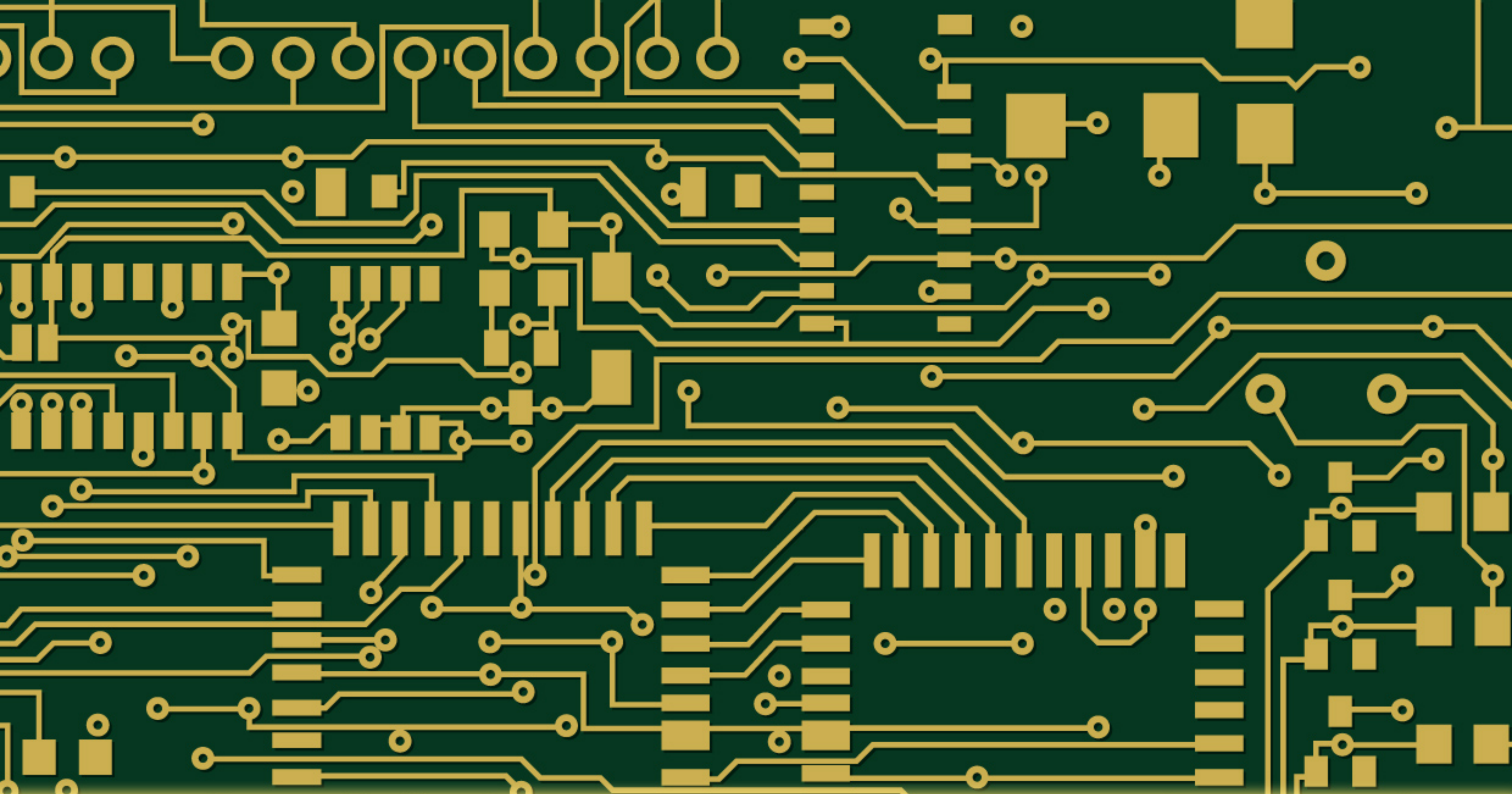


Summary of Voltage & Current Relationships

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\dfrac{dv}{dt}$	$\mathbf{V} = \dfrac{\mathbf{I}}{j\omega C}$

Example #4

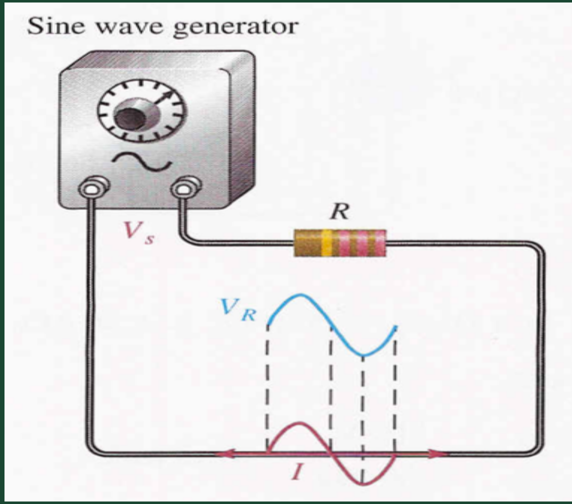
- ✓ The voltage $v=12\cos(60t + 45°)$ is applied to a 0.1H inductor. Find the steady-state current through the inductor.



AC Circuit Analysis

A.C. Analysis

- ✓ If a sinusoidal voltage is across a resistor there is a sinusoidal current.
- ✓ The current is zero when the voltage is zero and is maximum when the voltage is maximum.
- ✓ When the voltage changes polarity, the current reverses direction.
- ✓ As a result, the voltage and current are said to be in phase with each other.

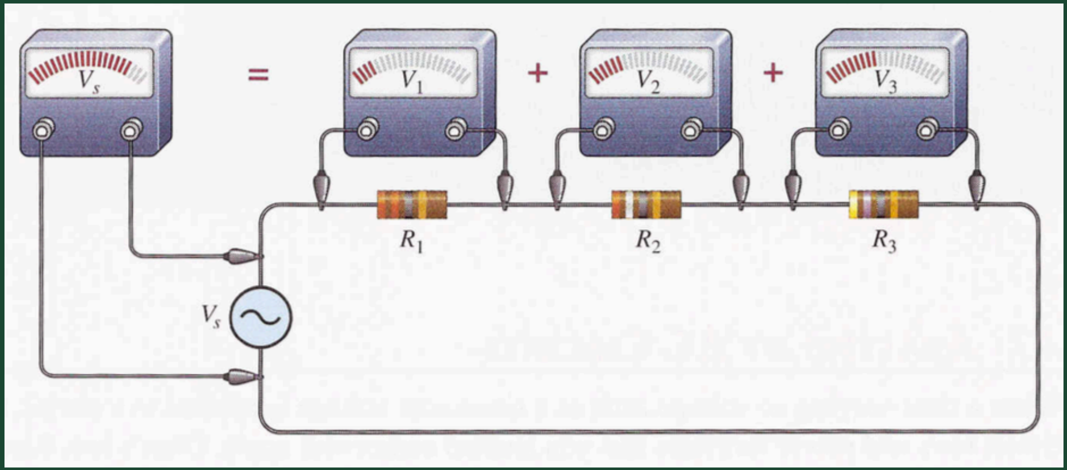


Ohm’s Law

- ✓ When ohm’s law is used in A.C. circuits, both the voltage and the current must be expressed consistently, i.e., both as peak values, both as rms values, both as average values and so on.

Kirchhoff's Voltage

- ✓ Kirchhoff's voltage law in a resistive circuit that has a sinusoidal voltage source is shown below:



- ✓ The source voltage is the sum of all the voltage drops across the resistors, just as in a dc circuit.

Power

- ✓ Power in resistive A.C. circuits is determined the same as for dc circuits except that you must use rms values of current and voltage.
- ✓ The general power formulas are restated for a resistive A.C. circuits as:

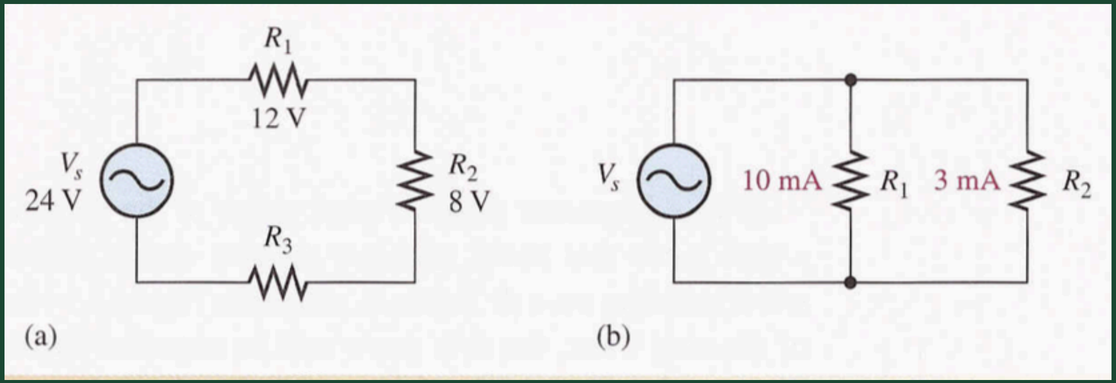
$$P = V_{rms} I_{rms}$$

$$P = \frac{V_{rms}^2}{R}$$

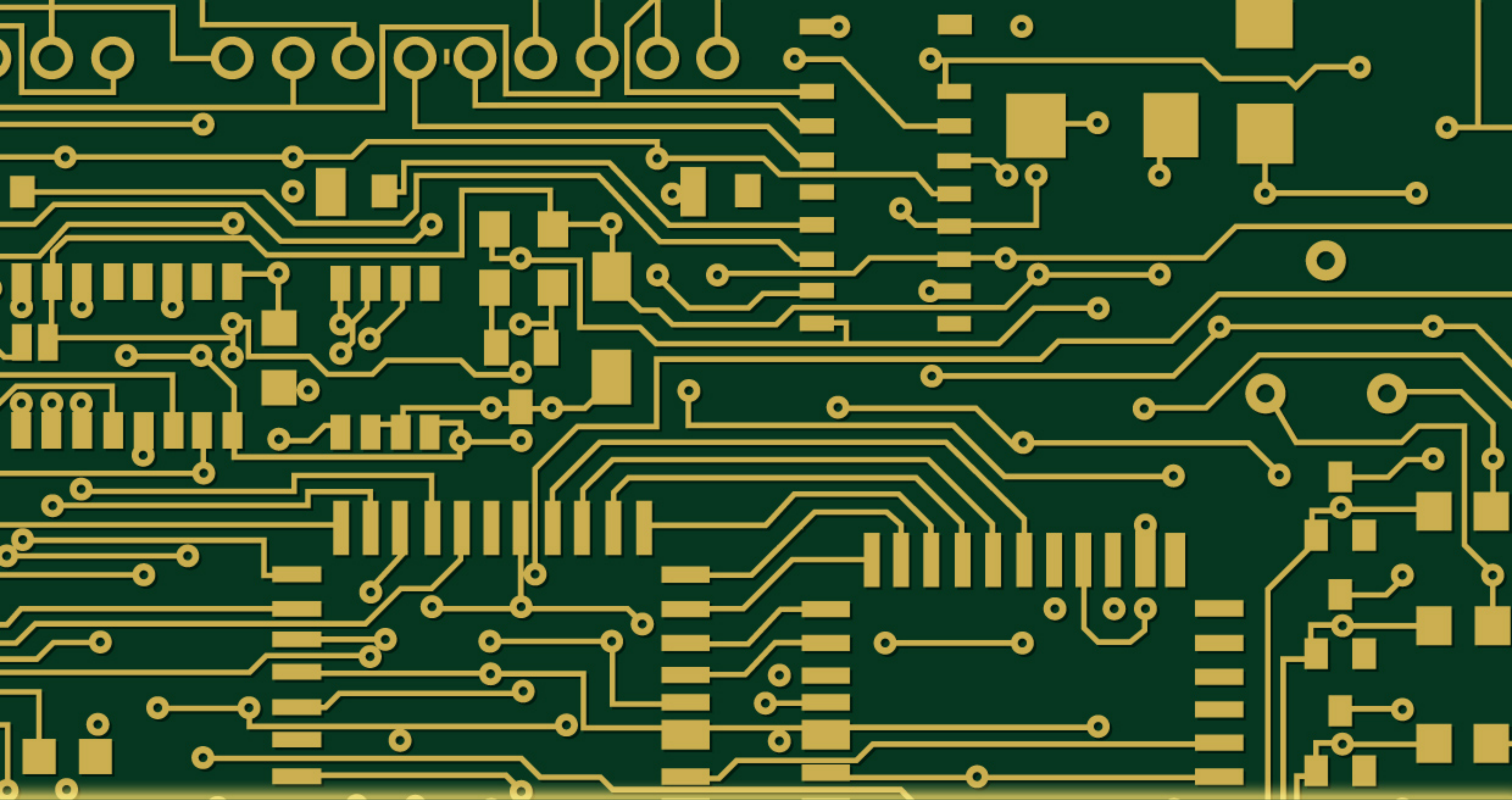
$$P = I_{rms}^2 R$$

Example #5

- ✓ Consider the figure below and calculate the following:



  - ✓ (a): Find the unknown peak voltage drop in fig: a.
  - ✓ (b): Find the total rms current in fig: b.
  - ✓ (c): Find the total power in fig: b if V_{rms}=24V.
- ✓ Note: All values in the circuits are given in rms.



Exercise Problems

Problem #1

- ✓ Given the sinusoidal voltage: v(t)= 50cos (30t + 10°) V.
  - ✓ What is the amplitude V_m?
  - ✓ What is the period T ?
  - ✓ The frequency f ?
  - ✓ Calculate v(t) at t = 10ms.

Problem #2

- ✓ Given $v_1 = 45 \sin(\omega t + 30°)$V and $v_2 = 50 \cos(\omega t - 30°)$V, determine the phase angle between the two sinusoids and which one lags the other.

Problem #3

- ✓ Find the phasors corresponding to the following signals:
  - ✓ (a): $v(t) = 21\cos(4t - 15°)$ V
  - ✓ (b): $i(t) = -8\sin(10t + 70°)$ mA

Problem #4

- ✓ Determine the current that flows through an 8-Ω resistor connected to a voltage source $v_s = 110\cos(377t)$ V.

Problem #5

- ✓ Given $i_1(t) = 4 \cos(\omega t + 30°)$ A and $i_2(t) = 5 \sin(\omega t - 20°)$ A, find their sum.

Thank You