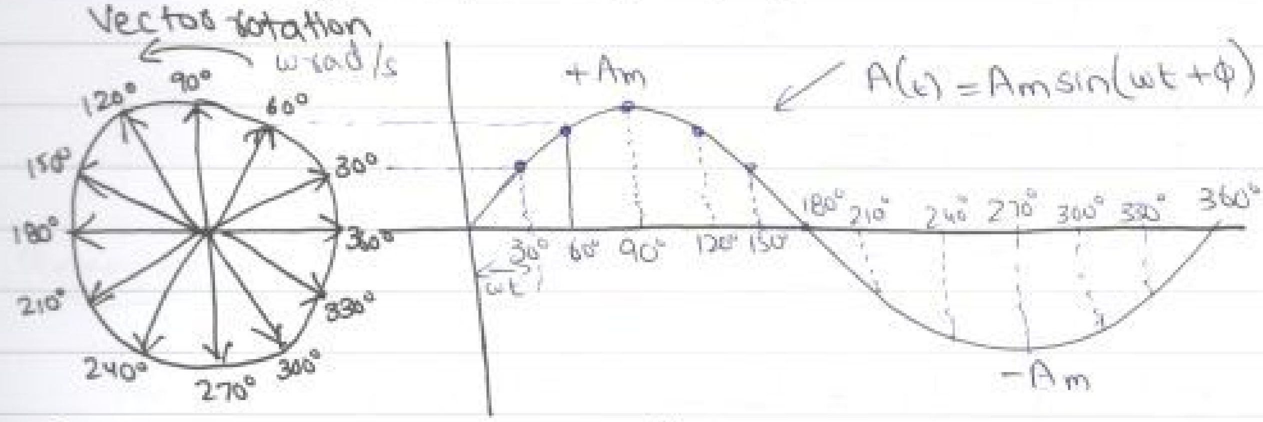
Day/Date Fri / 6-4-18

: Lecture #3 :-

Frequency f & Angular Frequency ω :



Rotating Phasor   Sinusoidal Waveform in the Time Domain

⇒ $\omega = 2\pi f$ rad/sec

⇒ ϕ phase ⇒ at which point in the cycle it is right now of $v(t)$ rad/sec

Phasors:-

⇒ A Phasor is a vector that represents a sinusoidally varying quantity by means of a line rotating about a point in a plane.

⇒ The length of the line being proportional to the magnitude of the quantity.

⇒ The angle between the line and a reference line being equal to the phase of the quantity.

Example #1 :

Find the amplitude, phase, period and frequency of the sinusoid

$$v(t) = 12 \cos(50t + 10^\circ)$$

Sol:

Comparing the given signal with

$$v(t) = V_m \cos(\omega t + \phi)$$

The amplitude is $V_m = 12V$

The phase is $\phi = 10^\circ$

The period is $T = \frac{2\pi}{\omega}$ $\quad \therefore \omega = 50 rad/s$

$$T = \frac{2\pi}{50} \Rightarrow 0.1257s$$

Frequency $f = \frac{1}{T} = \frac{1}{0.1257} \Rightarrow 7.958 Hz$

SIGNAL CONVERSION :-

$\Rightarrow$ From sine to cosine and vice versa.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$\sin(\omega t + 90^\circ) = \sin(\omega t + \frac{\pi}{2}) = \cos \omega t$

$\sin \omega t = \cos(\omega t - 90^\circ) = \cos(\omega t - \frac{\pi}{2})$

$\Rightarrow$ With these identities :-

$$\cos(\omega t) = \sin(\omega t + 90^\circ)$$
$$-\cos(\omega t) = \sin(\omega t - 90^\circ)$$
$$\sin(\omega t) = \cos(\omega t - 90^\circ)$$
$$-\sin(\omega t) = \cos(\omega t + 90^\circ)$$

EXAMPLE #2:

Convert the following into cosine form.

$$v(t) = 5\sin(4\pi t - 60^\circ)$$

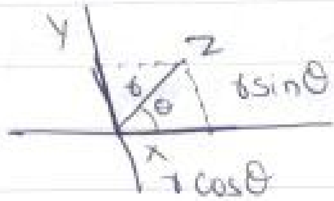
Sol:

$$\therefore \sin(\omega t) = \cos(\omega t - 90^\circ)$$
$$v(t) = 5\cos(4\pi t - 60^\circ - 90^\circ)$$
$$= 5\cos(4\pi t - 150^\circ)$$

⇒ We can write any complex number z as,

Rectangular form ⇒ $z = x + jy$ where $j = \sqrt{-1}$

$z = r(\cos\theta + j\sin\theta)$



Exponential form ⇒ $z = re^{j\theta}$ ∴ $e^{j\theta} = \cos\theta + j\sin\theta$

Polar form ⇒ $z = r\angle\theta$

REPRESENTING A PHASOR:

Time domain representation ⟺ Phasor domain representation

$v(t) = V_m \cos(\omega t + \phi)$ ⟺ $V = V_m\angle\phi$

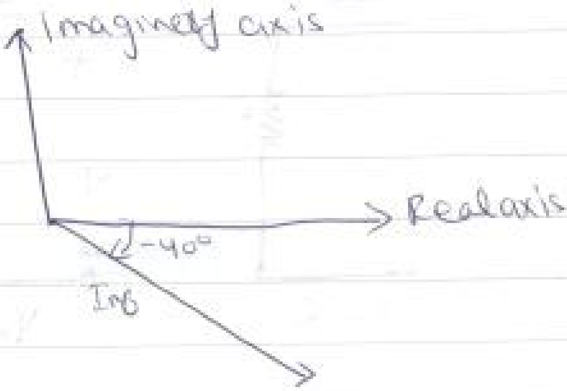
EXAMPLE #3:

Transform the sinusoid to phasor:

$$i = 6\cos(50t - 40^\circ)\,A$$

Sol:

$$I = 6\angle -40^\circ A$$



EXAMPLE #4:

Find the sinusoid representation by the phasor:

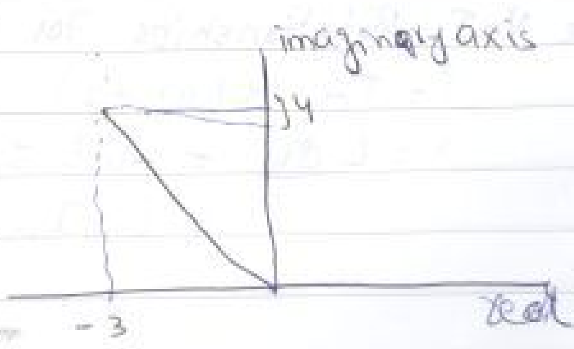
$$\bar{I} = -3 + j4A$$

Sol:

$\bar{I} = -3 + j4A$

$= 5\angle 126.87^\circ$ (Polar form)

$i(t) = 5\cos(\omega t + 126.87^\circ)\,A$



Phasor Relationships For Circuit Elements:-

⇒ While writing, you can write it as phasor with a bar over the letter.

$$\bar{V} = V_m \angle \phi$$

The V-I Relationships For Resistors:-

If $i = I_m \cos(\omega t + \phi)$

then $v = iR = Ri$

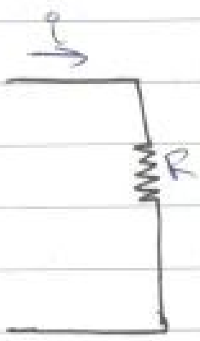
$$= R I_m \cos(\omega t + \phi)$$

⇒ Voltage and current are in same phase for resistor

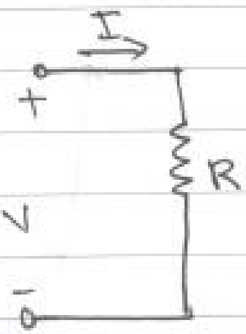
⇒ In Phasor

$$\bar{V} = R I_m \angle \phi$$

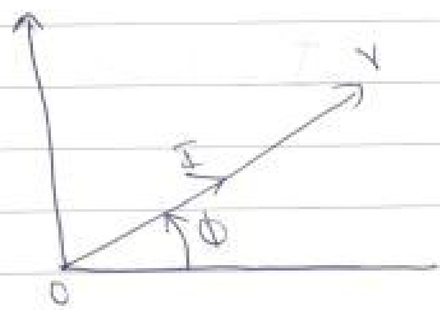
$$\bar{V} = R\bar{I}$$



$v = iR$



$V = IR$



The Phasor voltage and the Phasor current are in phase.

The V-I Relationships For Inductors:-

$$i = I_m \cos(\omega t + \phi)$$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

$$= \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

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$(\cos)' = -\sin$

In Phasor $\bar{V} = \omega L I_m e^{j(\phi + 90^\circ)}$

$= \omega L I_m e^{j\phi} e^{j90^\circ}$

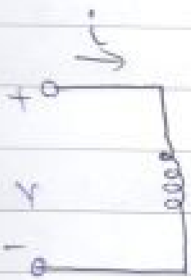
but $I_m e^{j\phi} = I_m \angle \phi = \bar{I}$

$e^{j90^\circ} = j$ (By Euler's formula)

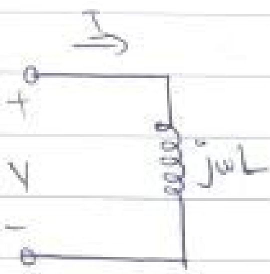
$e^{j90^\circ} = \cos 90^\circ + j \sin 90^\circ \Rightarrow 0 + j \Rightarrow j$

then $\bar{V} = j\omega L I$

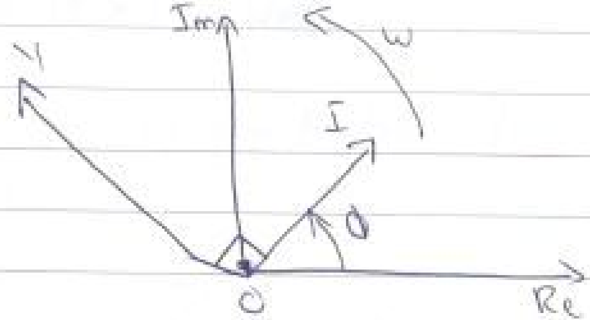
$\Rightarrow$ The 'j' sign indicates that there is a phase difference of 90° b/w V and I and voltage leads by 90°.



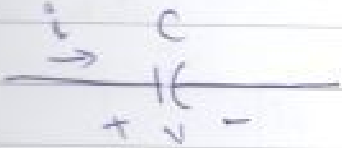
$v = L\frac{di}{dt}$



$\bar{V} = j\omega L \bar{I}$

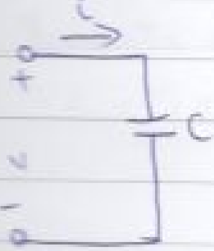


The V-I Relationships for Capacitor



$v = \frac{1}{C}\int i\,dt$

$\bar{V} = \frac{1}{j\omega C}\bar{I}$



$i = C\frac{dv}{dt}$

$\bar{I} = j\omega C \bar{V}$

$\bar{V} = \frac{-j}{\omega C}\bar{I}$

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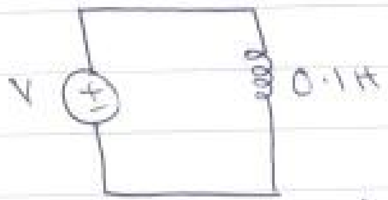
EXAMPLE #5:

The voltage $v = 12\cos(60t + 45^\circ)$ is applied to a 0.1-H inductor. Find the steady state current through the resistor.

Sol:

By Phasor Method



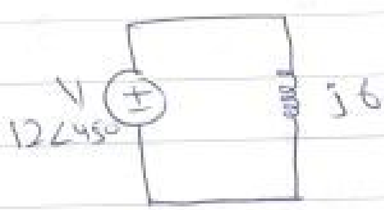
$v = 12\cos(60t + 45^\circ)$

$\bar{V} = V_m \angle\phi = 12\angle 45^\circ$

$\omega = 60$

$j\omega L = j \times 60 \times 0.1H \Rightarrow j6$

Phasor diagram



$$\therefore \bar{I} = \frac{\bar{V}}{j\omega L}$$

$$= \frac{12\angle 45^\circ}{j6} = \frac{12\angle 45^\circ}{6\angle 90^\circ}$$

$$= \frac{12\angle 45^\circ}{1\angle 90^\circ} \Rightarrow 2\angle -45^\circ.$$

$i = 2\cos(60t - 45^\circ)$ Amp.

AC CIRCUIT ANALYSIS:

KVL (KIRCHHOFFS VOLTAGE LAW)

⇒ In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop.

⇒ In other words the algebraic sum of all voltages with in the loop must be equal to zero. This idea is known as the Conservation of Energy.

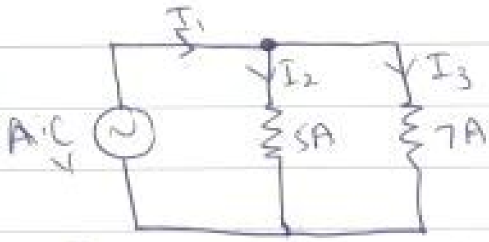
KIRCHHOFF'S CURRENT LAW (KCL):

⇒ The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost with in the node.

⇒ the ~~a~~ algebraic sum of ALL the currents entering and leaving node must be equal to zero, $I_{(exiting)} + I_{(entering)} = 0$.

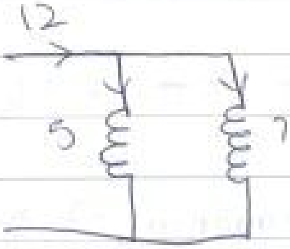
⇒ This idea is commonly known as conservation of charge.

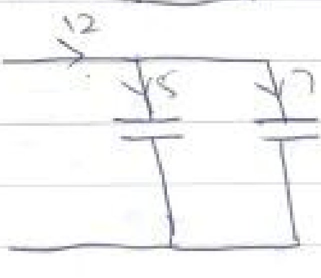
⇒ If we have same source in a circuit the kirchhoff's law will give correct value always. In terms of RMS, Instantaneous, Phasor (vector).



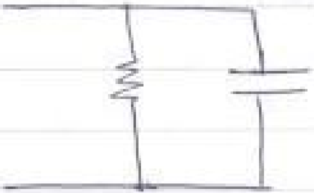
$I_1 = I_2 + I_3$

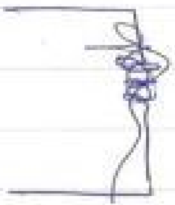
$12 = 5 + 7$

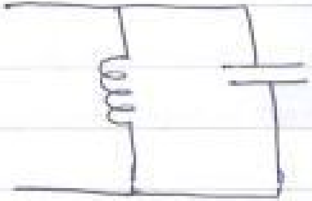




⇒ If load types are not same.







$$I_1 = I_2 + I_3$$
$$= 5 + 7$$

RMS ×

⇒ When types of load are different still Kirchhoff's law is valid in AC but we don't use RMS value. We use either Instantaneous or Phasor.

Exercise Problems:

Problem #1:

Given the sinusoidal voltage $v(t) = 50 \cos(30t + 10^\circ)$ V.

a) The amplitude $V_m = ?$

Sol:

$$\text{Amplitude}, V_m = 50V$$

b) The period T?

Sol:

$$T = \frac{2\pi}{\omega} \qquad \therefore \omega = 30$$

$$= \frac{2\pi}{30} \Rightarrow 0.209 \text{ s}$$

c) The frequency f?

Sol:

$$f = \frac{1}{T} = \frac{1}{0.209} \Rightarrow 4.77 \text{ Hz}$$

d) v(t) at t = 10 ms?

Sol:

$$v(t) = 50 \cos(30t + 10^\circ) V$$

$10^\circ \Rightarrow$ radian?

$= 0.174$

$t = 10\,ms$

$v(t) = 50\cos(30 \times (10 \times 10^{-3}) + 0.174)$

$= 50\cos(0.474) \Rightarrow 44.48V$

PROBLEM #2:-

Given $v_1 = 45\sin(\omega t + 30^\circ)V$ and $v_2 = 50\cos(\omega t - 30^\circ)V$, determine the phase angle between the two sinusoids and which one lags the other.

Sol:-

In order to compare v_1 and v_2, we must express them in the same form, If we express them sine form with positive amplitudes,

$v_1 = 45\sin(\omega t + 30^\circ)V \Rightarrow 45\cos(\omega t + 30^\circ - 90^\circ)$

$45\cos(\omega t - 60^\circ)$

$v_2 = 50\cos(\omega t - 30^\circ)$

$v_2 = 50\cos(\omega t - 30^\circ)V$

$= 50\sin(\omega t - 30^\circ \pm 90^\circ)$

$= 50\sin(\omega t \overset{-120^\circ}{\pm 60^\circ})$

$v_1 = 45\sin\omega t$ with a phase shift of $+30^\circ$.

$v_2 = 50\cos\omega t$ with a phase shift of -30°.

In order to compare v_1 and v_2, we must express them in the same form, Hence,

$$v_1 = 45 \sin(\omega t + 30^\circ) V$$
$$= 45 \cos(\omega t + 30^\circ - 90^\circ) V$$
$$v_1 = 45 \cos(\omega t - 60^\circ) V$$

$$v_2 = 50 \cos(\omega t - 30^\circ) V.$$

This indicates that the phase angle between the two signals is 30° and that v_1 lags v_2.

PROBLEM # 3:-

Find the phasors corresponding to the following signals:

a) $v(t) = 21 \cos(4t - 15^\circ) V$

Sol:-

$$\bar{V} = 21 \angle -15^\circ V.$$

b) $i(t) = -8 \sin(10t + 70^\circ) mA$

Sol:-

$$i(t) = -8 \sin(10t + 70^\circ)$$
$$= 8 \cos(10t + 70^\circ + 180^\circ - 90^\circ)$$
$$i(t) = 8 \cos(10t + 160^\circ)$$
$$\bar{I} = 8 \angle 160^\circ \ mA$$

PROBLEM # 5:-

Given $i_1(t) = 4 \cos(\omega t + 30^\circ) A$ and $i_2(t) = 5 \sin(\omega t - 20^\circ) A$, Find their sum.

Sol:-

Phasor form of current $i_1(t)$ is:-

$$\bar{I}_1 = 4 \angle 30^\circ$$

PROBLEM # 4:-

Determine the current that flows through an 8 Ω resistor connected to a voltage source $v_s = 110 \cos 377t$ V.

Sol:-

$$i(t) = \frac{v_s}{R}$$

$$= \frac{110 \cos(377t)}{8} \Rightarrow 13.75 \cos(377t) \text{A}$$

PROBLEM # 5 (cont):-

We need to express $i_2(t)$ in cosine form. The rule for converting sine to cosine is to subtract 90°. Hence,

$$i_2 = 5\cos(\omega t - 20° - 90°) \Rightarrow 5 \cos(\omega t - 110°)$$

and its phasor is:-

$$\bar{I}_2 = 5\angle -110°$$

If $i = i_1 + i_2$, then

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

$$= 4\angle 30° + 5\angle -110° \qquad \therefore x = r\cos\phi, \ y = r\sin\phi$$

$$4\angle 30° \Rightarrow x = r\cos\phi \Rightarrow 3.464, \ y = r\sin\phi \Rightarrow j2$$

$$5\angle -110° \Rightarrow x = r\cos\phi \Rightarrow -1.710, \ y = r\sin\phi \Rightarrow -j4.698$$

$$\bar{I} = 3.464 + j2 - 1.71 - j4.698$$

$$= 1.754 - j2.698$$

$$= 3.218\angle -56.97° \text{ A}$$

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Transforming this to the time domain, we get

$$i(t) = 3.218 \cos(\omega t - 56.97^\circ)\ A$$