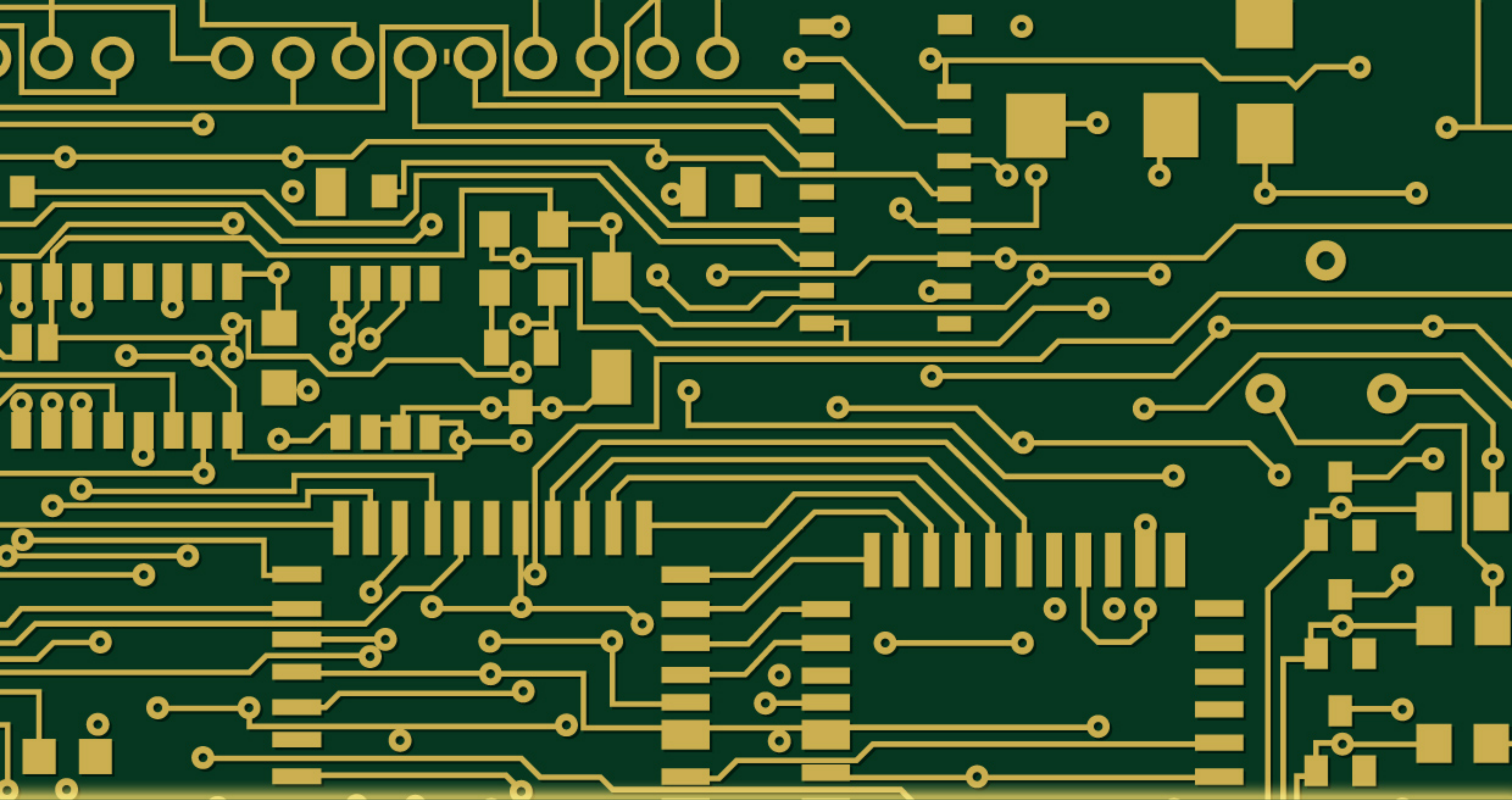


Circuit Analysis-II

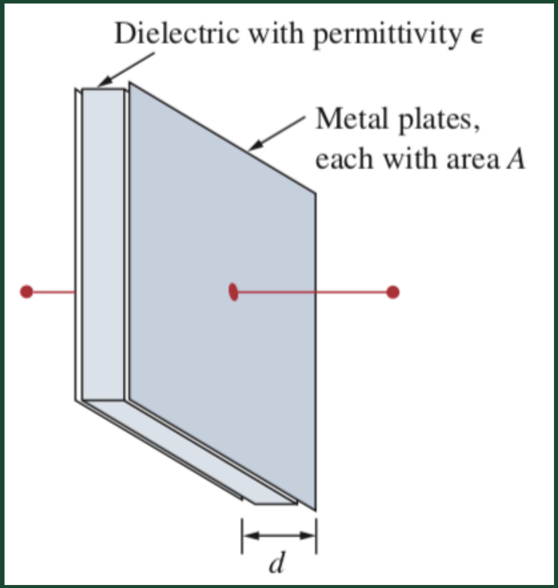
Capacitors & Inductors

Introduction

- ✓ The capacitors and inductors are passive linear circuit elements.
- ✓ Unlike resistors, which dissipate energy, capacitors and inductors do not dissipate but store energy.
- ✓ This is the reason capacitors and inductors are called storage elements.

Capacitors

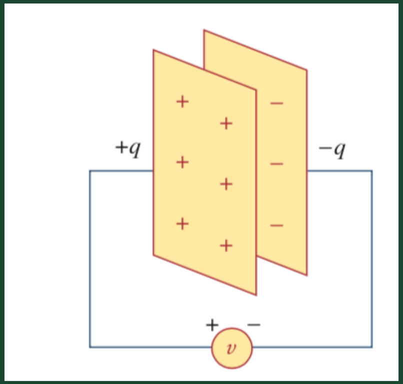
- ✓ A capacitor consists of two conducting plates separated by an insulator.



- ✓ The plates may be aluminum foil while the dielectric may be air, ceramic, paper or mica.

Capacitors (cont.)

- ✓ When a voltage source v is connected to the capacitor, the source deposits a positive charge q on on plate and a negative charge –q on the other.



- ✓ The capacitor is said to store the electric charge.
- ✓ The amount of charge stored, represented by q is directly proportional to the applied voltage v so that: $q = Cv$.

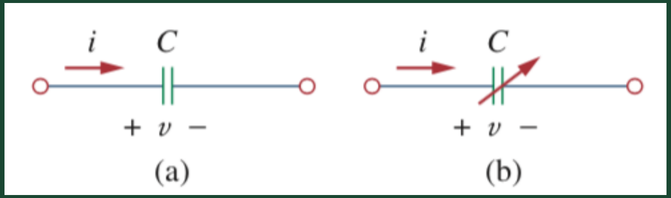
Capacitors (cont.)

- ✓ Where C is the constant of proportionality, is known as the capacitance of the capacitor.
- ✓ The unit of capacitance is the farad (F) in honor of the English physicist Michael Faraday.
- ✓ Capacitance is the ratio of the charge on on plate of a capacitor to the voltage difference between the two plates, measured in farads (F) i.e., 1 farad = 1 coulomb/volt.
- ✓ The capacitance of a capacitor depends on the physical dimensions of the capacitors.
- ✓ The parallel plate capacitor shown in the fig a, the capacitance is given by:

$$C = \frac{\varepsilon A}{d}$$

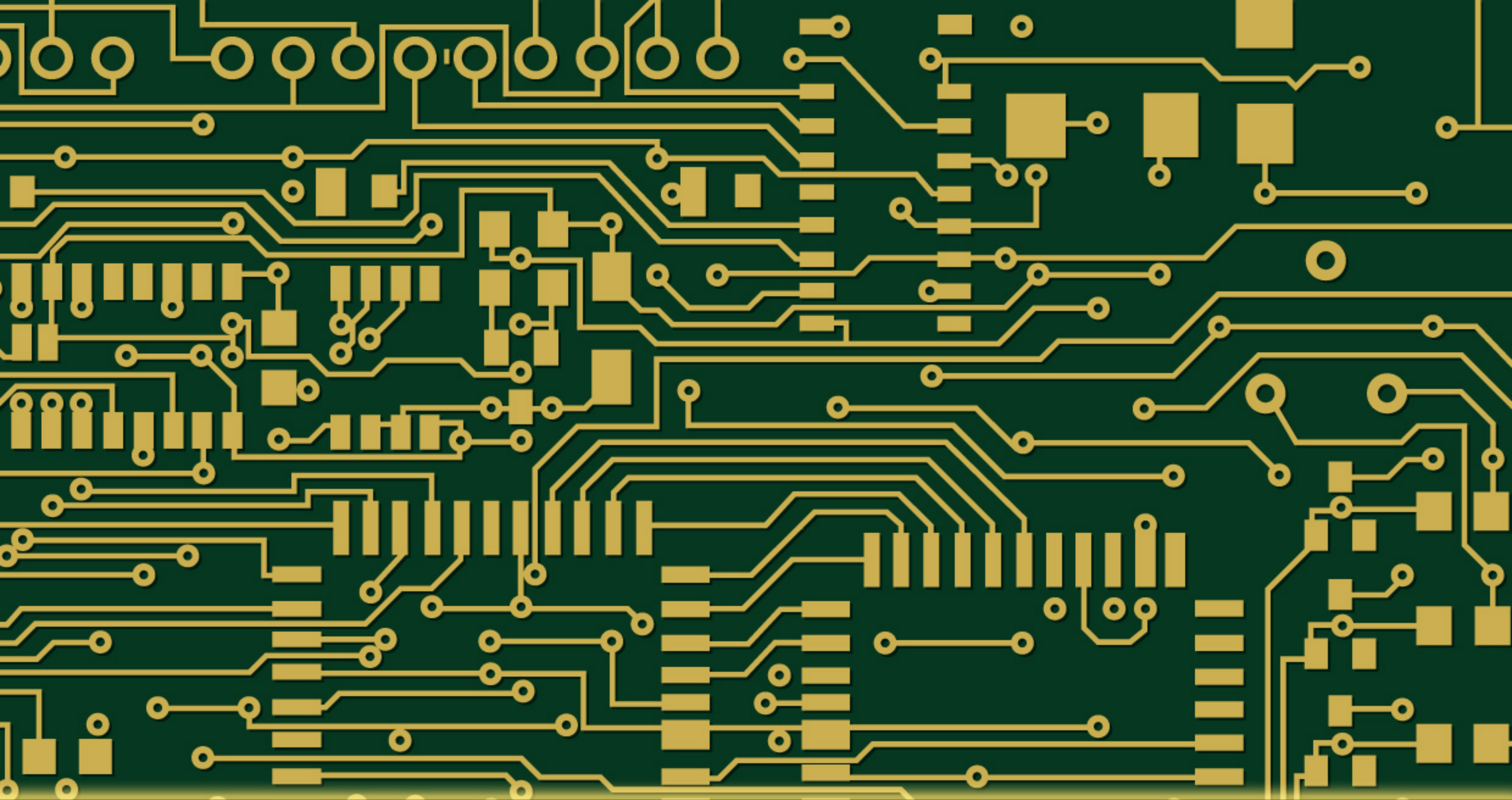
Capacitors (cont.)

- ✓ Where A is the surface area of each plate, d is the distance between the plates and ε is the permittivity of the dielectric material between the plates.
- ✓ Capacitors are commercially available in different values and types. They have the values in the picofarad(pF) to microfarad (μF) range.
- ✓ They are described by the dielectric material they are made of and whether they are fixed or variable type.



Capacitors (cont.)

- According to the passive sign convention:
  - If $v > 0$ and $i > 0$ or if $v < 0$ and $i < 0$, the capacitor is being charged.
  - If $v \cdot i < 0$, the capacitor is discharging.



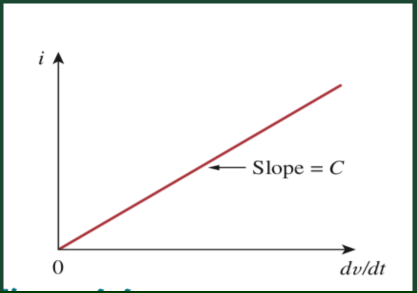
Current-Voltage Relationship

Current-Voltage Relationship of Capacitor

- ✓ To obtain the current-voltage relationship of the capacitor, we take the derivative of both sides of q=Cv. Since $i = \frac{dq}{dt}$
- ✓ Differentiating both sides of q =Cv gives:

$$i = C\frac{dv}{dt}$$

- ✓ Capacitors that satisfy the above equation are said to be linear and have straight line plot, as shown below:



Current-Voltage Relationship of Capacitor (cont.)

- ✓ For a non-linear capacitor, the plot of the current-voltage relationship is not a straight line.
- ✓ The voltage-current relation of the capacitor can be obtained by integrating both sides of the above equation. We get:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)\,d\tau$$

$$or$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$$

- ✓ Where $v(t_0) = q(t_0)/C$ is the voltage across the capacitor at time t_0.

Current-Voltage Relationship of Capacitor (cont.)

- ✓ The instantaneous power delivered to the capacitor is:

$$p = vi = Cv\frac{dv}{dt}$$

- ✓ The energy stored in the electric field that exists between the plates of the capacitor is represented as:

$$w = \frac{1}{2}Cv^2$$

Example #1

- (a): Calculate the charge stored on a 3-pF capacitor with 20V across it.
- (b): Find the energy stored in the capacitor.

Example #2

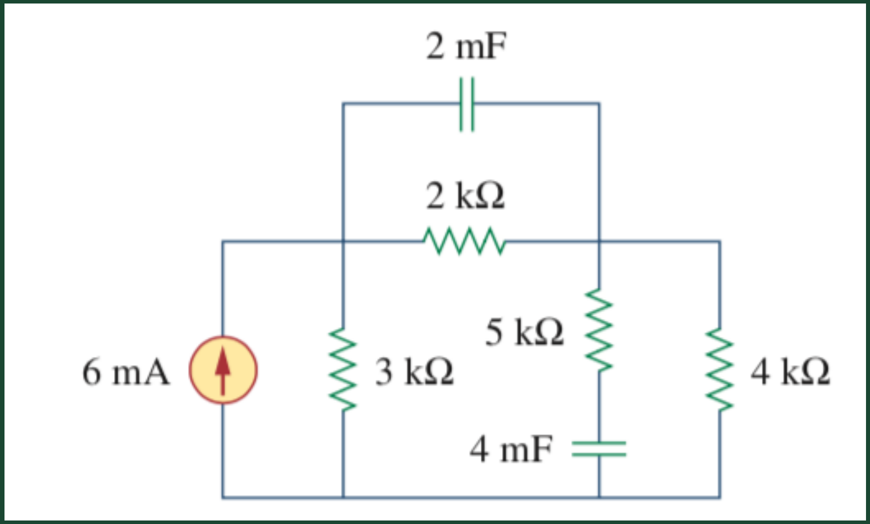
- ✓ The voltage across a 5-μF capacitor is:

$$v(t) = 10\cos(6000t)V$$

- ✓ Calculate the current through it.

Example #3

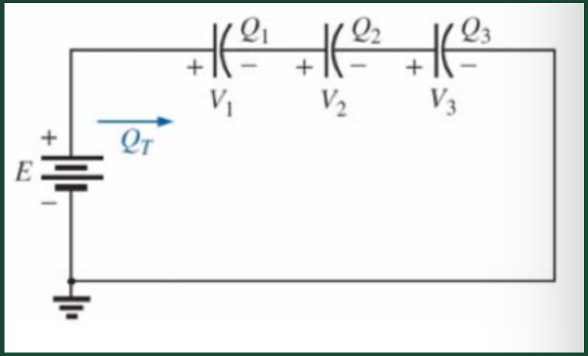
✓ Obtain the energy stored in the capacitor shown below under the dc conditions:



Series & Parallel Capcitors

Series Capacitors

✓ Capacitors in series are combined in the same manner as resistors in parallel.

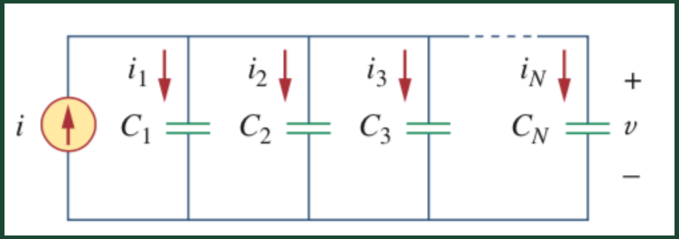


$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

✓ The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

Parallel Capacitors

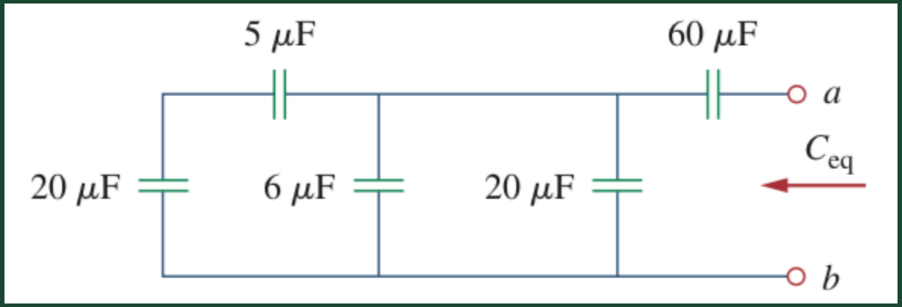
- ✓ The equivalent capacitance of N parallel-connected capacitors is the sum of the individual capacitances.

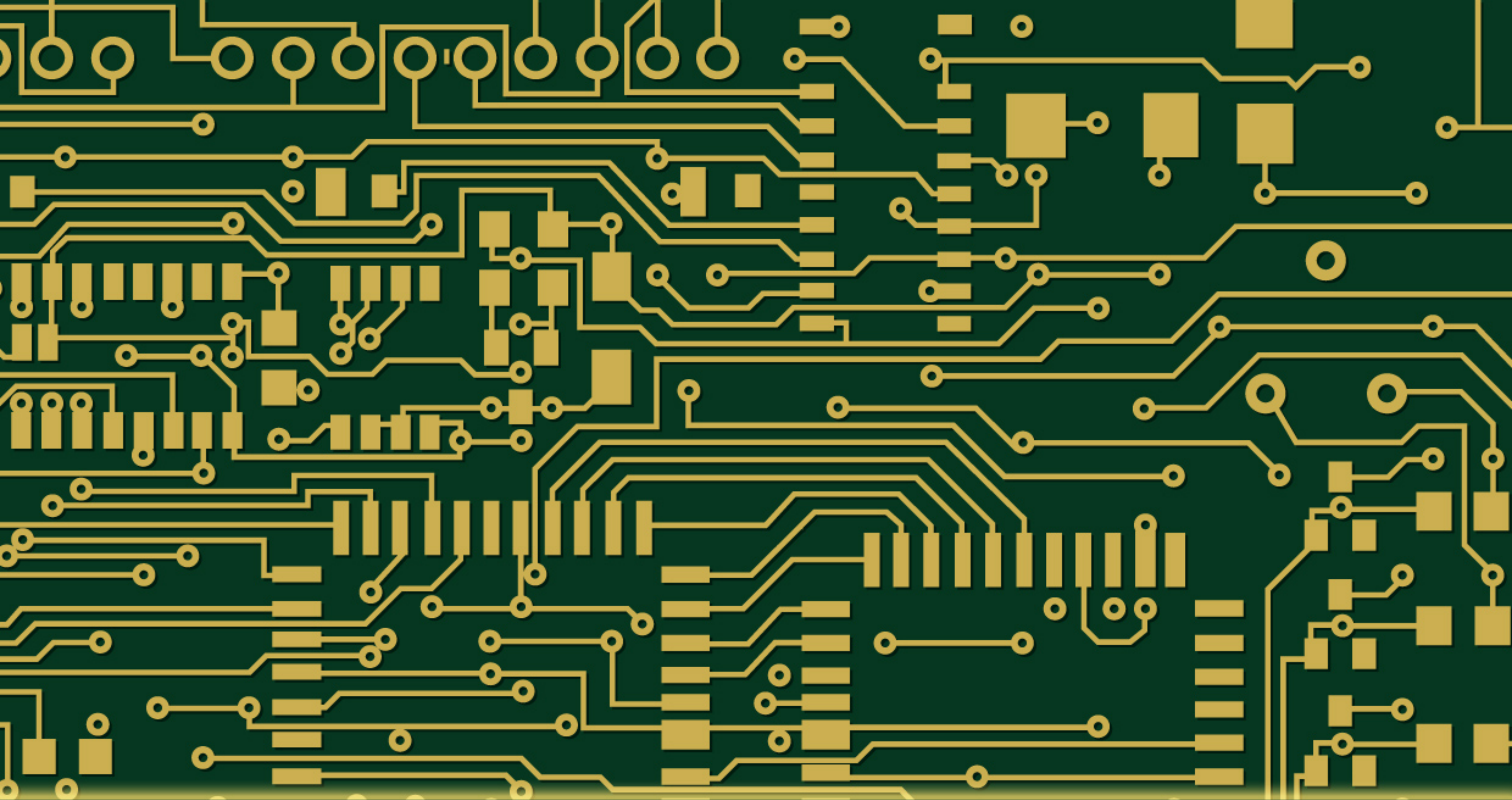


$$C_{eq} = C_1 + C_2 + C_3 + \dots\dots + C_N$$

Example #4

✓ Find the equivalent capacitance seen between terminals a and b of the circuit shown below:

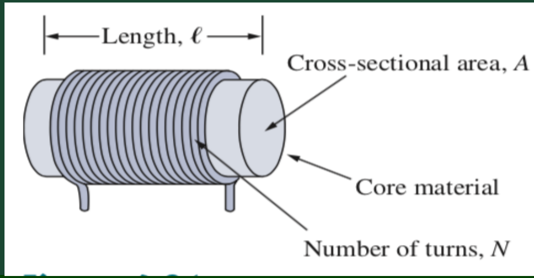




Inductors

Introduction

- ✓ An inductor is a passive element designed to store energy in its magnetic field.
- ✓ Inductors find numerous applications in electronic and power systems.
- ✓ Any conductor of electric current has inductive properties and may be regarded as an inductor.
- ✓ But in order to enhance the inductive effect, a practical inductor is usually formed into a cylindrical coil with many turns if conducting wire as shown below:

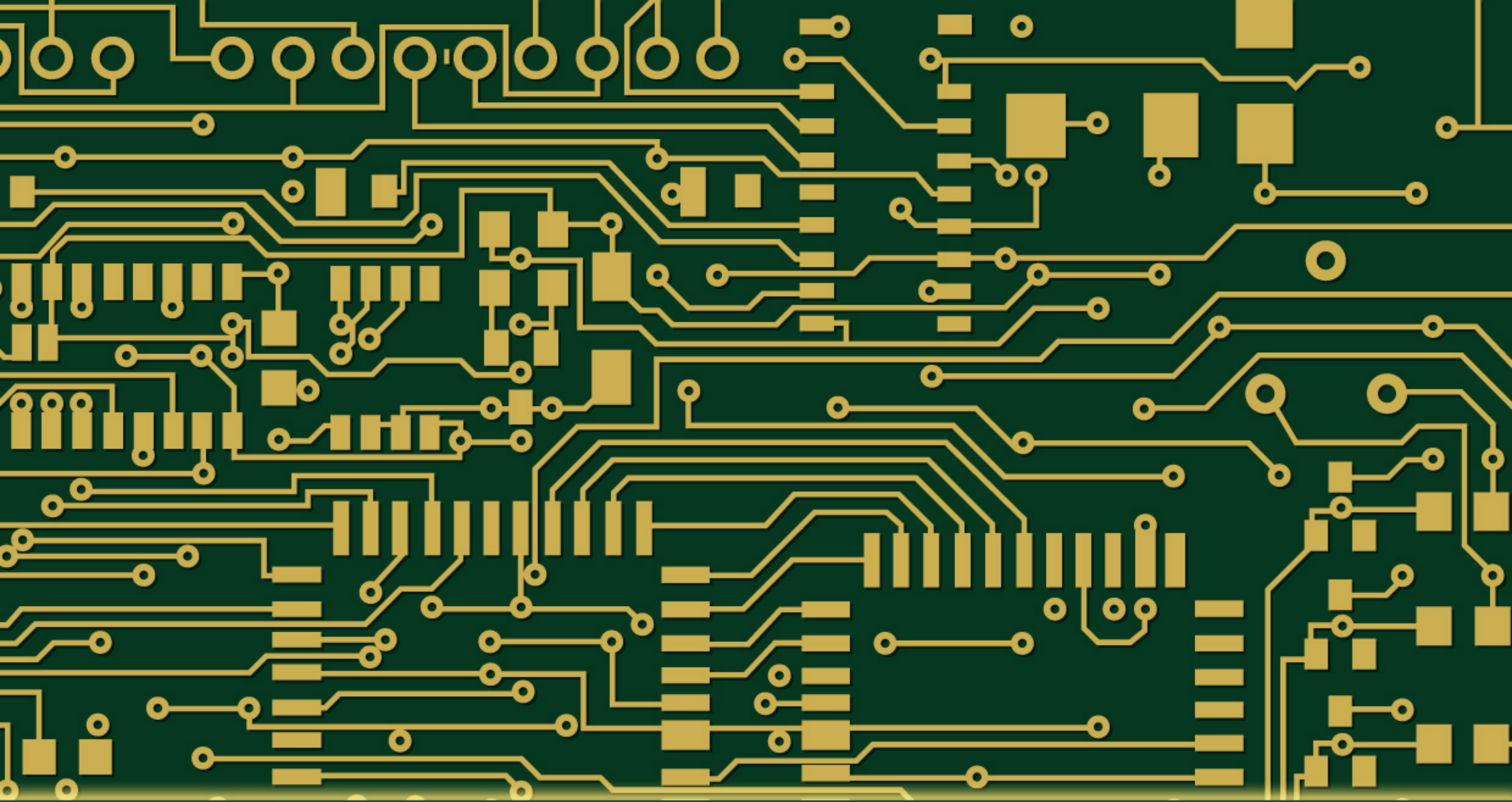


Inductor

- ✓ An inductor consists of a coil of conducting wire.
- ✓ If current is allowed to pass through an inductor, it is found that the voltage across the inductor is directly proportional to the time rate of change of the current.
- ✓ Using the passive sign convention: $v = L\frac{di}{dt}$
- ✓ Where L is the constant of proportionality called the inductance of the inductor.

Inductance

- ✓ Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it.
- ✓ The unit of inductance is the henry (H),named in honor of the American inventor Joseph Henry.
- ✓ 1 henry equals to 1 volt-second per ampere.
- ✓ Inductance can be increased by increasing the number of turns of coil, using material with higher permeability as the core, increasing the cross-sectional area or reducing the length of the coil.



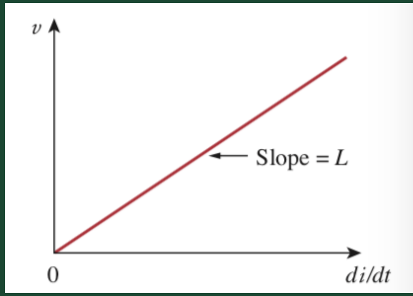
Current-Voltage Relationship

Current-Voltage Relation of Inductors

- ✓ The voltage-current relationship for an inductor is:

$$v = L\frac{di}{dt}$$

- ✓ The figure below shows this relationship graphically for an inductor whose inductance is independent of current.



- ✓ Such an inductor is known as linear inductor. For a nonlinear inductors the plot will not be a straight line because the inductance varies with current.

Current-Voltage Relation of Inductors (cont.)

- ✓ The current-voltage relationship is obtained as:

$$di = \frac{1}{L} v dt$$

- ✓ Integrating gives:

$$i = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\, d\tau$$

$$or$$

$$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i(t_0)$$

- ✓ Where $i(t_0)$ is the total current for $-\infty < t < t_0$ and $i(-\infty)=0$.

Current-Voltage Relation of Inductors (cont.)

- ✓ The idea of making $i(-\infty)=0$ is practical and reasonable, because there must be a time in the past when there was no current in the inductor.
- ✓ The power delivered to inductor is:

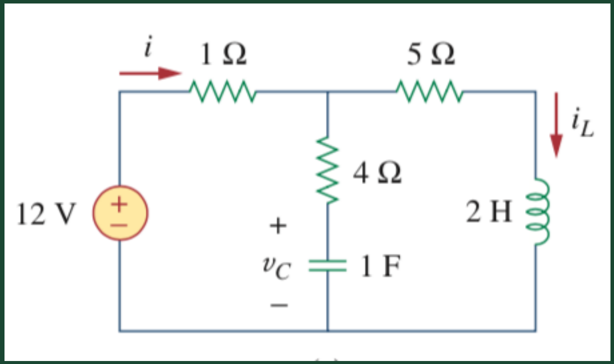
$$p = vi = \left(L\frac{di}{dt}\right)i$$

- ✓ The energy stored is:

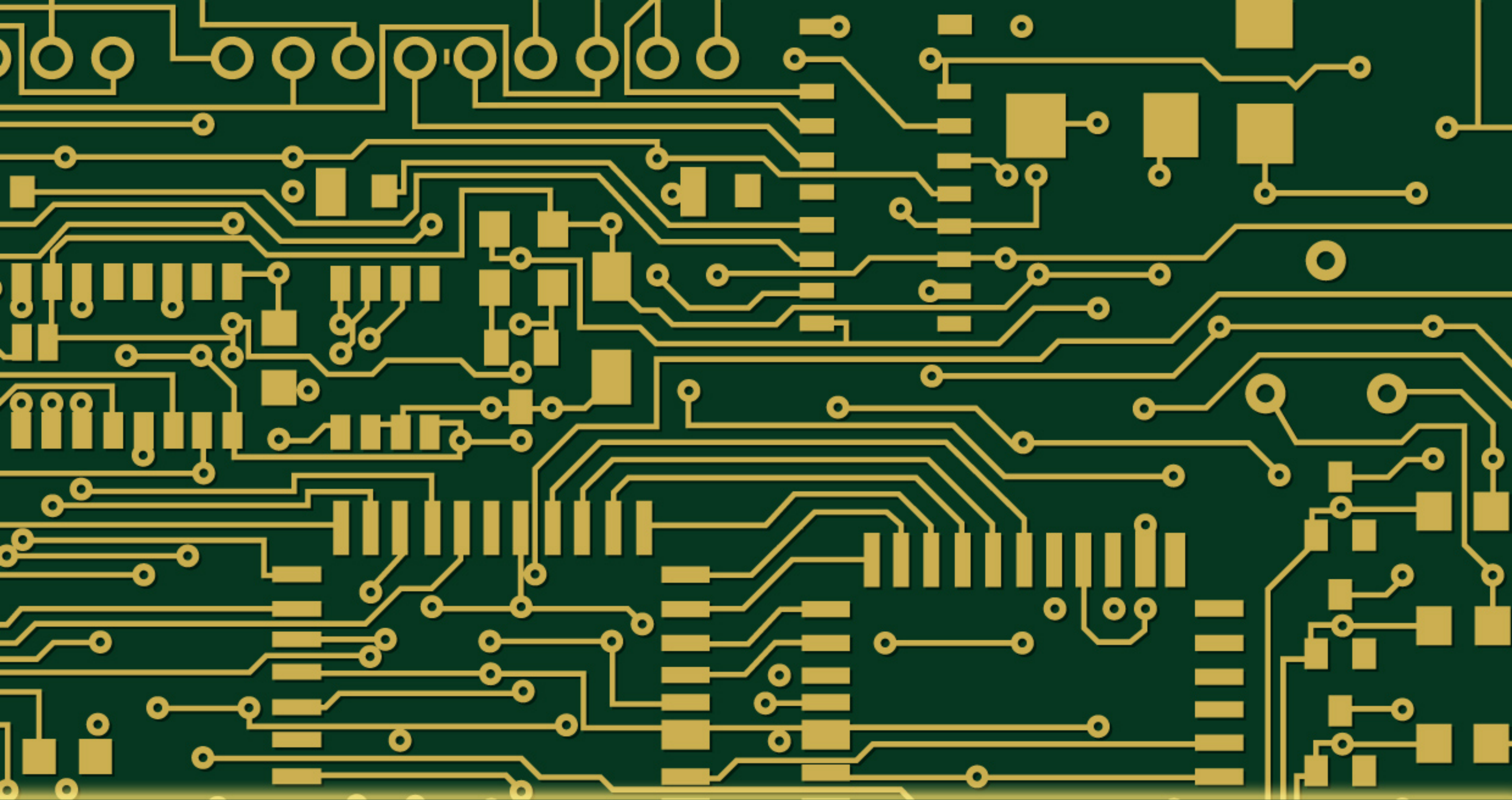
$$w = \frac{1}{2}Li^2$$

Example #5

- ✓ Consider the circuit in following figure.



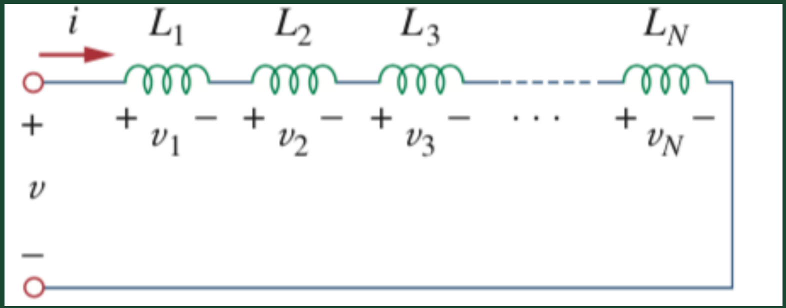
- ✓ Under dc conditions, find:
- ✓ (a): i, v_C, and i_L.
- ✓ (b): the energy stored in the capacitor and inductor.



Series & Parallel Inductors

Series Inductors

- ✓ Consider a series connection of N inductors shown below:

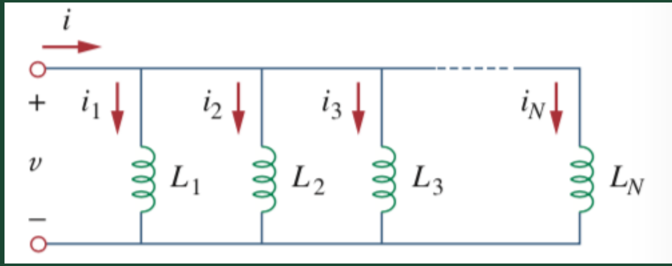


- ✓ The equivalent inductance of series-connected inductors is the sum of the individual inductances.

$$L_{eq} = L_1 + L_2 + L_3 + \ldots\ldots + L_N$$

Parallel Inductors

- ✓ The parallel connection of N inductors is shown below:



- ✓ The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

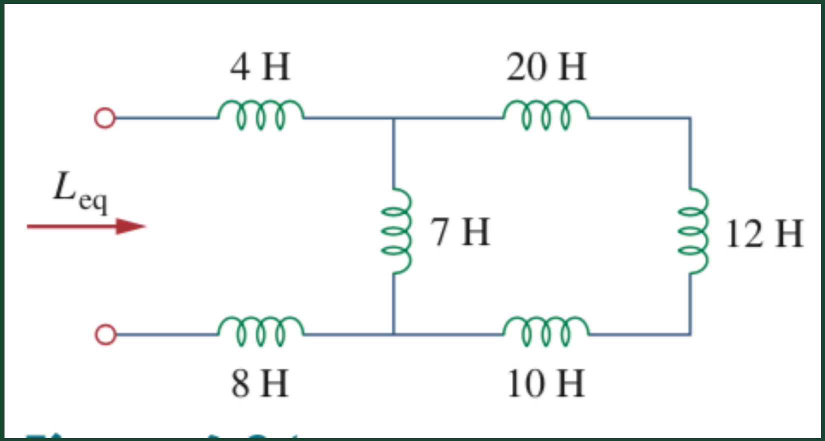
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots.. + \frac{1}{L_N}$$

Important Characteristics of the Basic Elements

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Example #6

- ✓ Find the equivalent inductance of the circuit shown below:



Thank You