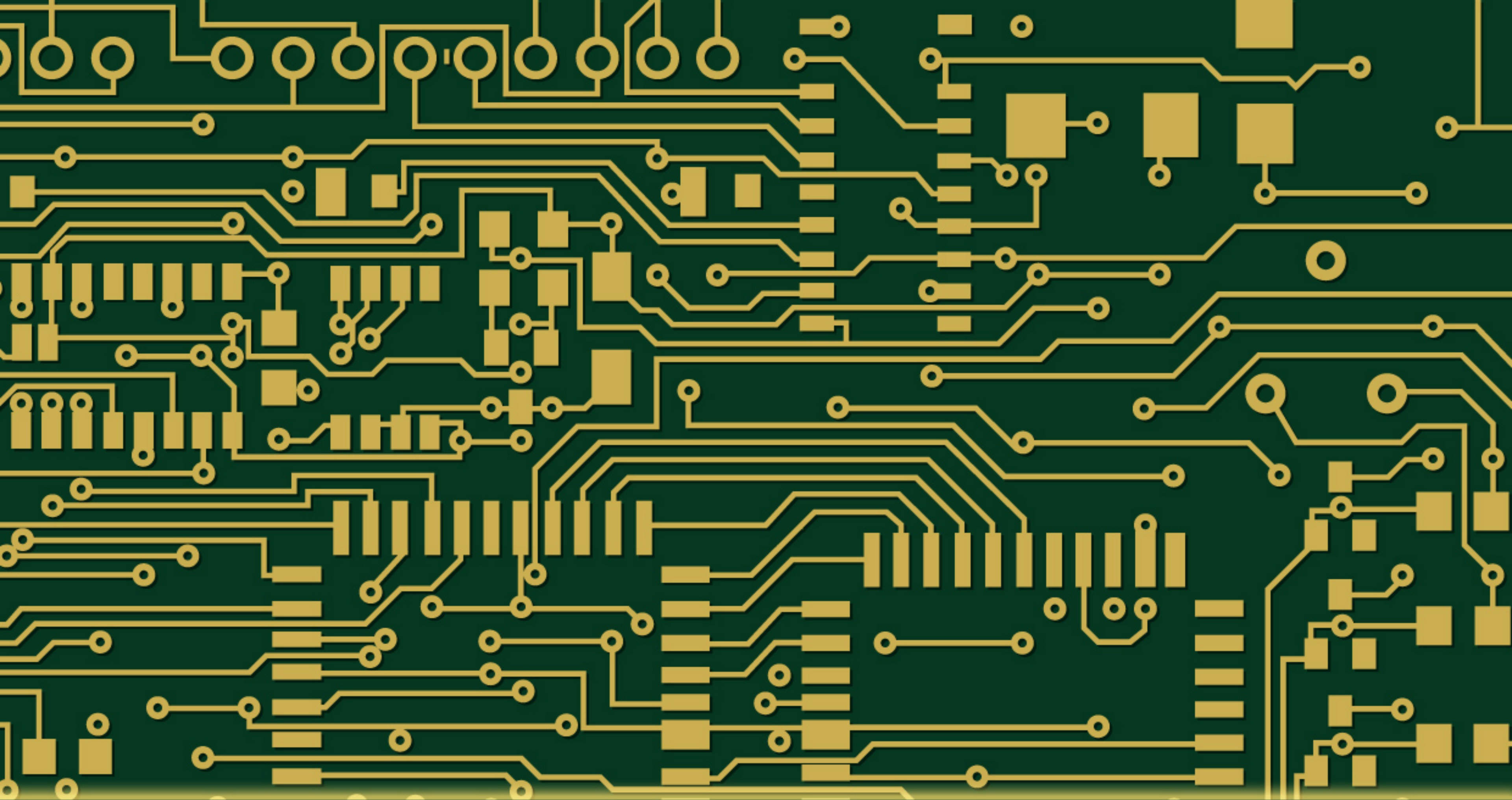


# Circuit Analysis-II

# Capacitors in AC Circuits

# Introduction

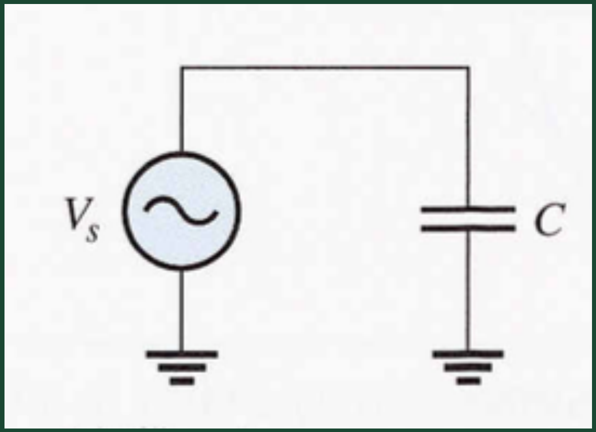
- ✓ The instantaneous capacitor current is equal to the capacitance times the instantaneous rate of change of the voltage across the capacitor.

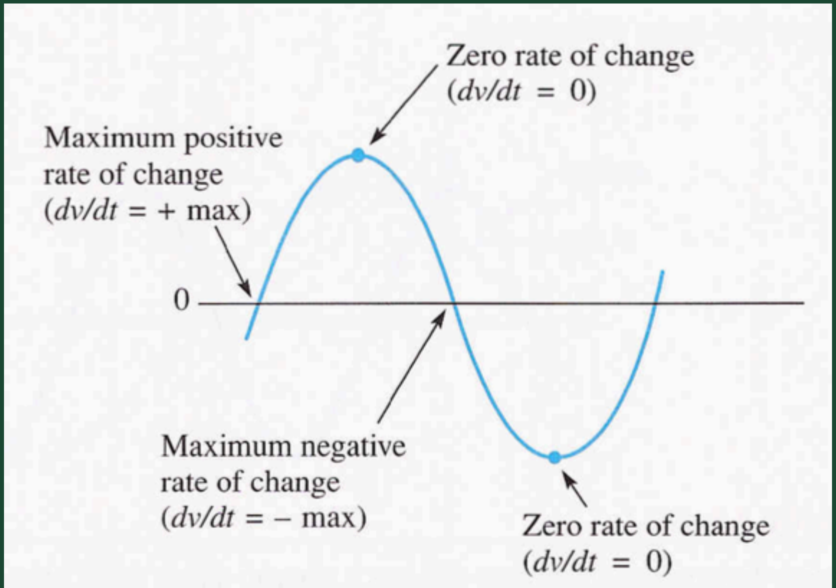
$$i = C\left(\frac{dv}{dt}\right)$$

- ✓ The faster the voltage across a capacitor changes, the greater the current.

# Phase relationship of Current and Voltage

- ✓ When a sinusoidal voltage is applied across a capacitor:
  - ✓ The voltage waveform has a maximum rate of change (dv/dt = max) at the zero crossings.
  - ✓ A zero rate of change (dv/dt = 0) at the peaks.



# Phase relationship of Current and Voltage (cont.)

- ✓ The phase relationship between the current and the voltage for the capacitor can be established from:

$$i = C\left(\frac{dv}{dt}\right)$$

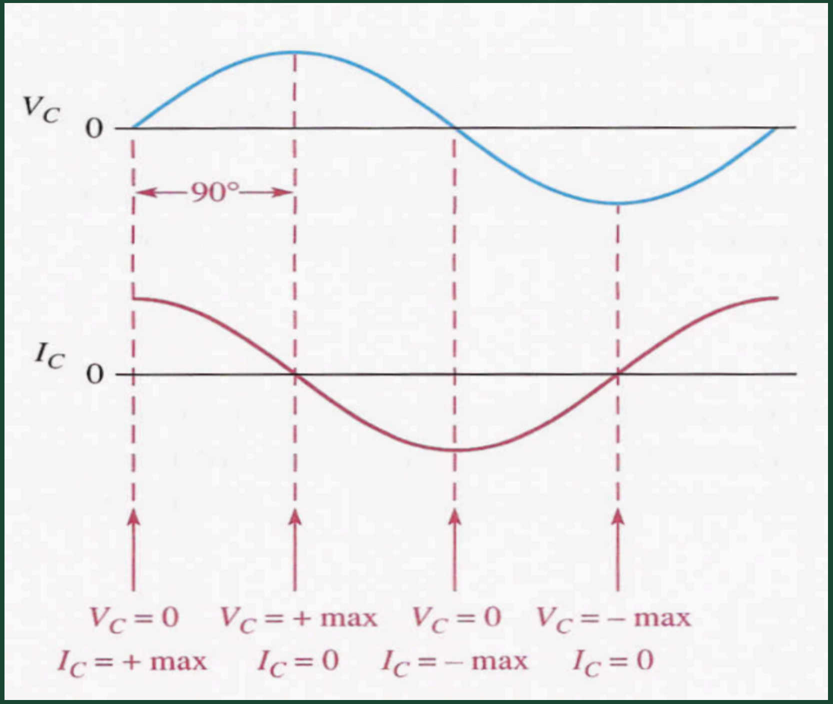
- ✓ When dv / dt = 0, i is also zero because:

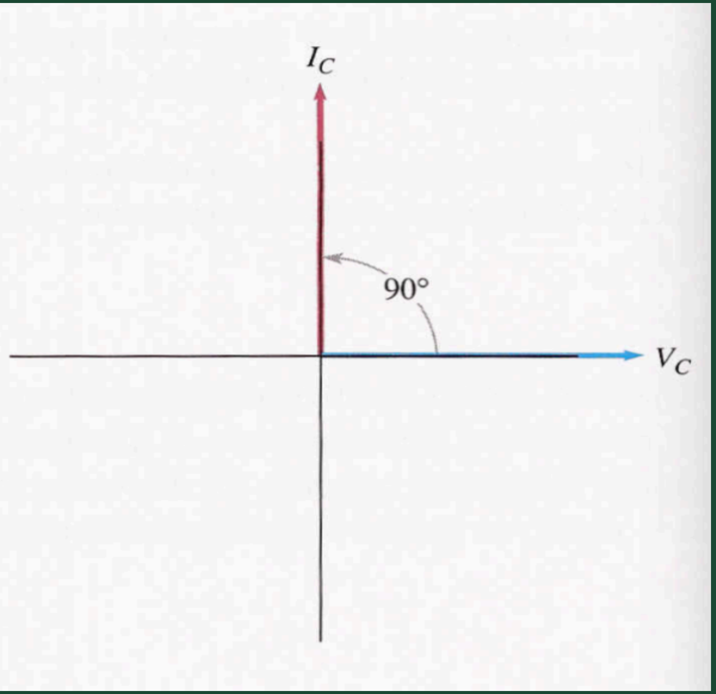
$$i = C\left(\frac{dv}{dt}\right) = C(0) = 0$$

- ✓ When dv / dt is a positive-going maximum, i is a positive maximum; when dv / dt is a negative-going maximum, i is a negative maximum.

# Phase relationship of Current and Voltage (cont.)

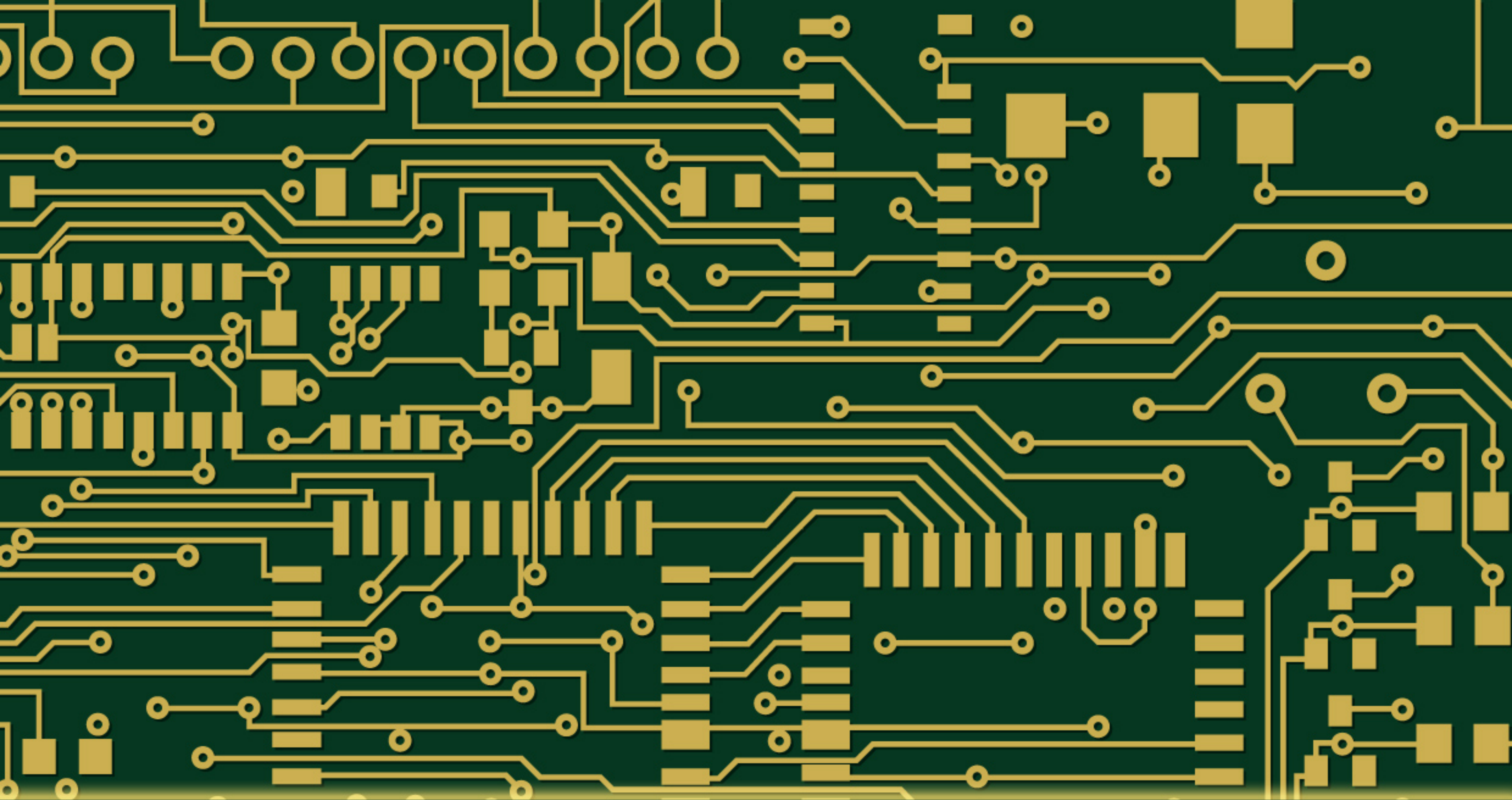
- ✓ A sinusoidal voltage always produces a sinusoidal current in a capacitive circuit.
- ✓ The voltage and current relationship is shown below:



# Phase relationship of Current and Voltage (cont.)

- ✓ The current leads the voltage in phase by 90°.
- ✓ This is always true in a purely capacitive circuit.



# Capacitive Reactance

# Capacitive Reactance

- ✓ Capacitive reactance is the opposition to sinusoidal current, expressed in ohms.
- ✓ The symbol for capacitive reactance is $X_C$.
- ✓ The rate of change of the voltage is directly related to frequency.
- ✓ The faster the voltage changes, the higher the frequency.
- ✓ When frequency increases, dv / dt increases, and thus i increases.
- ✓ When frequency decreases, dv / dt decreases, and thus i decreases.

$$\overset{\uparrow}{i} = C\left(\overset{\uparrow}{dv / dt}\right)$$

$$\underset{\downarrow}{i} = C\left(\underset{\downarrow}{dv / dt}\right)$$

# Capacitive Reactance (cont.)

- ✓ An increase in i means that there is less opposition to current ($X_C$ is less).
- ✓ A decrease in i means a greater opposition to current ($X_C$ is greater).
- ✓ Therefore, $X_C$ is inversely proportional to i and thus inversely proportional to frequency i.e., 1/f.
- ✓ If dv/dt is constant and C is varied, an increase in C produces an increase in i, and a decrease in C produces a decrease in i.

$$\overset{\uparrow}{i} = \overset{\uparrow}{C}(dv/dt) \quad and \quad \underset{\downarrow}{i} = \underset{\downarrow}{C}(dv/dt)$$

# Capacitive Reactance (cont.)

- ✓ An increase in i means less opposition ($X_C$ is less) and a decrease in i means greater opposition ($X_C$ is greater).
- ✓ Therefore, $X_C$ is inversely proportional to i and thus inversely proportional to capacitance.
- ✓ The capacitive reactance is inversely proportional to both f and C, i.e.,

$$X_C = \frac{1}{fC}$$

- ✓ Also,

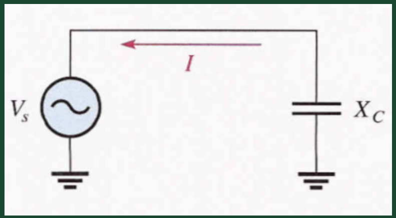
$$X_C = \frac{1}{2\pi fC}$$

# Capacitive Reactance (cont.)

- ✓ Capacitive reactance $X_C$ is in ohms when f is in hertz and C is in farads and 2π appears in the denominator as a constant of proportionality.

# Ohm's Law

- The reactance of a capacitor is analogous to the resistance of a resistor, as shown below:



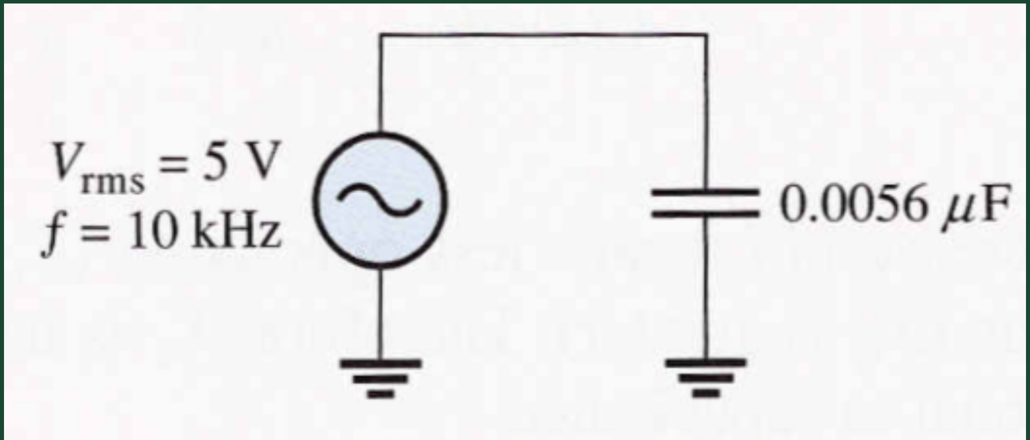

- Both are expressed in ohms. Since both R and $X_C$ are forms of opposition to current, ohm's law applies to capacitive circuits as well as to resistive circuits.

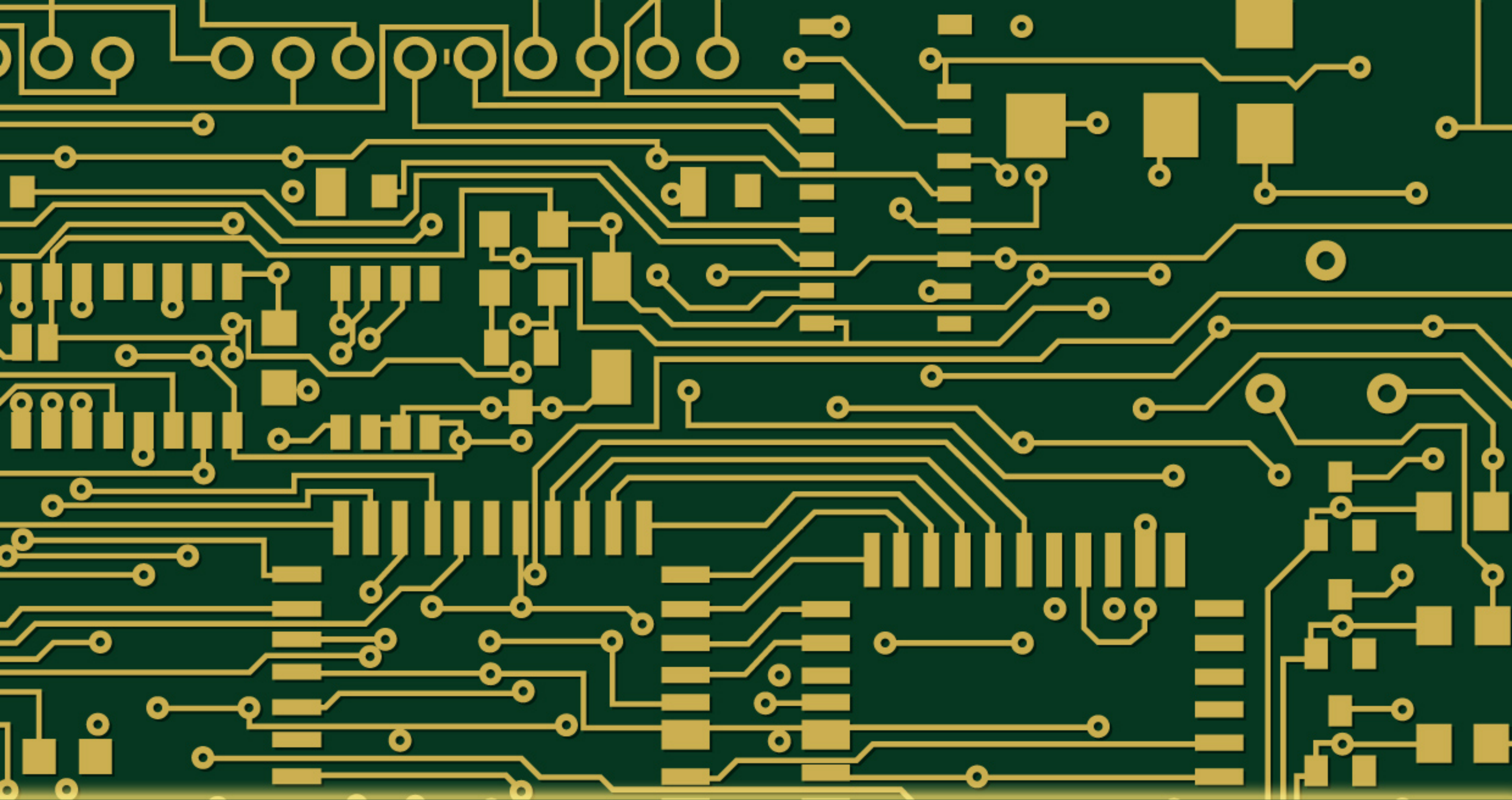
$$I = \frac{V}{X_C}$$

- When applying ohm's law in ac circuits, both the current and the voltage are expressed in the same way i.e., both in rms, both in peak and so on.

# Example #1

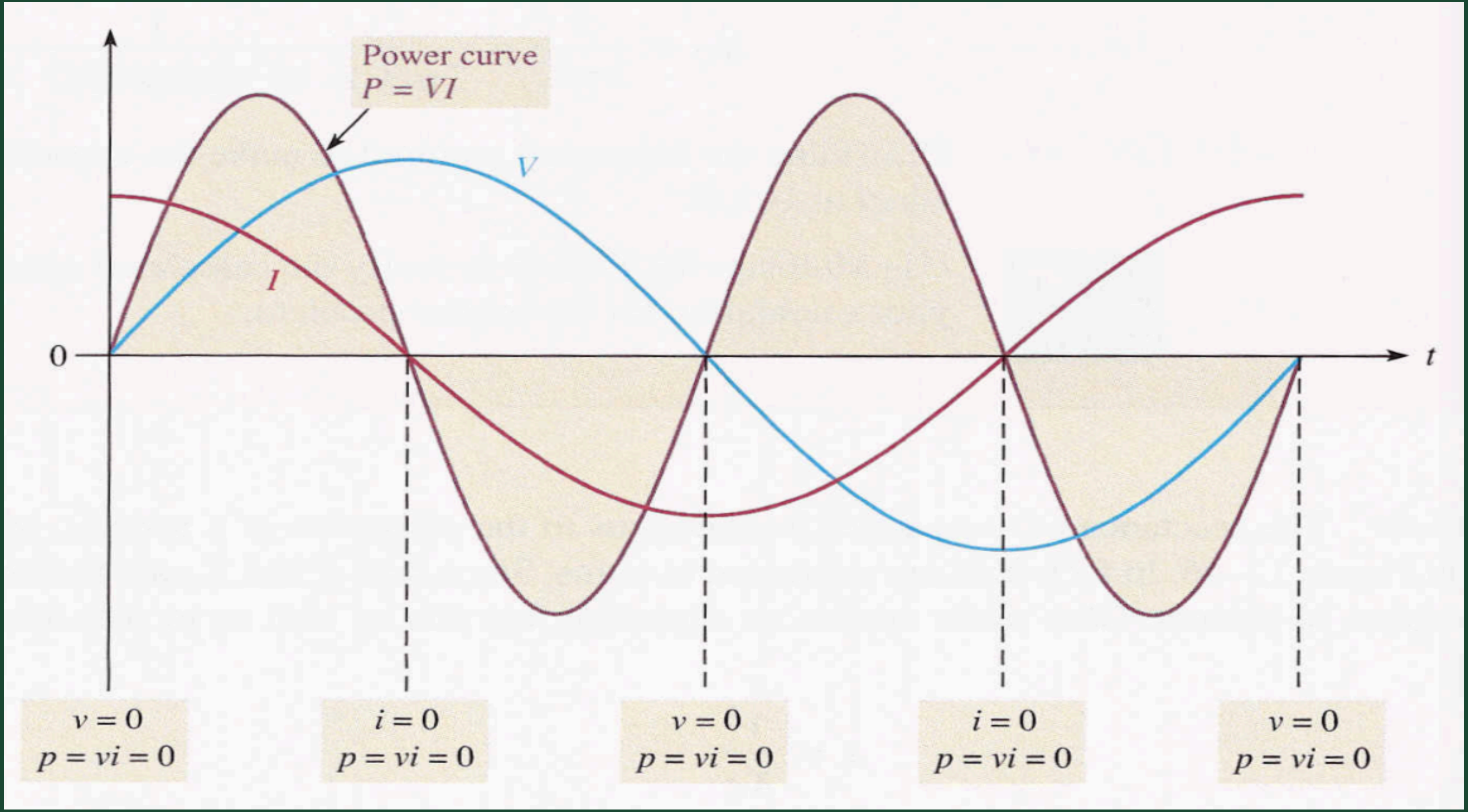
✓ Determine the rms current:





# Power in a Capacitor

# Power



# Instantaneous Power

- ✓ The product of v and i gives instantaneous power.
- ✓ Where v or i is zero, p is also zero, when both v and i are positive, p is also positive and when either v or i is positive and the other is negative, p is negative.
- ✓ When both v and i are negative, p is positive.
- ✓ Positive values of power indicate that energy stored by the capacitor.
- ✓ Negative values of power indicate that energy is returned from the capacitor to the source.

# True Power

- ✓ Ideally all of the energy stored by a capacitor during the positive portion of the power cycle is returned to the source during the negative portion.
- ✓ No net energy is lost due to conversion to heat in the capacitor, so the true power is zero.

# Reactive Power

- ✓ The rate at which a capacitor stores or returns energy is called its reactive power.
- ✓ The reactive power is a nonzero quantity, because at any instant in time, the capacitor is actually taking energy from the source or returning energy to it.
- ✓ Reactive power does not represent an energy loss.
- ✓ The following formulas apply:

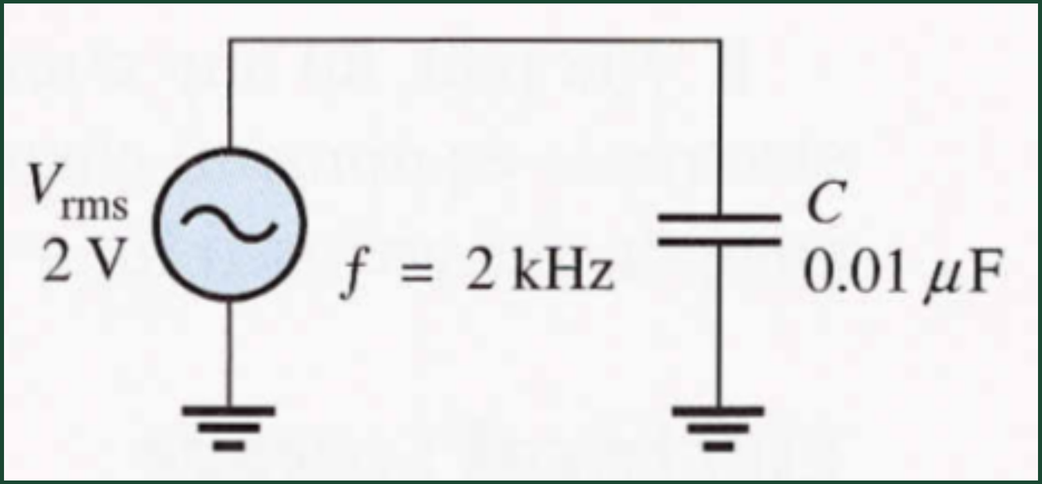
$$P_r = V_{rms} I_{rms}$$

$$P_r = \frac{V_{rms}^2}{X_C}$$

$$P_r = I_{rms}^2 X_C$$

# Example #2

✓ Determine the true power and the reactive power:

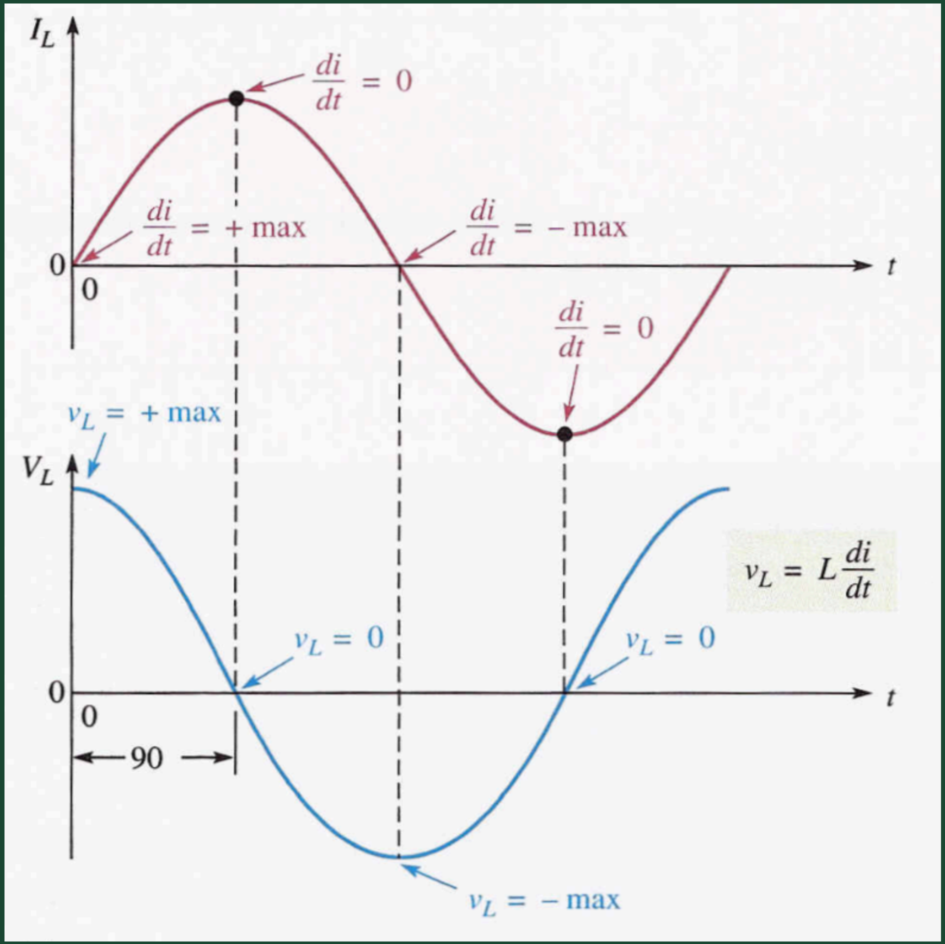


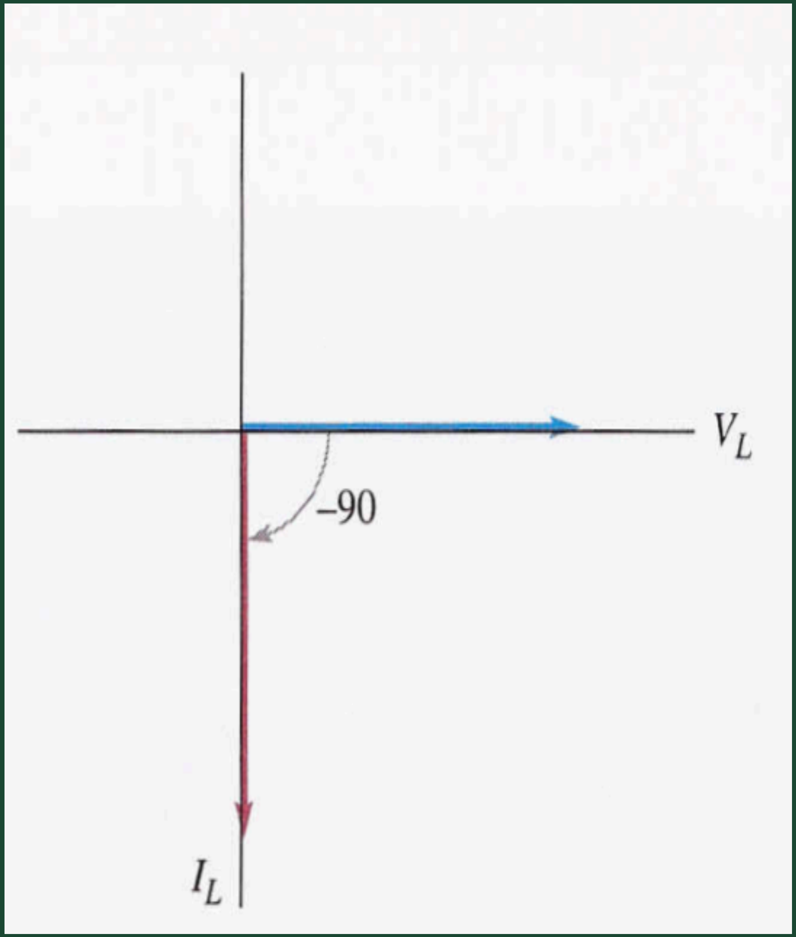
# Inductors in AC Circuits

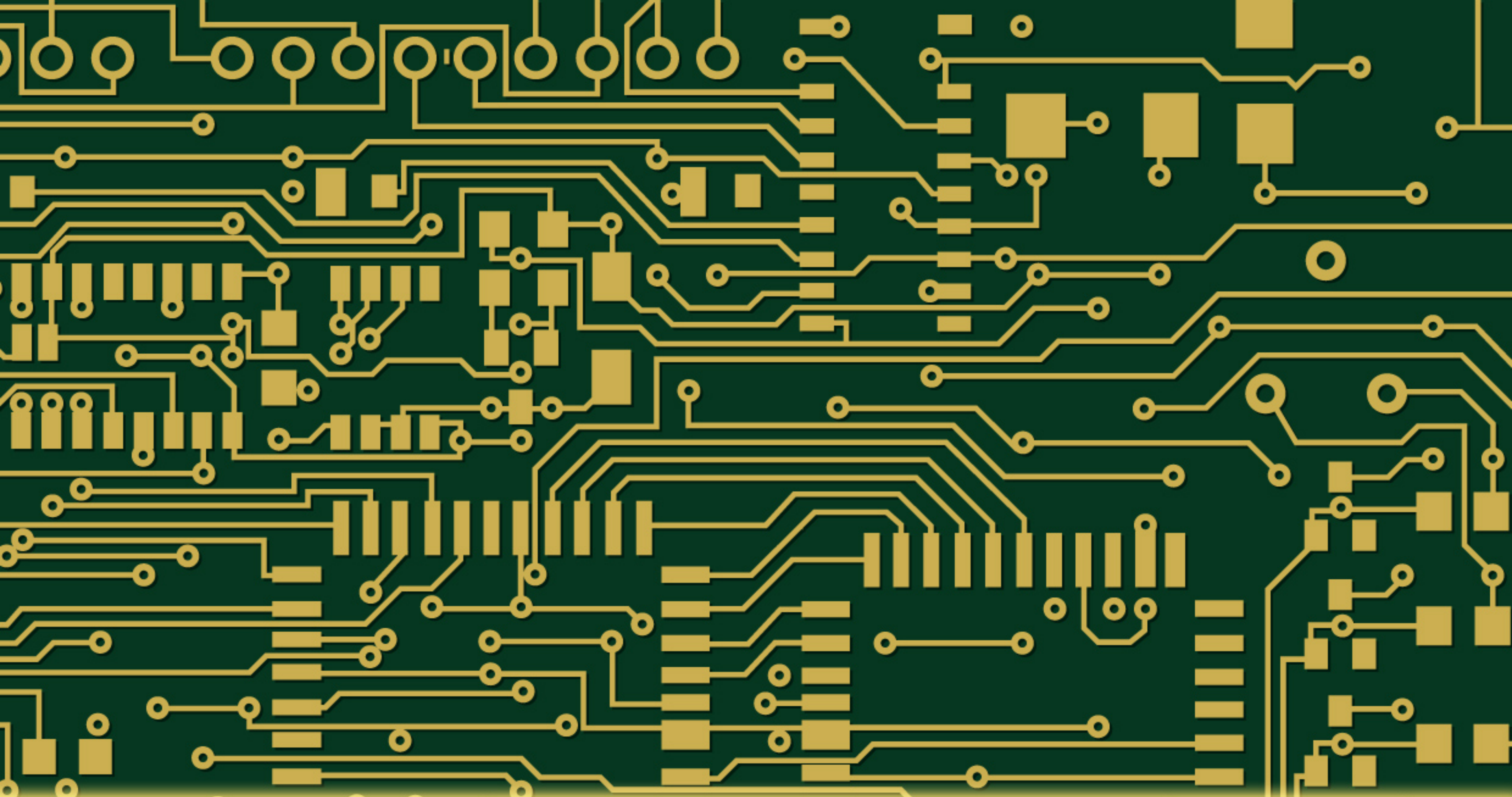
# Phase Relationship of Current and Voltage

- ✓ The faster the current through an inductor changes, the greater the induced voltage will be.
- ✓ A sinusoidal current always induces a sinusoidal voltage in inductive circuits.
- ✓ Therefore the plot of voltage with respect to the current can be plotted if the points on the current curve at which the voltage is zero and those at which it is maximum are known.
- ✓ The voltage leads the current by 90°.
- ✓ The phase relation of an inductor is shown below:

# Phase Relationship of Current and Voltage (cont.)



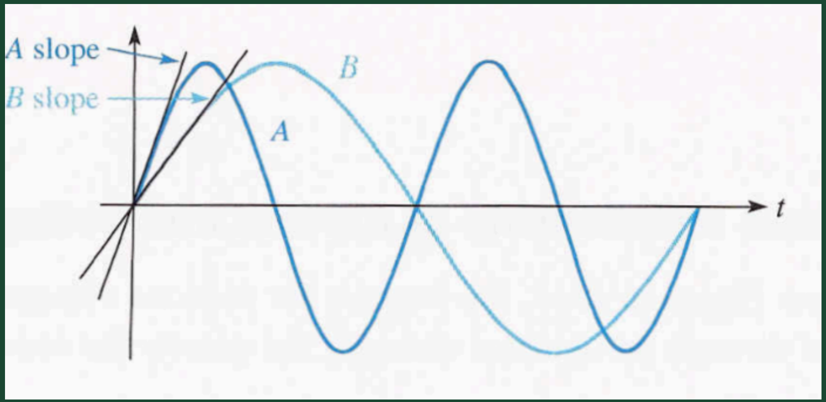





# Inductive Reactance

# Inductive Reactance

- ✓ Inductive reactance $X_L$ is the opposition to sinusoidal current, expressed in ohms.
- ✓ The symbol for inductive reactance is $X_L$.




- ✓ The rate of change of current is directly related to frequency.
- ✓ The faster the current changes, the higher the frequency.

# Inductive Reactance (cont.)

- ✓ When frequency increases, di / dt increases, and thus $v_{ind}$ increases.
- ✓ When frequency decreases, di / dt decreases, and thus $v_{ind}$ decreases.
- ✓ The induced voltage is directly dependent on frequency, i.e.,

$$\overset{\uparrow}{v}_{ind} = L(\overset{\uparrow}{di} / dt) \quad and \quad \underset{\downarrow}{v_{ind}} = L(\underset{\downarrow}{di} / dt)$$

- ✓ If di / dt is constant and the inductance is varied, an increase in L produces an increase in $v_{ind}$ and a decrease in L produces a decrease in $v_{ind}$.

$$\overset{\uparrow}{v}_{ind} = \overset{\uparrow}{L}(di / dt) \quad and \quad \underset{\downarrow}{v_{ind}} = \underset{\downarrow}{L}(di / dt)$$

# Inductive Reactance (cont.)

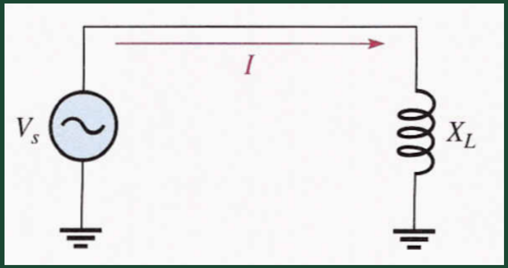
- ✓ Therefore, $X_L$ is directly proportional to induced voltage and thus directly proportional to inductance. Hence, $X_L$ is proportional to fL.

$$X_L = 2\pi fL$$

- ✓ Inductive reactance, $X_L$ is in ohms when f is in hertz and L is in henries.

# Ohm's Law

- ✓ The reactance of an inductor is analogous to the resistance of a resistor as shown below:

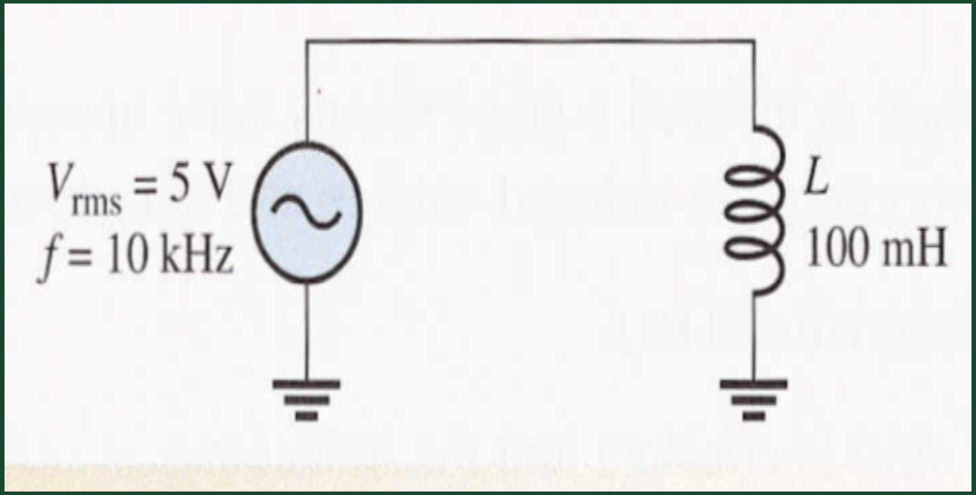



- ✓ Since inductive reactance is a form of opposition to current, ohm's law applies to inductive circuits as well as to resistive circuits and capacitive circuits and it is stated as follows:

$$I = \frac{V}{X_L}$$

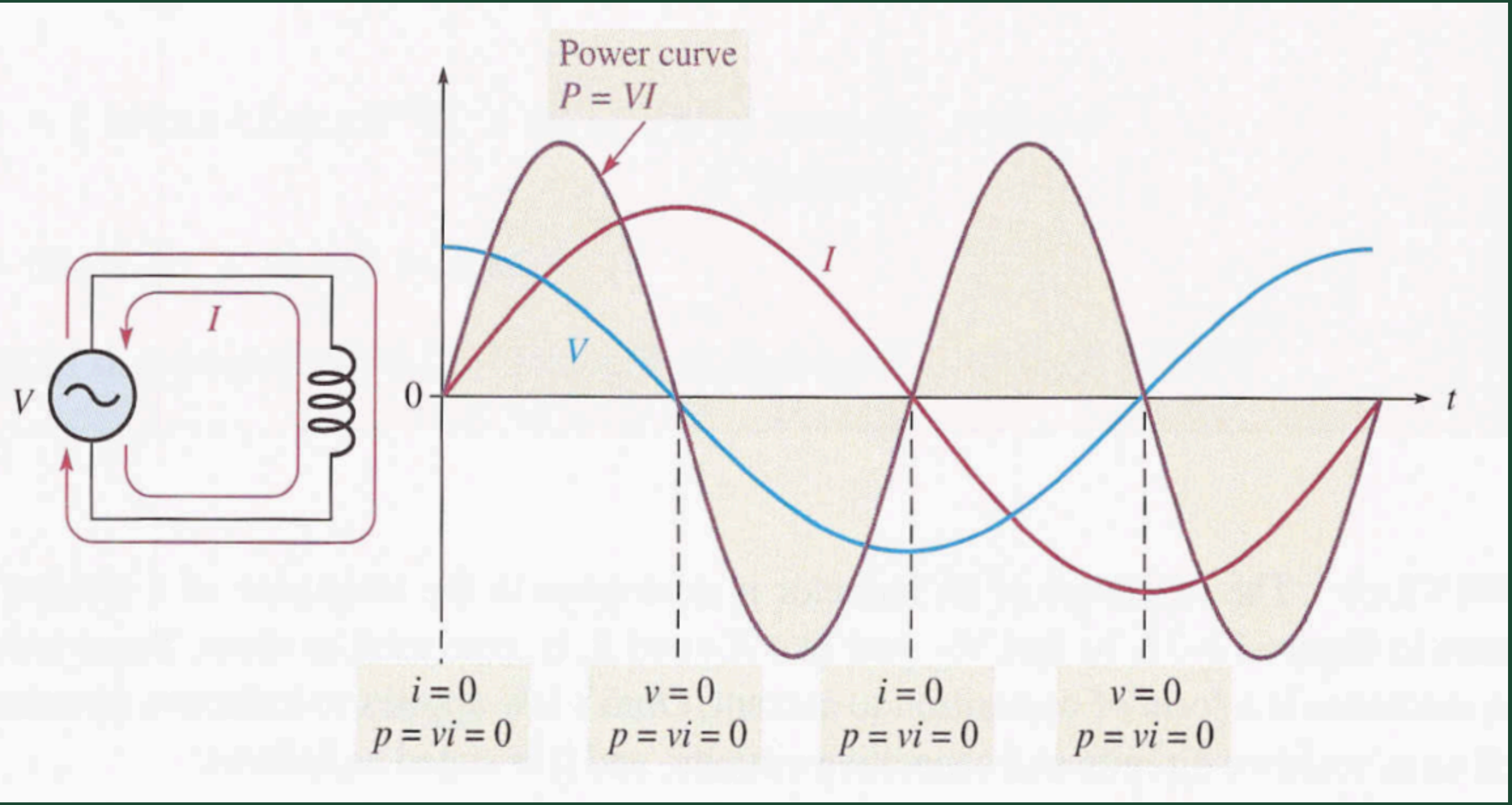
# Example #3

✓ Determine the rms current in figure below:



# Power in Inductor

# Power



# Instantaneous Power

- ✓ The product of v and i gives instantaneous power.
- ✓ At points where v or i is zero, p is also zero.
- ✓ When both v and i are positive, p is also positive.
- ✓ When either v or I is positive and the other negative, p is negative.
- ✓ When both v and i are negative, p is positive.

# True Power

- ✓ Ideally all of the energy stored by an inductor during the positive portion of the power cycle is returned to the source during the negative portion.
- ✓ No net energy is lost due to conversion to heat in the inductor, so the true power is zero.

$$P_{true} = (I_{rms})^2 R_W$$

# Reactive Power

- ✓ The rate at which an inductor stores or returns energy is called its reactive power, with the unit of VAR (volt-ampere reactive).
- ✓ The reactive power is a nonzero quantity because at any instant in time the inductor is actually taking energy from the source or returning energy to it.
- ✓ Reactive power does not represent an energy loss due to conversion to heat.
- ✓ The following formulas apply:

$$P_r = V_{rms} I_{rms}$$

$$P_r = \frac{V_{rms}^2}{X_L}$$

$$P_r = I_{rms}^2 X_L$$

# Quality Factor (Q) of a Coil

- ✓ The quality factor (Q) is the ratio of the reactive power in an inductor to the true power in the winding resistance of the coil or the resistance in series with the coil.
- ✓ It is a ratio of the power in L to the power in $R_W$.
- ✓ A formula for Q is as follows:

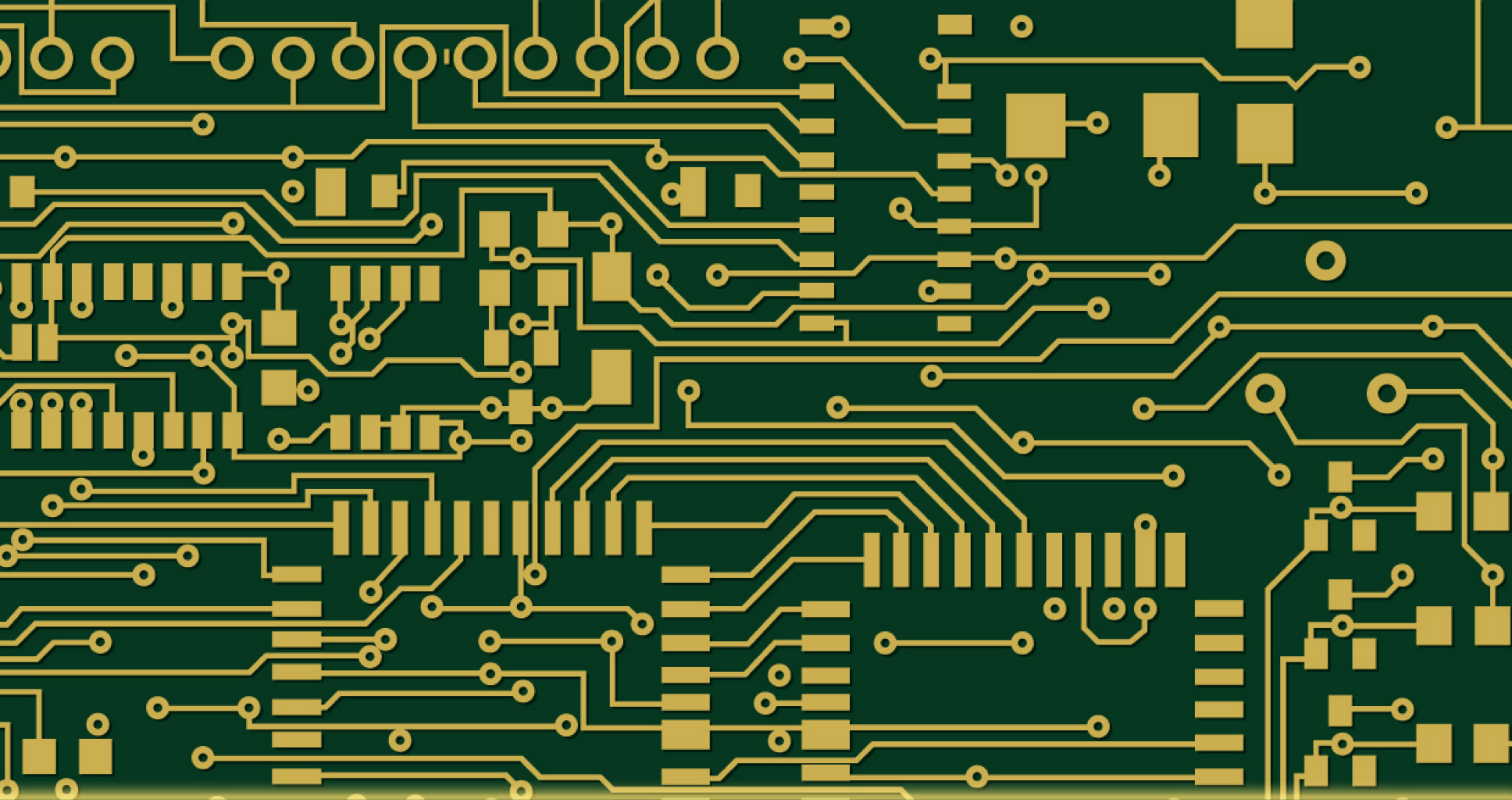
$$Q = \frac{reactive \quad power}{true \quad power} = \frac{I^2 X_L}{I^2 R_W}$$

- ✓ The current is the same in L and $R_w$; thus the $I^2$ terms cancel, leaving:

$$Q = \frac{X_L}{R_W}$$

# Example #4

- ✓ A 10 $V_{rms}$ signal with a frequency of 1kHz is applied to a 10mH coil with a negligible winding resistance. Determine the reactive power?



# Practice Problems

# Problem #1

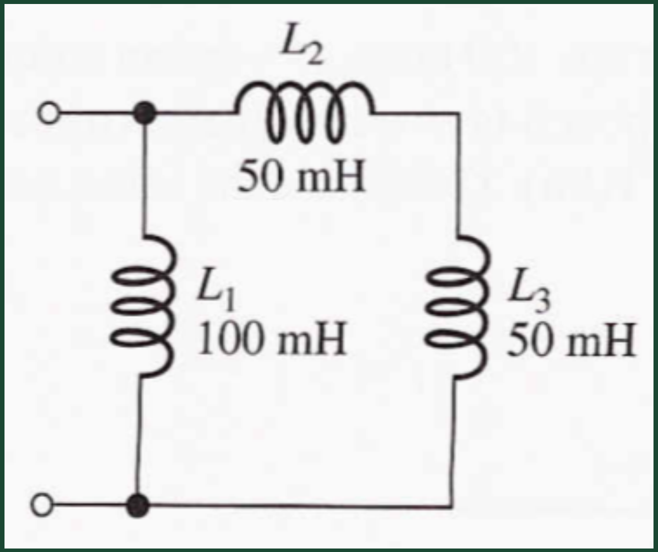
- ✓ Two series capacitors (one 1μF, the other of unknown value) are charged from a 12V source. The 1μF capacitor is charged to 8V and the other to 4V. What is the value of the unknown capacitor?

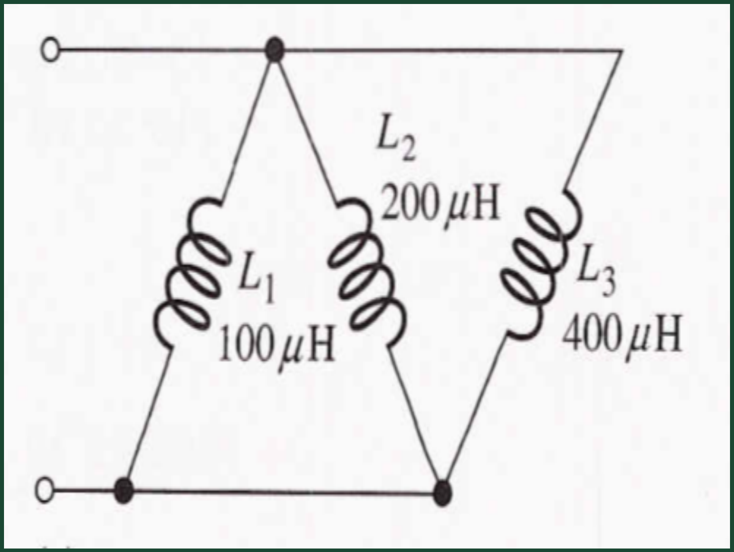
# Problem #2

- ✓ A sinusoidal voltage of 20V rms produces an rms current of 100mA when connected to a certain capacitor. What is the reactance?

# Problem #3

✓ Determine the total inductance of each circuit shown below:



# Thank You