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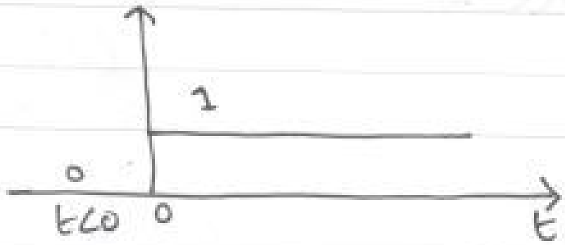
## CAUSALITY FOR LTI SYSTEM:-

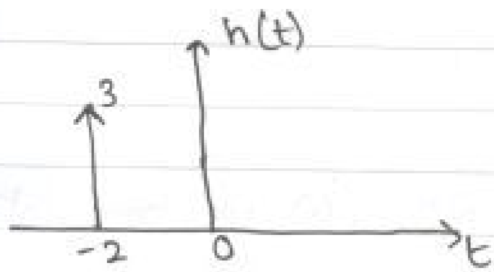
Example: $h(t) = u(t)$




Sol:-

when $t<0$ the $h(t)=0$, Hence the condition $h(t)=0$, $t<0$ is satisfied so the system is Causal in nature.

EXAMPLE #1:-




Sol:-

$t(-2) \Rightarrow 3$

$t<0$, $h(t) = 3$

Hence the condition is not satisfied so the system non-causal.

EXAMPLE #2:-

$h(t) = e^{-(t+1)} \cdot u(t)$

Sol:-

$$u(t) = \begin{cases} 0 & , t<0 \\ 1 & , t \geq 0 \end{cases}$$

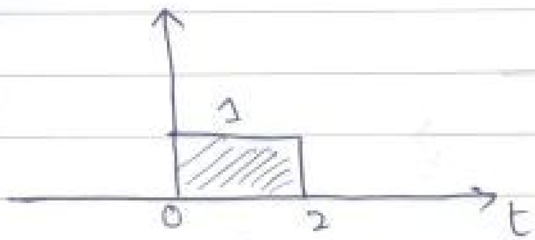
$$h(t) = \begin{cases} 0 & , t<0 \\ e^{-(t+1)} & , t \geq 0 \end{cases}$$

Hence the condition is satisfied so the given system is causal

Example #3:

$$h(t) = u(t) - u(t-2)$$

Sol:



$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{0} 0 dt + \int_{0}^{2} 1 dt + \int_{2}^{\infty} 0 dt$$

$$= 0 + (t)_{0}^{2} + 0 \Rightarrow 2-0 \Rightarrow 2 < \infty$$

Hence, the system is stable.

Time Delay:

Examples:- $u(t-1) * u(t-2) = ?$

Sol:

⇒ Whenever something is convoluted with $u(t)$ we have integrate only the first signal.

$$u(t) * u(t) = \int_{-\infty}^{t} u(\tau) d\tau = r(t)$$

incase of delayed signals, $u(t-1) * u(t-2) = r(t-3)$ Ans

Time Scaling:-

Example:-

$$(t^2+t) * t = y(t)$$

Sol:

the result $y(t) = \frac{1}{3} y(3t)$ then find out the convoluted functions

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$$? * ? = \frac{1}{3} y(3t)$$

$a = 3$ $\qquad \therefore x_1(at) * x_2(at) = \frac{1}{|a|} y(at), a \neq 0$

$x_1(3t) * x_2(3t)$

$x_1(t) = t^2 + t \Rightarrow x_1(3t) = (3t)^2 + 3t$

$\qquad\qquad\qquad x_1(3t) = 9t^2 + 9t$

$x_2(t) = 3 \Rightarrow x_2(3t) = 3t$

$$(9t^2 + 3t) * 3t = \frac{1}{3} y(3t)$$

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EXAMPLE #48:-

$x(t) = e^{-at} u(t)$ and $h(t) = e^{-bt} u(t)$

Sol:-

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) e^{-b(t-\tau)} u(t-\tau)\, d\tau$$

$$= \int_0^t e^{-a\tau} e^{-b(t-\tau)}\, d\tau$$

$$= \int_0^t e^{-a\tau} e^{-bt} e^{b\tau}\, d\tau$$

$$= e^{-bt} \int_0^t e^{-\tau(a-b)}\, d\tau$$

$$= \frac{e^{-bt}}{\phantom{a-b}} \cdot \frac{e^{-(a-b)\tau}}{-(a-b)} \Big|_0^t$$

$$= \frac{e^{-bt}}{-(a-b)} \left[ e^{-(a-b)t} - e^{-(a-b)0} \right] = \frac{e^{-bt}}{-(a-b)} \left[ e^{-(a-b)t} - 1 \right]$$

$$= \frac{e^{-bt}}{a-b} \left[ 1 - e^{-(a-b)t} \right] u(t)$$

$$= \frac{e^{-bt}}{a-b} \left[ 1 - e^{-at} e^{bt} \right] u(t) = \frac{1}{a-b} \left[ e^{-bt} - e^{-at} \cancel{e^{bt-bt}} \right] u(t)$$

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$$y(t) = \frac{1}{a-b}\left[e^{-bt} - e^{-at}\right]u(t) \quad , \quad a \neq b$$