Signal & Systems
Lecture #5

03rd April 18

System Properties & Impulse Response

LTI Systems With & Without Memory

- A system is memoryless if the output depends on the current input only.
- An LTI system is memoryless if and only if h[n]=0 for n≠0.
- In this case the impulse response has the form: h[n]=Kδ[n] where K=h[0] is a constant and the convolution sum reduces to the relation:

$$y[n] = Kx[n]$$

- If a discrete time LTI system has an impulse repsonse h[n] that is not identically zero for n≠0, then the system has memory.
- A continuous time LTI system is memoryless if h(t)=0 for t≠0, and such a memoryless LTI system has the form:y (t)= Kx(t) for some constant K and has the impulse response: $h(t) = K\delta(t)$

LTI Systems With & Without Memory (cont.)

- Note that if K=1 in discrete and continuous time impulse response then these systems become identity systems with output equal to the input and with unit impulse response equal to the unit impulse.

- In this case the convolution sum and integral formulas imply that:

$$x[n] = x[n] * \delta[n]$$

$$and$$

$$x(t) = x(t) * \delta(t)$$

LTI Systems With & Without Memory (cont.)

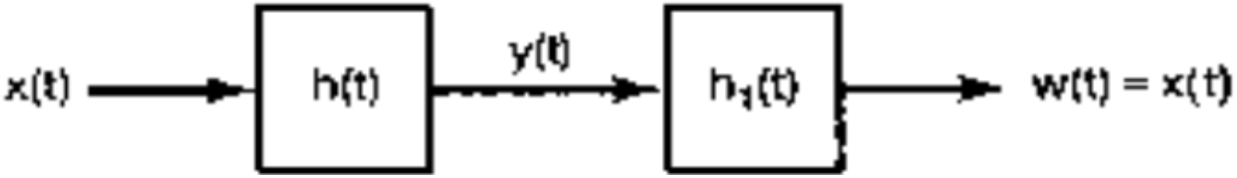
- Which reduces to the sifting properties of the discrete and continuous time unit impulses:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

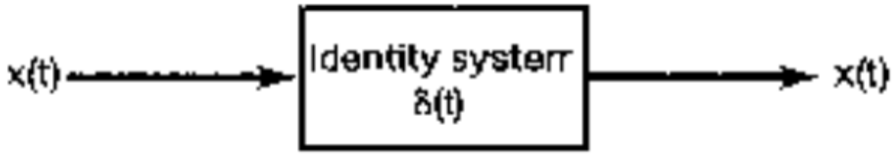
Invertibility of LTI Systems

- A system is invertible if and only if an inverse system exists that, when connected in series with the original system, produces an output equal to the input to the first system.
- If an LTI system is invertible, then it has an LTI inverse.
- Suppose we have a system with impulse response h(t). The inverse system with impulse response $h_1(t)$ results in w(t) = x(t) as shown below in series interconnection.



Invertibility of LTI Systems (cont.)

- This system is identical to the identitv svstem shown below:



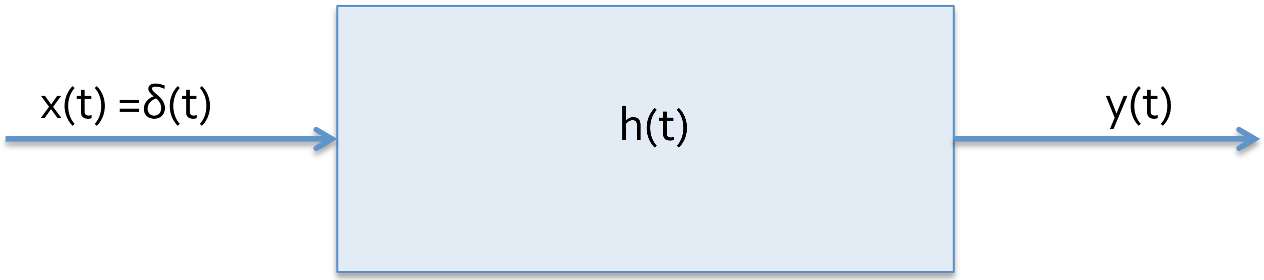
- Since the overall impulse response shown in first figure is h(t) * h_1(t), we have the condition that h_1(t) must satisfy for it to be the impulse response of the inverse system, i.e.,

$$h(t) * h_1(t) = \delta(t)$$

- Similarly in discrete time the impulse response h_1[n] of the inverse system for an LTI system with impulse response h[n] must satisfy:

$$h[n] * h_1[n] = \delta[n]$$

Causality for LTI Systems



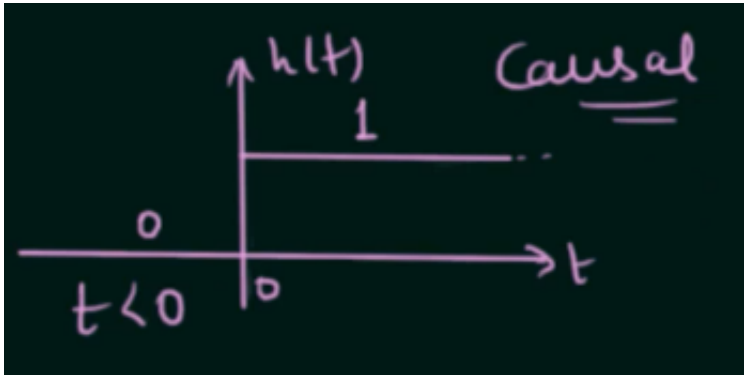
- Where

$$y(t) = \int_{-\infty}^{\infty} h(\tau).x(t-\tau)\,d\tau$$

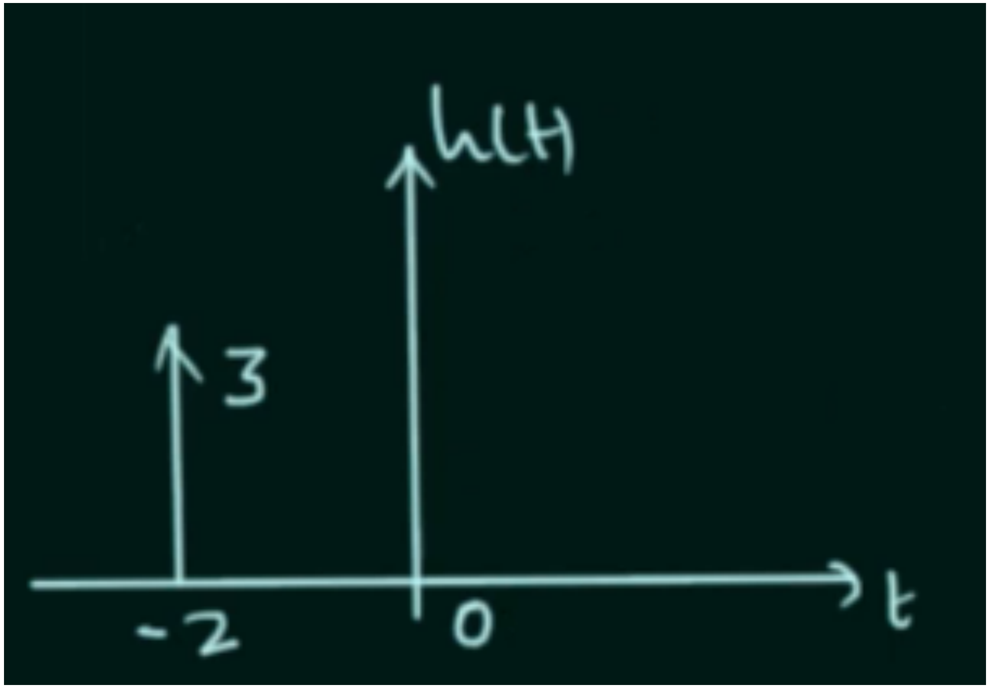
- The output of a causal system depends only on the present and past values of the input to the system.
- $\tau<0 \rightarrow x(t-\tau)$ will become the future value of i/p.
- When our system is causal our o/p y(0) will become zero.

Causality for LTI Systems (cont.)

- Whenever τ<o, impulse response h(τ)=o. i.e., h(t)=o, t<o. then our system will be causal.
- Example: h(t) = u(t)



Example #1



Example #2

- $h(t) = e^{-(t+1)}.u(t)$

Stability for LTI Systems

- A system is stable if every bounded input produces a bounded output.
- Inorder to determine conditions under which LTI systems are stable, consider an input x[n] that is bounded in magnitude:

$$\left|x[n]\right| < B, \quad for \quad all \quad n$$

- Suppose that this input is applied to an LTI system with unit impulse response h[n].
- Then, using the convolution sum, we obtain an expression for the magnitude of the output: $\left|y[n]\right| = \left|\sum_{k=-\infty}^{\infty} h[k]x[n-k]\right|$

Stability for LTI Systems (cont.)

- Since the magnitude of the sum of a set of numbers is no longer than the sum of the magnitudes of the numbers, it follows from the above equation that:

$$\left|y[n]\right| \le \sum_{k=-\infty}^{\infty} \left|h[k]\right| \left|x[n-k]\right|$$

- As $\left|x[n-k]\right| < B$ or all values of k and n, then:

$$\left|y[n]\right| \le B \sum_{k=-\infty}^{\infty} \left|h[k]\right|, \quad for \quad all \quad n$$

- From above equation we can conclude that if the impulse response if absolutely summable that is, if:

$$\sum_{k=-\infty}^{\infty} \left|h[k]\right| < \infty$$

Stability for LTI Systems (cont.)

- Then y[n] is bounded in magnitude and hence the system is stable.
- If above equation is not satisfied, there are bounded inputs that result in unbounded outputs.
- Thus in continuous time case the system is stable if the impulse response is absolutely integrable, i.e., if:

$$\int_{-\infty}^{\infty} \left| h(\tau) \right| d\tau < \infty$$

Example #3

- $h(t)=u(t) - u(t-2)$

Convolution Properties

Property of Delta Function

- Delta function is impulse function.

$$x(t) * \delta(t - t_1) = x(t - t_1)$$

- When t_1 =0 then,

$$x(t) * \delta(t) = x(t)$$

- If coefficient with delta is not 1 then,

$$x(t) * A\delta(t - t_1) = Ax(t - t_1)$$

Examples of Delta Property

- Example #1:
  - $x(t) = r(t)$; $r(t) * \delta(t-2) = r(t-2)$
- Example #2:
  - $u(t+3) * \delta(t-1)$
  - $u(t-1+3) = u(t+2)$

Time Delay

- $x_1(t) * x_2(t) = y(t)$
- Let's provide delays in $x_1(t)$ and $x_2(t)$ then, $x_1(t-t_1) * x_2(t-t_2) = y[t-(t_1+t_2)]$.
- Example : $u(t-1) * u(t-2) =?$

Time Scaling

- $x_1(t)*x_2(t) = y(t)$
- After peforming time scaling we have: $x_1(at) * x_2(at) = (1/|a|)\, y(at)$, $a \neq 0$.
- Example: $(t^2+t) * t = y(t)$

Convolution Example

Example #4

- $x(t) = e^{-at} u(t)$ and $h(t) = e^{-bt} u(t)$.
- Find $y(t) = x(t) * h(t)$ by evaluating the convolution integral.

Thank You!