Day/Date MONDAY / 29 April, 18

: LECTURE # 6 :

REVISION SESSION :-

PROBLEM # 1 :-

Calculate the period of each of the following values of frequency:

a) 60 Hz

Sol:-

$$T = \frac{1}{f} = \frac{1}{60\,Hz} \Rightarrow 0.016\,s$$

b) 1 kHz

Sol:-

$$T = \frac{1}{f} = \frac{1}{1000} \Rightarrow 1 \times 10^{-3}\,s$$

PROBLEM # 2 :-

At what speed of rotation must a four-pole generator be operated to produce a 400 Hz sinusoidal voltage?

Sol:-

$$f = (\text{number of pole pairs})(rps)$$

$$400\,Hz = (2)(rps)$$

$$\text{Speed} \Rightarrow rps = \frac{400}{2} \Rightarrow 200\,rps.$$

b) 135°

Problem #4 Part b

Sol:

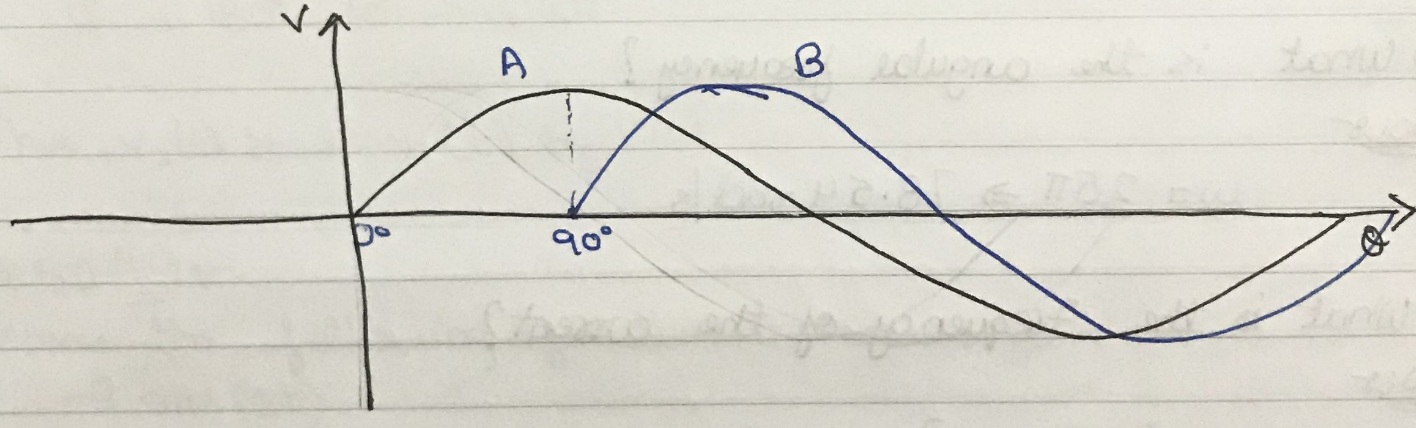
$$rad = \left(\frac{\pi\, rad}{180°}\right) \times degrees$$

$$= \left(\frac{\pi\, rad}{180°}\right) \times 135° \Rightarrow 2.35\ rad$$

PROBLEM #5:

Make a sketch of two sine waves as follows: Sine wave A is the reference and sine wave B lags A by 90°. Both have equal amplitude.

Sol:



PROBLEM #6:

For a particular 0° reference sinusoidal current, the peak value is 100m

Determine the instantaneous value at each of the following points:

a) 35°.

Sol:

$I_p = 100mA$

$$i = I_p \sin(35°)$$

$$= 100 \sin(35°) \Rightarrow 57.4mA$$

b) 95°

Sol:

$$i = I_p \sin(95^\circ)$$
$$= 100\,mA \sin(95^\circ) \Rightarrow 99.6\,mA$$

PROBLEM # 7:

A current source in a linear circuit has:-

$$i_s = 15\cos(25\pi t + 25^\circ)\,A$$

a) What is the amplitude of the current?

Sol:-

Amplitude $\Rightarrow$ 15 A

b) What is the angular frequency?

Sol:-

$$\omega = 25\pi \Rightarrow 78.54\ rad/s$$

c) What is the frequency of the current?

Sol:-

$\cancel{f = \omega/2\pi}$ $\quad f = \dfrac{1}{T}$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{78.54} \Rightarrow 0.07999\,s$$

$$f = \frac{1}{T} = \frac{1}{0.07999} \Rightarrow 12.5\ Hz$$

d) Calculate i_s at $t = 2ms$.

Sol:

$$i_s(2ms) = 15\cos\left[25\pi(2\times10^{-3}) + 25^\circ\right]A$$
$$= 15\cos\left[0.157 + 25^\circ\right]A$$

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$i_s(2ms) = 15\cos(25.157°)\,A \Rightarrow 13.57A.$

PROBLEM #8:-

For the following pair of sinusoid determine which one leads and by how much?

~~Sol~~: ~~v(t) = ...~~ $v_1(t) = 4\cos(377t + 10°)V$

$v_2(t) = -20\cos(377t)V$

Sol:-

$v_1(t) = 4\cos(377t + 10°)V$

$v_2(t) = -20\cos(377t)V \Rightarrow 20\cos(377t + 180°)$

$\because -\cos wt = \cos(wt + 180°)$

$\phi = 180° - 10° \Rightarrow 170°$

Thus, $v_2(t)$ leads $v_1(t)$ by $170°$.

PROBLEM #9:-

a) Express the following function in cosine form:

$-9\sin(8t)$

Sol:-

$\because -\sin wt = \cos(wt + 90°)$

$-9\sin(8t) \Rightarrow 9\cos(8t + 90°)$

b) Express the following function in sine form:-

$-10\cos(wt + 50°)$

Sol:-

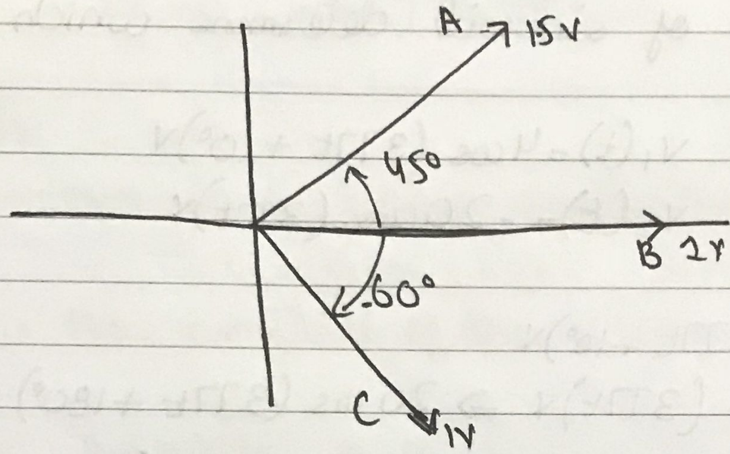
$\because -\cos(wt) = \sin(wt - 90°)$

$-10\cos(wt + 50°) = 10\sin(wt + 50° - 90°)$

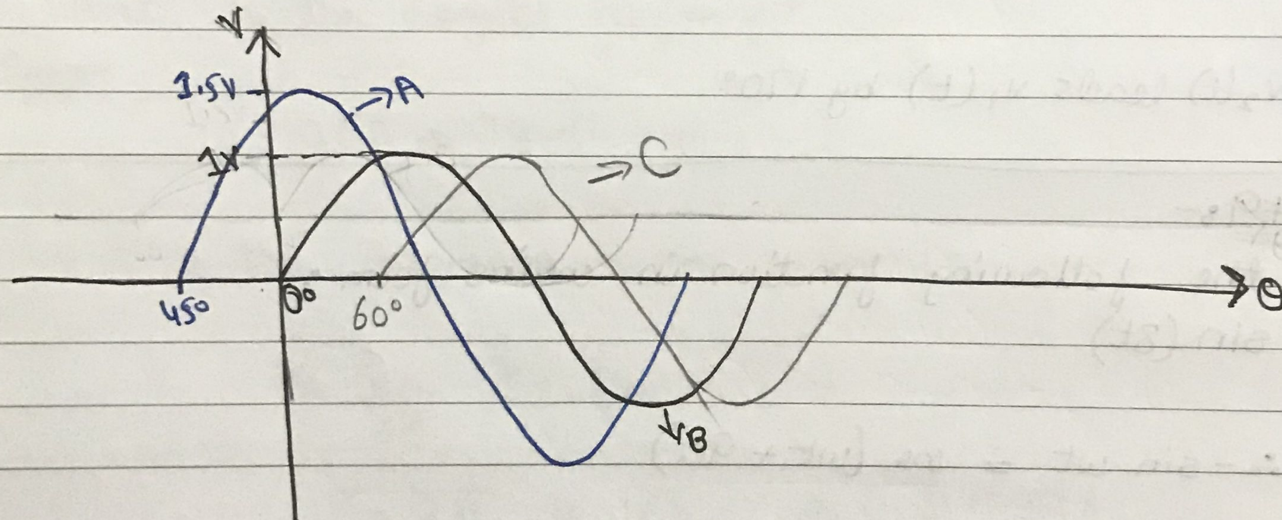
$\Rightarrow 10\sin(wt - 40°)$

PROBLEM #10:

Draw the sine waves represented by the phasor diagram shown below. The phasor lengths represent peak values.



Sol:



PROBLEM #11:

If $f(\phi) = \cos\phi + j\sin\phi$, show that $f(\phi) = e^{j\phi}$.

Sol:

If, $f(\phi) = \cos\phi + j\sin\phi$

$$\frac{df}{d\phi} = -\sin\phi + j\cos\phi$$ taking j common

$$= j(\cos\phi + j\sin\phi) = j f(\phi)$$

$$\frac{dz}{z} = j\,d\phi$$

Integrating both sides:

$$\int \frac{dz}{z} = j\int d\phi$$

$$\ln f = j\phi + \ln A \qquad \therefore \text{taking exponent on both sides}$$

$$f = Ae^{j\phi} = \cos\phi + j\sin\phi$$

$$z(0) = A = 1 \qquad \therefore e^{0} = 1, \; \cos(0) + j\sin(0) \Rightarrow 1$$

$$z(\phi) = e^{j\phi} = \cos\phi + j\sin\phi$$

PROBLEM #12:

Find the phasors corresponding to the following signals:

a) $v(t) = 21\cos(4t - 15^\circ)\,V$

Sol:

$$\bar{V} = 21\angle -15^\circ\ V.$$

b) $i(t) = -8\sin(10t + 70^\circ)\,mA$

Sol:

$$i(t) = +8\sin(10t + 70^\circ + 180^\circ) \Rightarrow 8\cos(10t + 70^\circ + 180^\circ - 90^\circ)$$

$$= 8\cos(10t + 160^\circ)$$

$$\bar{I} = 8\angle 160^\circ\ mA$$

PROBLEM #13:

Let $\bar{X} = 4\angle 40^\circ$ and $\bar{Y} = 20\angle -30^\circ$. Evaluate $(X+Y)/X$ and express your result in polar form:

Sol:

To evaluate $(\bar{X} + \bar{Y})$ change $\bar{X}$ and $\bar{Y}$ in rectangular form.

$\bar{X} = 4 \angle 40°$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 4\cos 40°$, $y = 4\sin 40°$

$x \Rightarrow 3.06$, $y \Rightarrow 2.57$

$\bar{X} = 3.06 + j2.57$

$\bar{Y} = 20 \angle -30$

$x = 20\cos(-30°)$, $y = 20\sin(-30°)$

$x \Rightarrow 17.32$, $y \Rightarrow -10$

$\bar{Y} = 17.32 - j10.$

$(\bar{X}+\bar{Y}) = (3.06 + j2.57) + (17.32 - j10)$

$= (3.06 + 17.32) + (j2.57 - j10)$

$\cancel{= 20.38 + j} -$

$(\bar{X}+\bar{Y}) = (3.06 + j2.57) + (17.32 - j10)$

$= (3.06 + 17.32) + j(2.57 + (-10))$

$= 20.38 + j(-7.43) \Rightarrow 20.38 - j7.43$

$r = \sqrt{x^2+y^2} \Rightarrow \sqrt{(20.38)^2 + (-7.43)^2} \Rightarrow 21.69$

$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{-7.43}{20.38} \Rightarrow -20.03$

$(\bar{X}+\bar{Y}) = 21.69 \angle -20.03$

Now, $(\bar{X}+\bar{Y})/\bar{X} = (21.69 \angle -20.03°)/(4 \angle 40°)$

$= \frac{21.69}{4} \angle (-20.03° - 40°)$

$\Rightarrow 5.4225 \angle -60.03°$

$$(\bar{X}+\bar{Y})/\bar{X} \Rightarrow 5.4\angle -60^\circ$$

PROBLEM #14:-

Simplify the following expressions :-

a) $\frac{2+j3}{1-j6} + \frac{7-j8}{-5+j11}$

Sol: Change all in polar form

$2+j3 \Rightarrow r = \sqrt{2^2+3^2} \Rightarrow 3.60$
$\theta = \tan^{-1}\frac{3}{2} \Rightarrow 56.3^\circ$ } $3.60\angle 56.3^\circ$

$1-j6 \Rightarrow r = \sqrt{1^2+(-6)^2} \Rightarrow 6.08$
$\theta = \tan^{-1}\frac{-6}{1} \Rightarrow -80.5^\circ$ } $6.08\angle -80.5^\circ$

$7-j8 \Rightarrow r = \sqrt{7^2+(-8)^2} \Rightarrow 10.6$
$\theta = \tan^{-1}\frac{-8}{7} \Rightarrow -48.8^\circ$ } $10.6\angle -48.8^\circ$

$-5+j11 \Rightarrow r = \sqrt{(-5)^2+(11)^2} \Rightarrow 12.08$
$\theta = \tan^{-1}\frac{11}{-5} \Rightarrow -65.5^\circ$ } $12.08\angle -65.5^\circ$

Now,

$$\frac{2+j3}{1-j6} + \frac{7-j8}{-5+j11} = \left[\frac{3.60}{6.08}\angle 56.3^\circ - (-80.5^\circ)\right]$$
$$+\left[\frac{10.6}{12.08}\angle -48.8^\circ - (-65.5^\circ)\right]$$
$$= [0.59\angle 136.8^\circ] + [0.877\angle 16.7^\circ]$$

again change the answer in rectangular form:-

$0.59\angle 136.8^\circ \Rightarrow x = r\cos\theta \quad , \quad y = r\sin\theta$
$= 0.59\cos(136.8^\circ) \quad , \quad y = 0.59\sin(136.8^\circ)$
$x \Rightarrow -0.43 \quad , \quad y \Rightarrow 0.404$

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$-0.43 + j0.404.$

$0.877 \angle 16.7^\circ \Rightarrow x = 0.877 \cos 16.7^\circ , y = 0.877 \sin 16.7^\circ$

$x \Rightarrow 0.84 \quad , \quad y \Rightarrow 0.252$

Now,

$(-0.43 + j0.404) + (0.84 + j0.252) = [-0.43 + 0.84]$
$+ j(0.404 + 0.252)$

$\Rightarrow 0.41 + j0.656$

PROBLEM # 15:-

Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$ V and $v_2 = 12\cos(50t + 30^\circ)$ V. Find v:-

Sol:-

$v_1 = 10\cos(50t - 60^\circ)$ V , $v_2 = 12\cos(50t + 30^\circ)$ V -

$\bar{V}_1 = 10\angle -60^\circ$, $\bar{V}_2 = 12\angle 30^\circ$

$\bar{V}_1 \Rightarrow x + jy$, $\bar{V}_2 = x + jy$

$x = 10\cos(-60^\circ) \Rightarrow 5$, $x = 12\cos(30^\circ) \Rightarrow 10.39$

$y = 10\sin(-60^\circ) \Rightarrow -8.66$, $y = 12\sin(30^\circ) \Rightarrow 6$

$\bar{V}_1 = 5 - 8.66j$, $\bar{V}_2 = 10.39 + j6$

$\bar{V} = \bar{V}_1 + \bar{V}_2 = (5 - 8.66j) + (10.39 + j6)$

$= (5 + 10.39) + j(-8.66 + 6)$

$\bar{V} \Rightarrow 15.39 + j(-2.66) \Rightarrow 15.39 - j2.66$

Now changes in ~~phasor~~ polar form

$\bar{V} = r\angle\theta$

$r = \sqrt{x^2 + y^2} \Rightarrow \sqrt{(15.39)^2 + (-2.66)^2} \Rightarrow 15.62$

$\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{-2.66}{15.39} \Rightarrow -9.805^\circ$

$V = 15.62 \cos(50t - 9.805^\circ)$

PROBLEM #16:-

Obtain the sinusoids corresponding to each of the following Phasors:

a) $\bar{V} = 60 \angle 15^\circ V$; $\omega = 1$

Sol:-

~~$V = 60 \sin(\omega t + 15^\circ) V \Rightarrow 60 \sin(t + 15^\circ) V$~~

$V = 60 \cos(\omega t + 15^\circ) V \Rightarrow 60 \cos(t + 15^\circ) V$

b) $\bar{I} = 2.8 e^{-j\pi/3} A$, $\omega = 377$

Sol:-

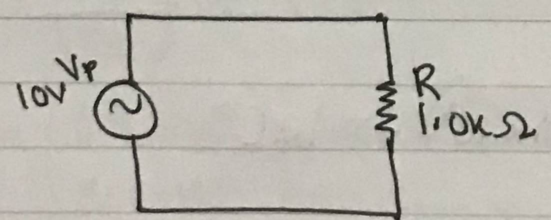
$\bar{I} = 2.8 \angle -\pi/3 \, A$

$i = 2.8 \cos(377t - \pi/3)$

$V = IR \quad I =$

PROBLEM #17:-

A sinusoidal voltage is applied to the resistive circuit shown below. Determine the following:



a) I_{rms}

Sol:

$V_{rms} = 0.707 V_p = 0.707(10) \Rightarrow 7.07 V$

From Ohm's law, $I_{rms} = \frac{V_{rms}}{R} \Rightarrow \frac{7.07}{1K} \Rightarrow 7.07 mA$

b) I_{avg}

Sol:

$$I_{avg} = 0.637 I_p$$

$$I_p = 1.414 I_{rms} = 1.414(7.07mA) \Rightarrow 9.99mA$$

c) I_{pp}

Sol:-

$$I_{pp} = 2.828 I_{rms}$$
$$= 2.828(7.07mA) \Rightarrow 19.99mA$$

PROBLEM #18:

a) Find the capacitance when $Q = 50\mu C$ and $V = 10V$.

Sol:

$$C = \frac{Q}{V}$$

$$C = \frac{50 \times 10^{-6}}{10} \Rightarrow 5\mu C$$

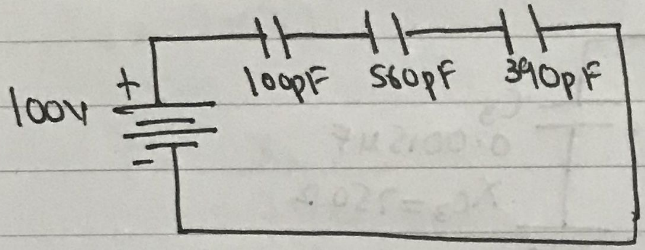
b) Find the charge Q when $C = 0.001\mu F$ and $V = 1kV$.

Sol:

$$Q = CV$$
$$= (0.001 \times 10^{-6}) \times (1 \times 10^{3}) \Rightarrow 1\mu C$$

PROBLEM #19:-

For the circuit shown below, determine the voltage across each capacitor :-



Sol:-

$V_T = 100V$

$C_1 = 100pF$, $C_2 = 560\,pF$, $C_3 = 390\,pF$

$$V_1 = \left(\frac{C_T}{C_1}\right) V_T$$

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{1\times10^{-14} + 1.78\times10^{-15} + 2.56\times10^{-15}}$$

$$C_T = \frac{1}{1.434\times10^{-14}} \Rightarrow 6.97\times10^{13}\,F$$

$$V_1 = \left(\frac{6.97\times10^{13}}{100\times10^{12}}\right)100V = (0.697)\,100V \Rightarrow 69.7V.$$

$$V_2 = \left(\frac{C_T}{C_2}\right) V_T$$

$$= \left(\frac{6.97\times10^{13}}{560\times10^{12}}\right)100V = (0.124)\,100V \Rightarrow 12.4V$$

$$V_3 = \left(\frac{C_T}{C_3}\right) V_T$$

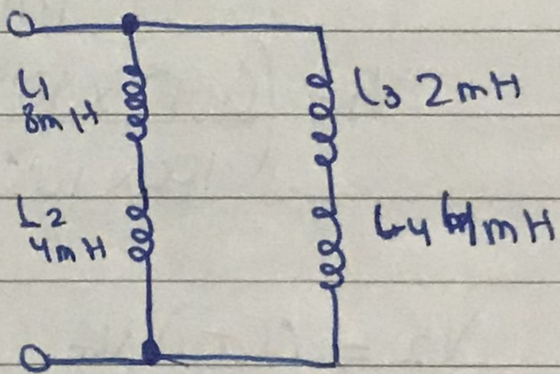
$$= \left(\frac{6.97\times10^{13}}{390\times10^{12}}\right)100V = (0.178)\,100 \Rightarrow 17.8\,V$$

PROBLEM # 20:-

What frequency will produce 500 mA total rms ~~of~~ current in the following circuit with an rms & input voltage of 10V?

Sol:-

$I_{rms} = 500\ mA$



$V_{rms} = 10V$

$f = ?$

$$X_L = \frac{V_{rms}}{I_{rms}}$$

$$X_L = \frac{10V}{500\times10^{-3}} \Rightarrow 20\,\Omega$$

$L_T = $ (L_3 in series with L_4) parallel with (series of L_1 and L_2)

$= (2mH + 4mH)$ parallel $(8mH + 4mH)$

$= (6mH)$ parallel $(12mH)$

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$$L_T = \frac{1}{\frac{1}{6mH} + \frac{1}{12mH}} = \frac{1}{166.66 + 83.33} \Rightarrow 4\,mH$$

$$X_L = 2\pi f L$$

$$f = \frac{X_L}{2\pi L}$$

$$f = \frac{20}{2\pi(4mH)} = 796Hz$$

PROBLEM # 3:-

A sine wave has a peak value of 12V. Determine the following values:-

a) Peak-to-Peak

Sol:-

$V_p = 12V$

$V_{rms} = 0.707V_p$

$= 0.707(12) \Rightarrow 8.484 V_{rms}$

$V_{PP} = 2.828 V_{rms}$

$= 2.828(8.484) \Rightarrow 23.99V$

b) Average.

Sol:-

$\cancel{V_{avg}} = V_{avg} = 0.637(12) \Rightarrow 7.644V$

PROBLEM # 4:-

Convert the following angular values from degrees to radians:-

a) $45°$

Sol:-

$rad = \left(\frac{\pi \, rad}{180°}\right) \times degrees$

$= \left(\frac{\pi \, rad}{180°}\right) 45° \Rightarrow 0.785 \, rad$

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