22-May-18 / TUESDAY

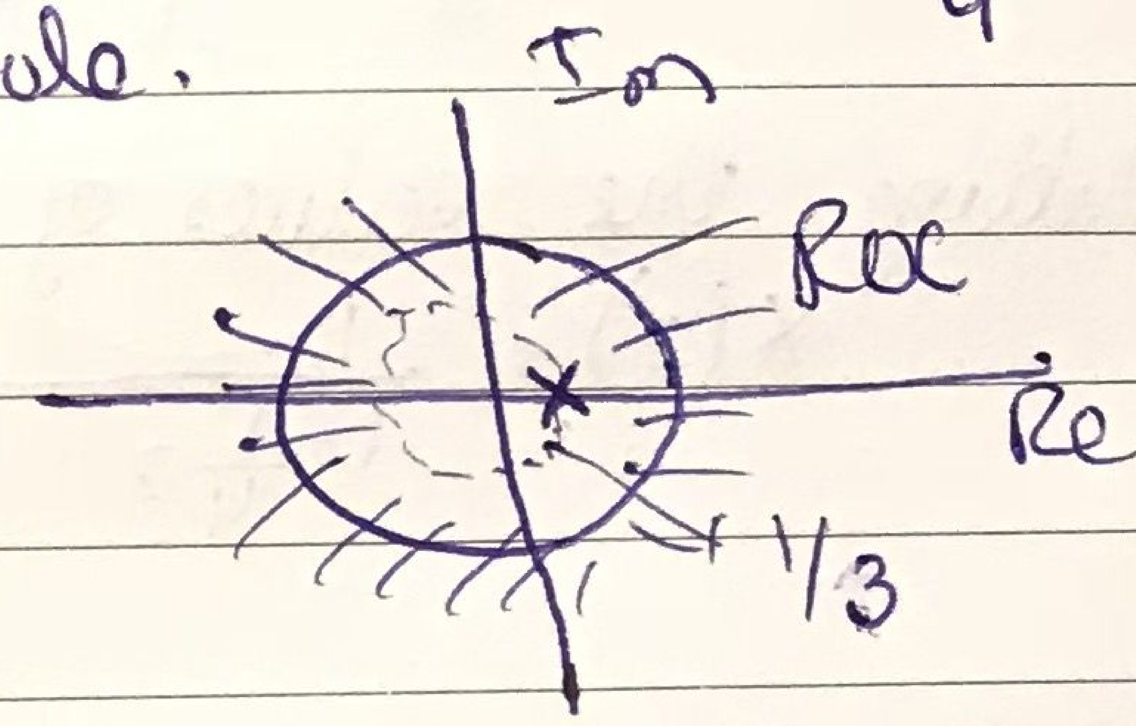
-: LECTURE # 10 :-

EXAMPLE # 4 :-

$$X(z) = \frac{3-\frac{5}{6}z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}, \quad |z| > \frac{1}{3}$$

Sol:

There are two poles, one at $z=\frac{1}{3}$, and one at $z=\frac{1}{4}$ and the ROC lies outside the outermost pole.



To solve this let's expand using partial fractions.

$$X(z) = \frac{3-5/6\, z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} \Rightarrow \frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1-\frac{1}{3}z^{-1}} \rightarrow ①$$

Cross multiplication gives

$$3-\frac{5}{6}z^{-1} = A\left(1-\frac{1}{3}z^{-1}\right) + B\left(1-\frac{1}{4}z^{-1}\right) \rightarrow ②$$

$$-\frac{1}{3}z^{-1} = -1 \quad ; \quad -\frac{1}{4}z^{-1} = -1$$

$$z^{-1} = 3 \quad ; \quad z^{-1} = 4$$

Put $z^{-1} = 3$ in equ ①

$$3-\frac{5}{6_2}(3) = A\left[1-\frac{1}{3}(3)\right] + B\left[1-\frac{1}{4}(3)\right]$$

$$3-\frac{5}{2} = A\left[1-1\right] + B\left[1-\frac{3}{4}\right]$$

$$\frac{6-5}{2} = A(0) + B\left[\frac{4-3}{4}\right]$$

$$\frac{1}{2} = B\left[\frac{1}{4}\right], \quad B = \frac{1}{2} \times 4 \Rightarrow 2$$

Now put $z^{-1} = 4$ in equ ②

$$3 - \frac{5}{\cancel{6}_3}\left(\cancel{4}^{2}\right) = A\left[1 - \frac{1}{3}(4)\right] + B\left[1 - \frac{1}{\cancel{4}}(\cancel{4})\right]$$

$$3 - \frac{10}{3} = A\left[1 - \frac{4}{3}\right] + B[0]$$

$$\frac{9-10}{3} = A\left[\frac{3-4}{3}\right]$$

$$-\frac{1}{3} = A\left[-\frac{1}{3}\right], \quad A = -\frac{1}{\cancel{3}}(\cancel{-3}) \Rightarrow 1$$

Putting the values of A & B in equ ①

$$X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{2}{1 - \frac{1}{3}z^{-1}} \rightarrow ③$$

→ Before determinging the inverse, we must specify the ROC associated with each of the term.

→ As ROC lies outside the outermost pole, The ROC ~~associated~~ for each individual term in equ(3) must lie outside the pole associated with that term.

→ Now , $x[n] = x_1[n] + x_2[n]$

$$x_1[n] \overset{z}{\longleftrightarrow} \frac{1}{1 - \frac{1}{4}z^{-1}}, \quad |z| > \frac{1}{4}$$

$$x_2[n] \overset{z}{\longleftrightarrow} \frac{2}{1 - \frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}$$

→ By inspection,

$$\therefore a^n u[n] \overset{z}{\longleftrightarrow} \frac{1}{1 - az^{-1}}$$

$$x_1[n] = \left(\frac{1}{4}\right)^n u[n], \quad x_2[n] = 2\left(\frac{1}{3}\right)^n u[n]$$

$$x[n] = \left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$$

EXAMPLE #5:

$$X(z) = \frac{1}{1-az^{-1}}, \quad |z| > |a|$$

Expand in a power series by long division.

Sol:

$$X(z) = \frac{1}{1-az^{-1}}$$

By using long division

$$\begin{array}{r|l}
 & 1 + az^{-1} + a^2z^{-2} + \cdots \\
\hline
1-az^{-1} & \cancel{1} \\
 & -\cancel{1} + az^{-1} \\
\hline
 & a\cancel{z}^{-1} \\
 & -\cancel{az^{-1}} + a^2z^{-2} \\
\hline
 & a^2\cancel{z}^{-2} \\
 & -\cancel{a^2z^{-2}} + a^3z^{-3} \\
\hline
 & a^3z^{-3} \\
 & \vdots
\end{array}$$

or

$$\frac{1}{1-az^{-1}} = 1 + az^{-1} + a^2z^{-2} + \cdots$$

→ The series expansion converges since $|z| > |a|$ or equivalently $|az^{-1}| < 1$

→ By matching terms in power of z, we see that $x[n] = 0$ $n < 0$, $x[0] = 1$, $x[1] = a$, $x[2] = a^2$ and in general $x(n) = a^n u(n)$