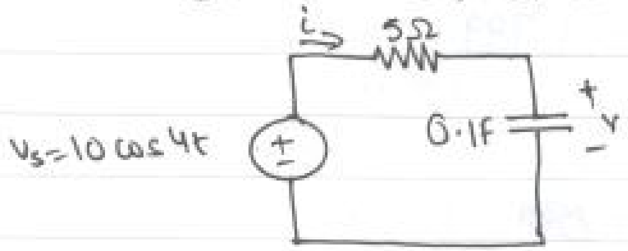
Day/Date Monday / 14th May, 18

Lecture # 7

Example # 1

Find $v(t)$ and $i(t)$ in the circuit shown below:



Sol:

From the voltage source, $\bar{V}_s = 10\angle 0° V$

The impedance is:

$$\bar{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1}$$

$$= 5 - j2.5\,\Omega$$

Hence the current

$$\bar{I} = \frac{\bar{V}_s}{\bar{Z}} = \frac{10\angle 0°}{5 - j2.5\,\Omega}$$

$\bar{Z} = 5 - j2.5\,\Omega$ convert in polar form

$$|Z| = \sqrt{R^2 + X^2} \quad , \quad \theta = \tan^{-1}\frac{X}{R}$$

$$= \sqrt{5^2 + (-2.5)^2} \Rightarrow 5.59 \quad , \quad = \tan^{-1}\frac{-2.5}{5} = -26.5°$$

$$\bar{I} = \frac{\bar{V}_s}{\bar{Z}} = \frac{10\angle 0°}{5.6\angle -26.5°}$$

$$= \frac{10}{5.6}\angle 0° - (-26.5°)$$

$$\bar{I} = 1.78\angle 26.5°$$

The voltage across the capacitor is :

$$\bar{V} = \bar{I} Z_C$$

$$= \frac{\bar{I}}{j\omega C} = \frac{1.789 \angle 26.57^\circ}{j4 \times 0.1}$$

$$= \frac{1.789 \angle 26.57^\circ}{j0.4}$$

$$j0.4 \Rightarrow |Z| = \sqrt{(0)^2 + (0.4)^2} \Rightarrow 0.4$$

$$\theta = 90^\circ$$

$$\bar{V} = \frac{1.789 \angle 26.57^\circ}{0.4 \angle 90^\circ}$$

$$= \frac{1.789}{0.4} \angle 26.57 - 90^\circ \Rightarrow 4.47 \angle -63.43$$

Converting $\bar{I}$ and $\bar{V}$ to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) A$$

$$v(t) = 4.47 \cos(4t - 63.43^\circ) V$$

Example #28

For the circuit below write the phasor expression for the impedance in both rectangular and polar form.:-

Sol:-

$$Z = R - jX_C \quad \text{: Rectangular form}$$

$$= 56 - j100 \Omega$$

The impedance in polar form is:-

$$|Z| = \sqrt{R^2+X^2}$$
$$= \sqrt{(56)^2+(+100)^2}$$
$$|Z| \Rightarrow 114.6 \simeq 115$$

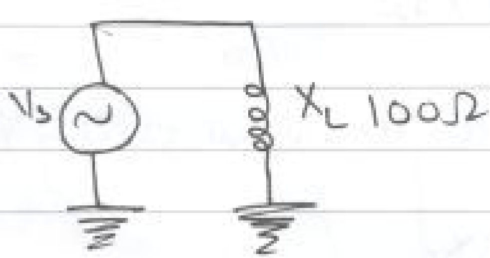
$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$\theta = \tan^{-1}\left(-\frac{100}{56}\right) \Rightarrow -60.8^\circ$$

$$\bar{Z} = |Z|\angle\theta$$
$$= 115\angle -60.8^\circ\ \Omega$$

EXAMPLE # 3:-

For the circuit below write the phasor expression for the impedance in both rectangular and polar form:



Sol:-

The impedance in rectangular form is

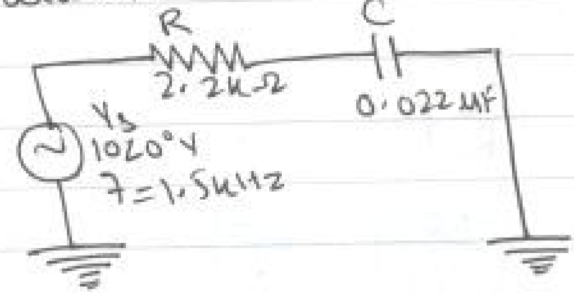
$$\mathbf{Z} = 0 + jX_L \Rightarrow j100\ \Omega \quad \because R=0$$

The impedance in polar form is:-

$$|Z| = \sqrt{0^2+100^2} \Rightarrow 100$$
$$\theta = 90^\circ$$
$$\mathbf{Z} = 100\angle 90^\circ\ \Omega$$

Example #4:-

Determine the current in the circuit shown below, and draw a phasor diagram showing the relation b/w source voltage and current.



Sol:-

The magnitude of the capacitive reactance is,

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi(1.5\,kHz)(0.022\times10^{-6})}$$

$$X_C = 4.82\,k\Omega$$

The total impedance in rectangular form is,

$$Z = R - jX_C$$

$$= 2.2\,k\Omega - j4.82\,k\Omega$$

Converting Z in polar form.

$|Z| = \sqrt{R^2 + X_C^2}$, $\theta = \tan^{-1}\frac{X_C}{R}$

$= \sqrt{(2.2k)^2 + (4.82k)^2}$, $= \tan^{-1}\frac{-4.82}{2.2}$

$|Z| \Rightarrow 5.30k$, $\theta \Rightarrow -65.5°$

$$Z = 5.30\angle -65.5\,k\Omega.$$

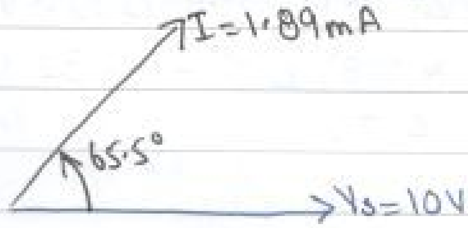
Use Ohm's law to determine the current

$$\bar{I} = \frac{\bar{V}}{Z} = \frac{10\angle 0^\circ \text{ V}}{5.30\angle -65.5 \text{ k}\Omega}$$

$$= \frac{10}{5.30} \angle 0 - (-65.5)$$

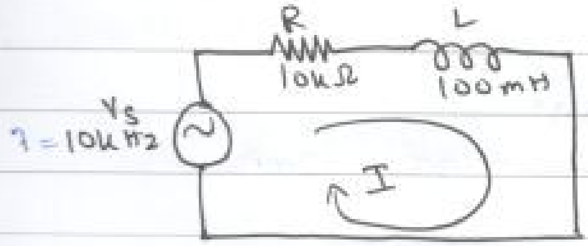
$$\bar{I} = 1.89\angle 65.5^\circ \text{ mA}$$

The magnitude of the current is 1.89 mA. The positive phase angle of 65.5° indicates that the current leads the voltage by that amount, as shown in the phasor diagram below



Example #5:

$$\bar{I} = 0.2\angle 0^\circ \text{ mA.}$$



Sol:

The magnitude of the inductive reactance is:

$$X_L = 2\pi f L$$

$$= 2\pi(10\times10^3)(100\times10^{-3}) \Rightarrow 6.28\text{k}\Omega$$

The impedance in rectangular form is

$$Z = R + jX_L$$

$$= 10\text{k}\Omega + j\,6.28\text{k}\Omega$$

Converting to polar fields

$$Z = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \sqrt{10k^2 + (6.28k)^2} \angle \tan^{-1}\left(\frac{6.28}{10}\right)$$

$$Z \Rightarrow 11.8 \angle 32.1^\circ k\Omega$$

Use Ohm's law to determine the source voltage.

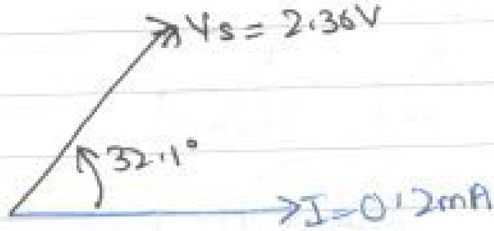
$$V_s = \bar{I} Z$$

$$= (0.2\angle 0^\circ mA)(11.8 \angle 32.1^\circ k\Omega)$$

$$= (0.2 \times 11.8) \angle 0^\circ mA + 32.1^\circ k\Omega$$

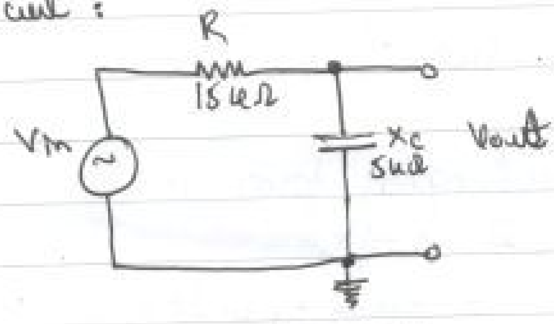
$$V_s = 2.36 \angle 32.1^\circ V$$

The magnitude of the source voltage is 2.36V at an angle of 32.1° with respect to the current. The voltage leads the current by 32.1°.



EXAMPLE #6:-

Determine the amount of phase $\angle$ lag from input to output in following lag circuit:



Day/Date

Sol:

For the lag circuit

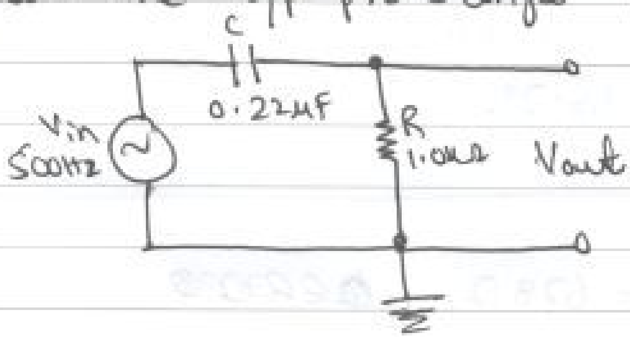
$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{15k\Omega}{5k\Omega}\right)$$

$$\phi = -71.6^\circ$$

The output lags the input by 71.6°.

EXAMPLE #7:

Calculate the o/p phase angle for the following circuit:



Sol:

First determine the capacitive reactance.

$$X_C = \frac{1}{2\pi f C}$$

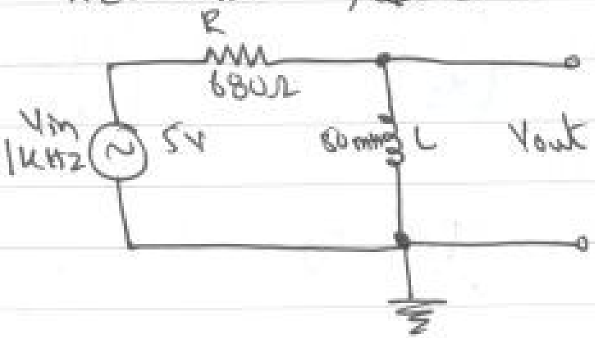
$$= \frac{1}{2\pi(500Hz)(0.22\times10^{-6})} \Rightarrow 1.45k\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1.45k\Omega}{1.0k\Omega}\right) \Rightarrow 55.4^\circ$$

The o/p leads by 55.4°.

Example #8:-

$X_L = 314\,\Omega$,



Sol:-

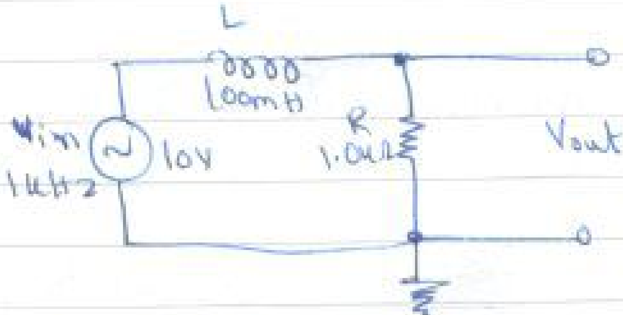
$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{680\,\Omega}{314\,\Omega}\right) \Rightarrow 65.2^\circ$$

The o/p leads the i/p by 65.2°.

Example #9:-

$V = 10V$, $X_L = 628\,\Omega$,

Sol:-



$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= -\tan^{-1}\left(\frac{628\,\Omega}{1k\Omega}\right) \Rightarrow -32.1^\circ$$

The o/p lags the input by 32.1°.