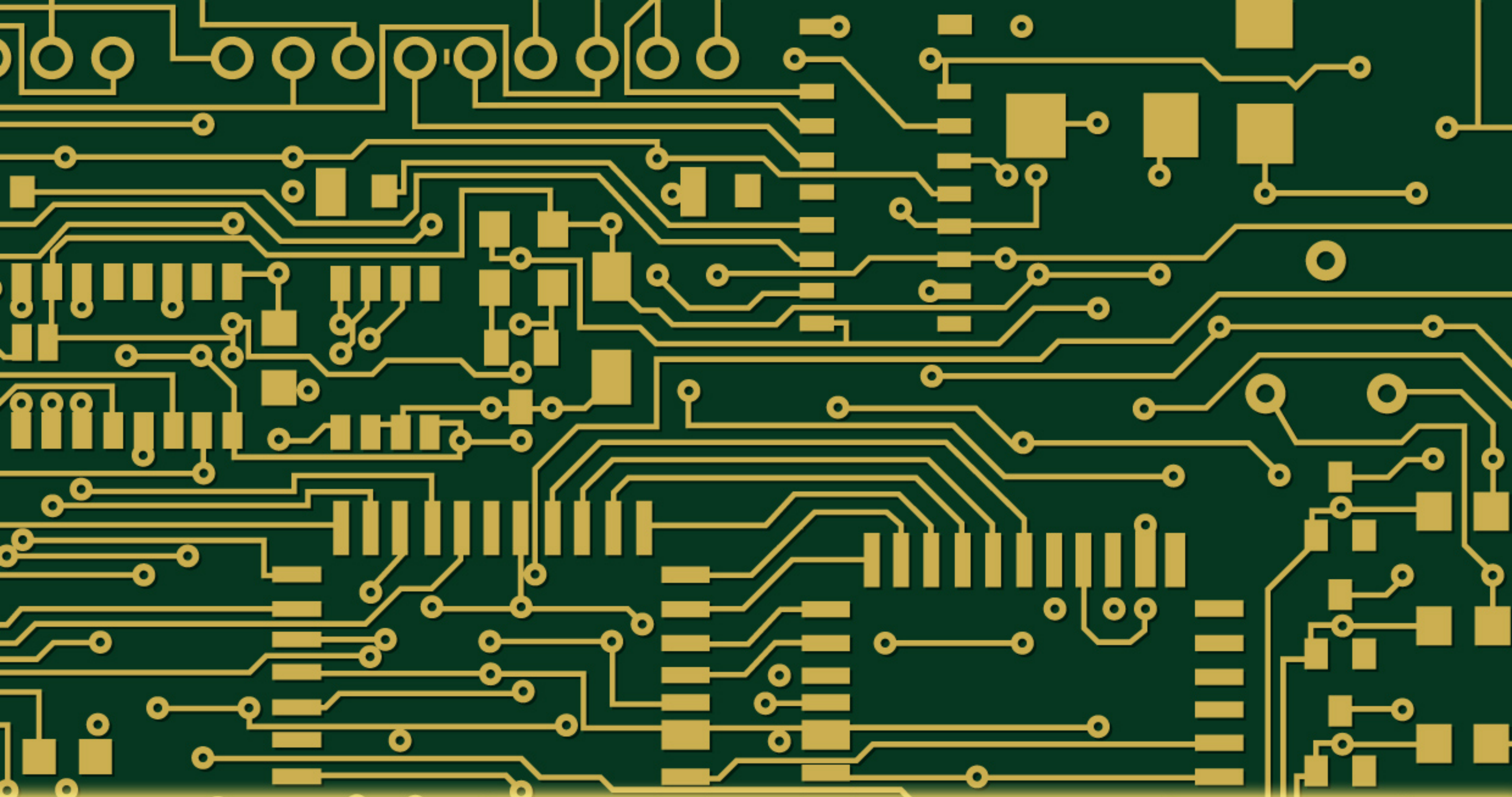
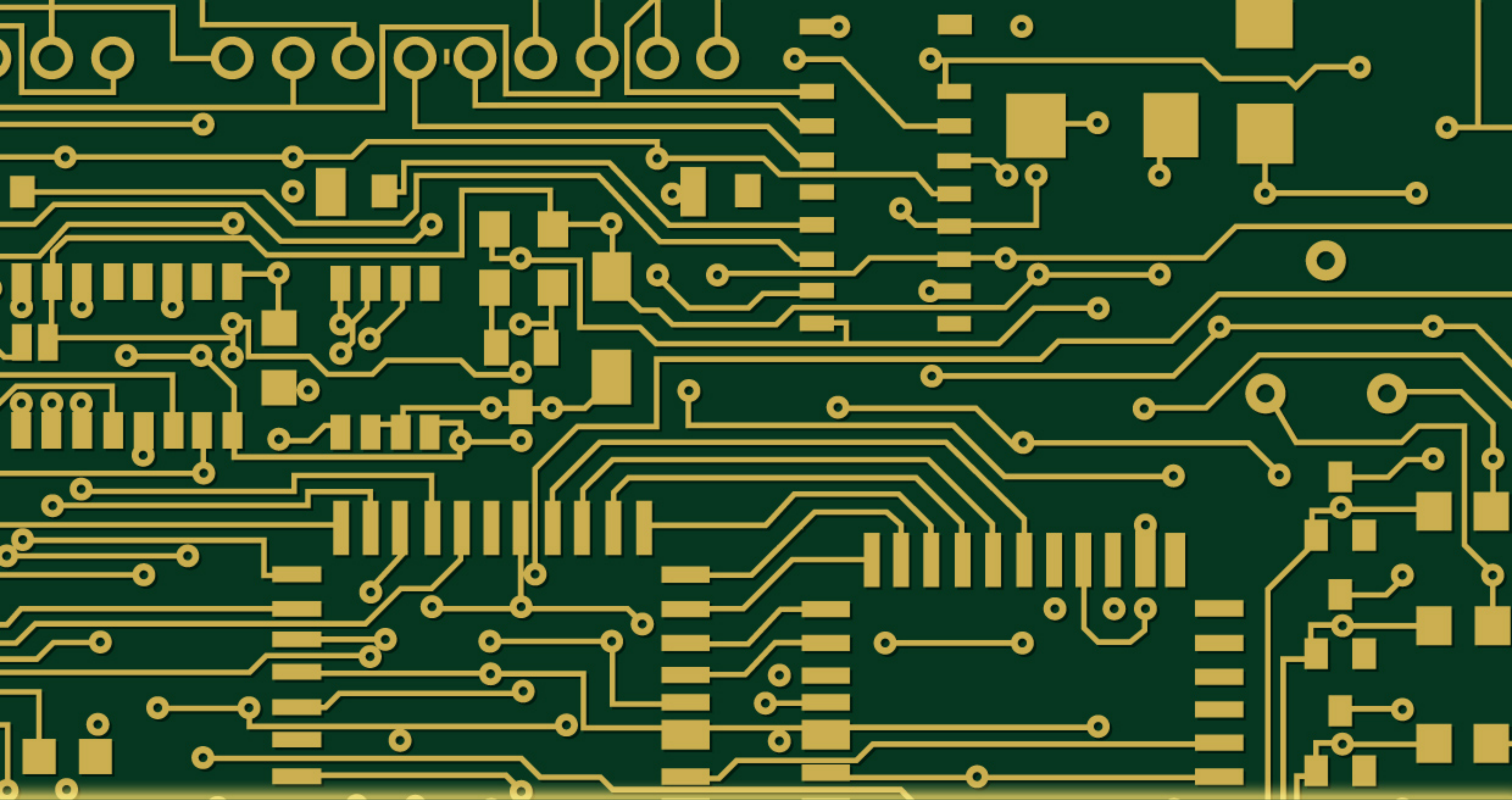


Circuit Analysis-II



Impedance & Admittance

Impedance

- ✓ The voltage-current relations for the three passive elements are:

$$\bar{V} = R\bar{I}, \quad \bar{V} = j\omega L\bar{I}, \quad \bar{V} = \frac{\bar{I}}{j\omega C}$$

- ✓ These equations may be written in terms of the ratio of the phasor voltage to the phasor current as:

$$\frac{\bar{V}}{\bar{I}} = R, \quad \frac{\bar{V}}{\bar{I}} = j\omega L, \quad \frac{\bar{V}}{\bar{I}} = \frac{1}{j\omega C}$$

- ✓ From these three expressions we obtain Ohm's law in phasor form for any type of element:

$$Z = \frac{\bar{V}}{\bar{I}} \quad or \quad \bar{V} = Z\bar{I}$$

Impedance (cont.)

- ✓ Where Z is a frequency-dependent quantity known as impedance, measured in ohms.
- ✓ The impedance Z of a circuit is the ratio of the phasor voltage **V** to the phasor current **I,** measured in ohms Ω.
- ✓ The impedance represents the opposition that the circuit exhibits to the flow of sinusoidal current.
- ✓ The impedance is the ratio of two Phasors, but it is nit a phasor because it dose not corresponds to a sinusoidal varying quantity.
- ✓ Consider two extreme cases of angular frequency:
  - ✓ When $\omega=0$ $Z_L=0$ and $Z_C \rightarrow \infty$ confirming that inductor act as a short circuit while capacitor acts like an open circuit.
  - ✓ When $\omega \rightarrow \infty$ $Z_L \rightarrow \infty$ and $Z_C=0$, indicates that the inductor is an open circuit while capacitor is a short circuit.

Impedance (cont.)

- ✓ As a complex quantity impedance can be expressed as:

$$Z = R + jX$$

$$Z = |Z| \angle \theta$$

$$\textit{where} \quad |Z| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1}\frac{X}{R}$$

$$\textit{and} \quad R = |Z|\cos\theta, \quad X = |Z|\sin\theta$$

- ✓ Where R=Re **Z** is the resistance and X=Im **Z** is the reactance.
- ✓ The reactance X may be positive or negative.
- ✓ The impedance is inductive when X is positive or capacitive when X is negative.
- ✓ Thus, impedance Z=R+jX is said to be inductive or lagging since current lags voltage.

Impedance (cont.)

- ✓ While impedance Z=R-jX is capacitive or leading because current leads voltage.
- ✓ Impedances and admittances of passive elements:

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

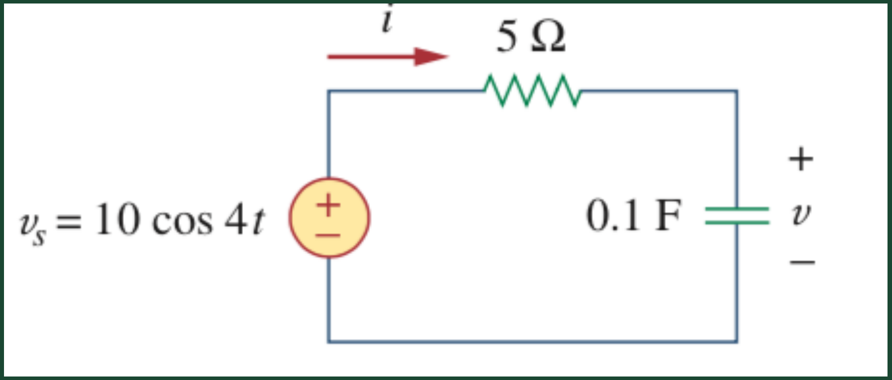
Admittance

- ✓ The admittance Y is the reciprocal of impedance, measured in siemens (S).
- ✓ The admittance Y of an element is the ratio of the phasor current through it to the phasor voltage across it:

$$Y = \frac{1}{Z} = \frac{\bar{I}}{\bar{V}}$$

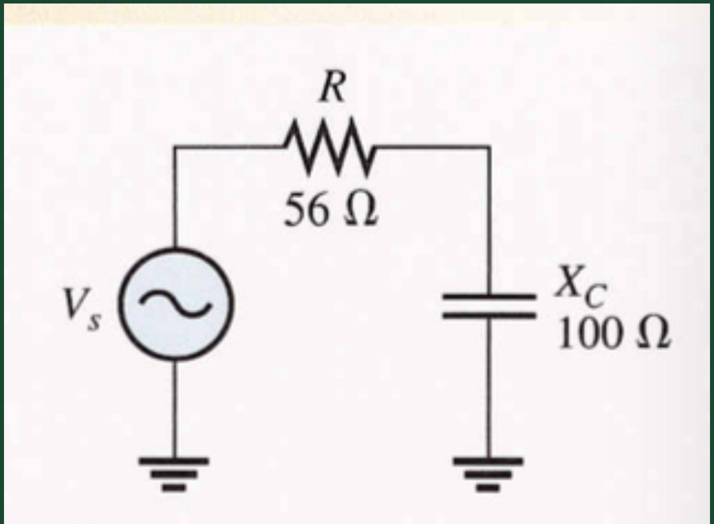
Example #1

✓ Find v(t) and i(t) in the circuit shown below:



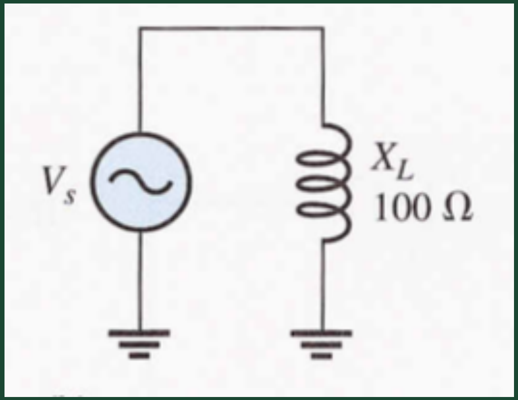
Example #2

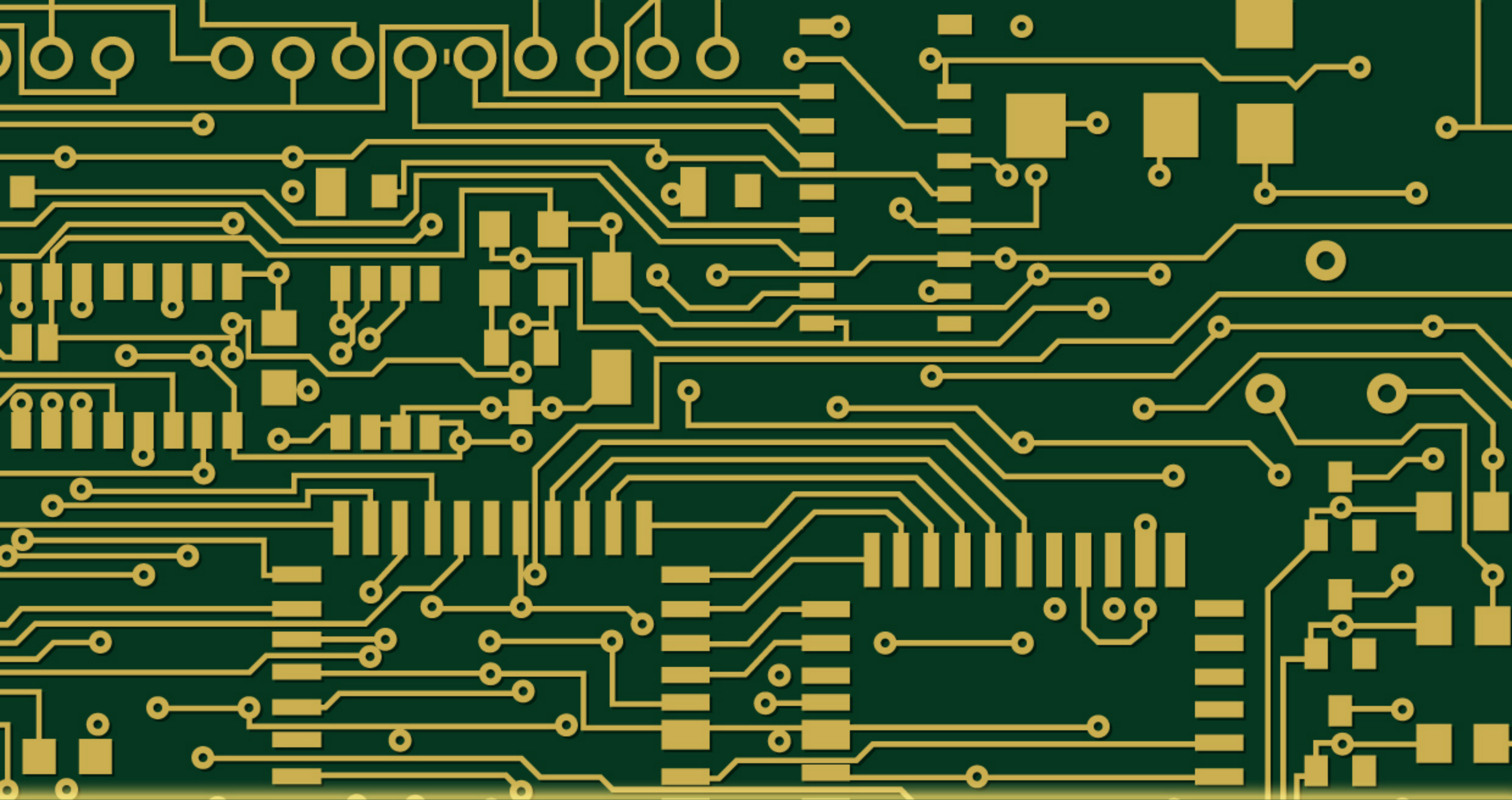
- ✓ For the circuit below write the phasor expression for the impedance in both rectangular and polar form:



Example #3

✓ For the circuit below write the phasor expression for the impedance in both rectangular and polar form:





Analysis of Series Circuits

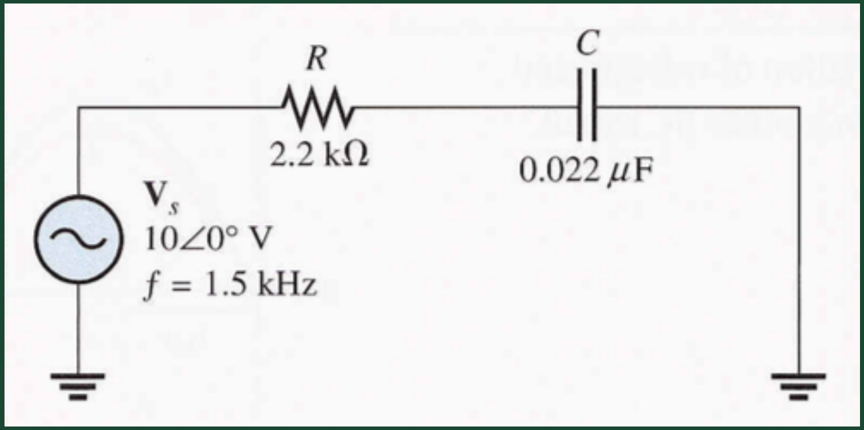
Series RC & RL Circuits

- ✓ For the analysis of series RC circuits Ohm's law can be used, which involves the phasor quantities of **Z**, **V**, and **I.**
- ✓ The three equivalent forms of Ohm's law are as follows:

$$\bar{V} = \bar{I}Z, \quad \bar{I} = \frac{\bar{V}}{Z}, \quad Z = \frac{\bar{V}}{\bar{I}}$$

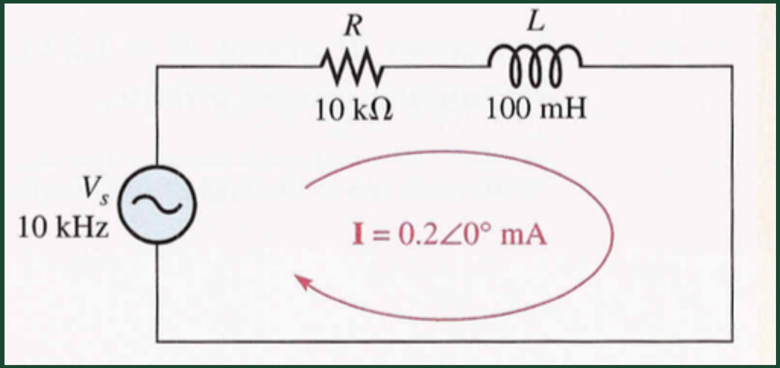
Example #4

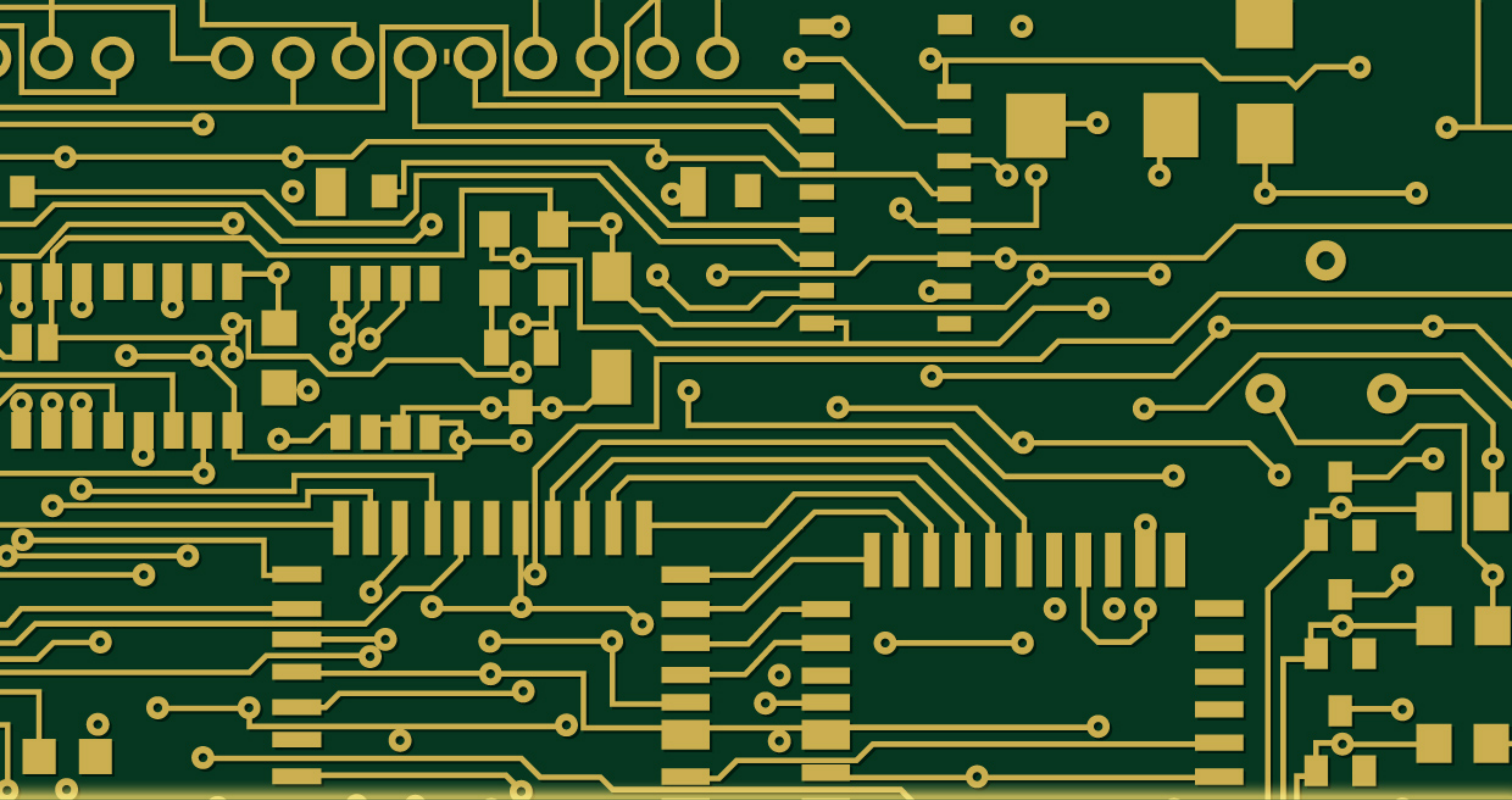
✓ Determine the current in the circuit shown below, and draw a phasor diagram showing the relation between source voltage and current.



Example #5

✓ The current in figure below is expressed in polar form as $\bar{I} = 0.2\angle 0° mA$. Determine the source voltage expressed in polar form, and draw a phasor diagram showing relationship between the source voltage and the current.

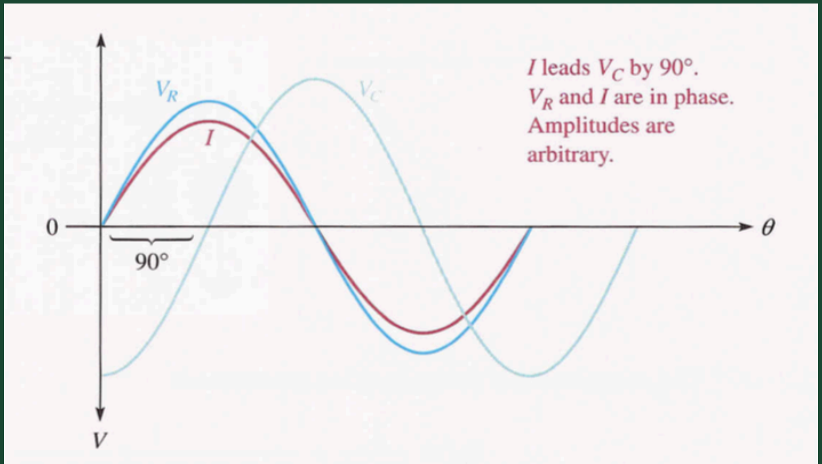




Phase Relationships of Current & Voltages

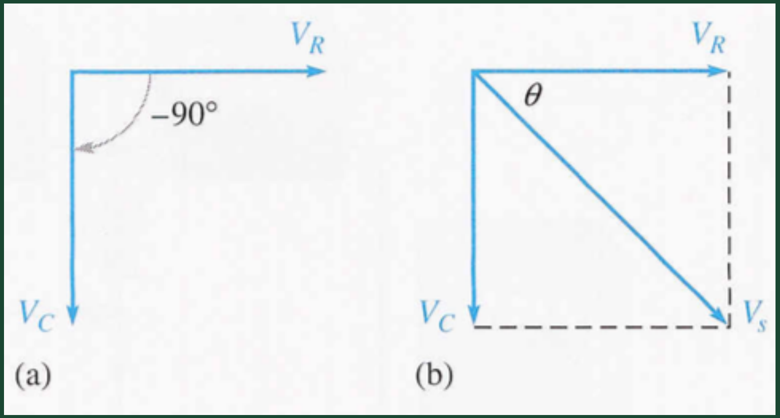
Series RC Circuit

- ✓ In a series RC circuit, the current is the same through both the resistor and capacitor.
- ✓ Thus, the resistor voltage is in phase with the current and the capacitor voltage lags the current by 90°.
- ✓ Therefore, there is a phase difference of 90° between the resistor voltage, V_R and the capacitor voltage V_C.

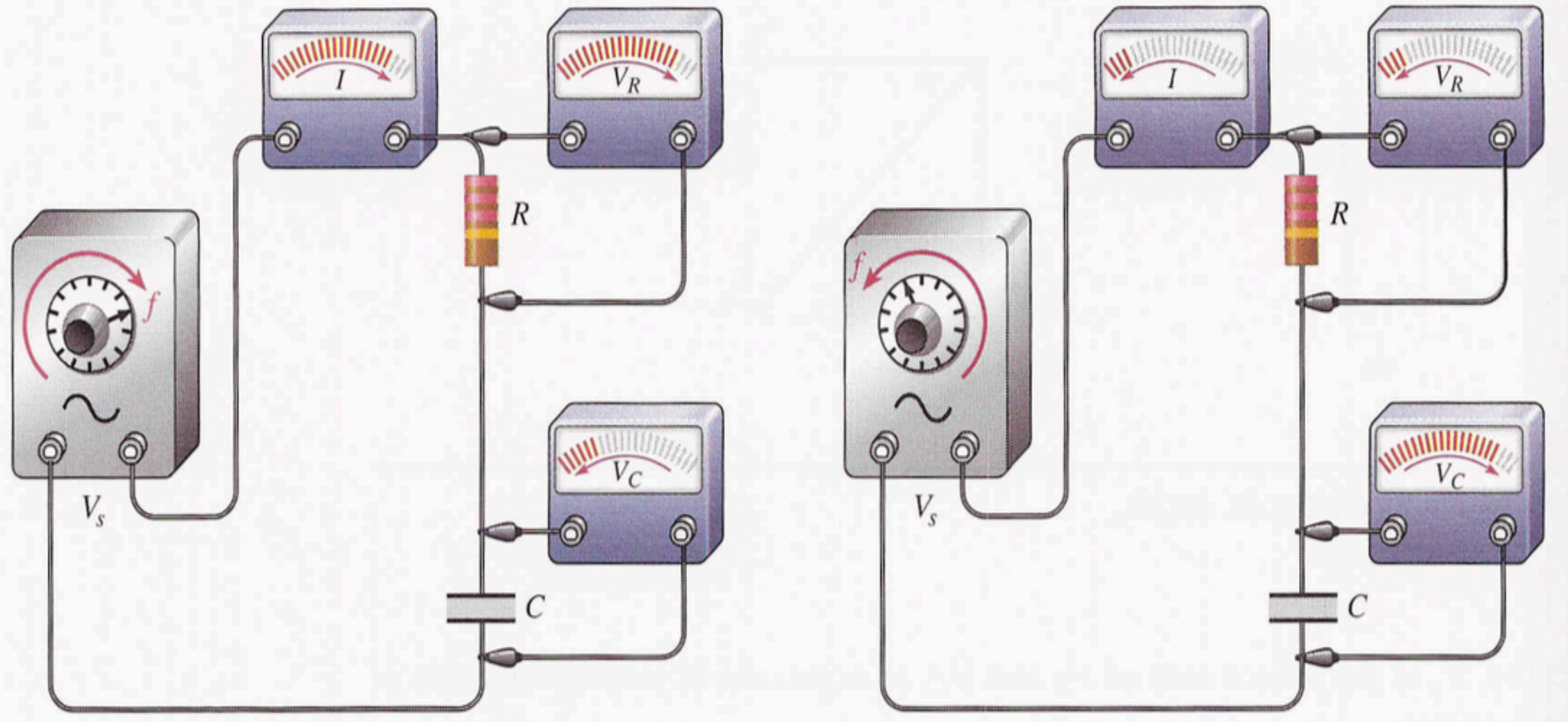


Series RC Circuit (cont.)

- ✓ From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage.
- ✓ Since V_R and V_C are not in phase with each other they must be added as phasor quantities.
- ✓ Voltage phasor diagram for a series RC circuit is as follows:



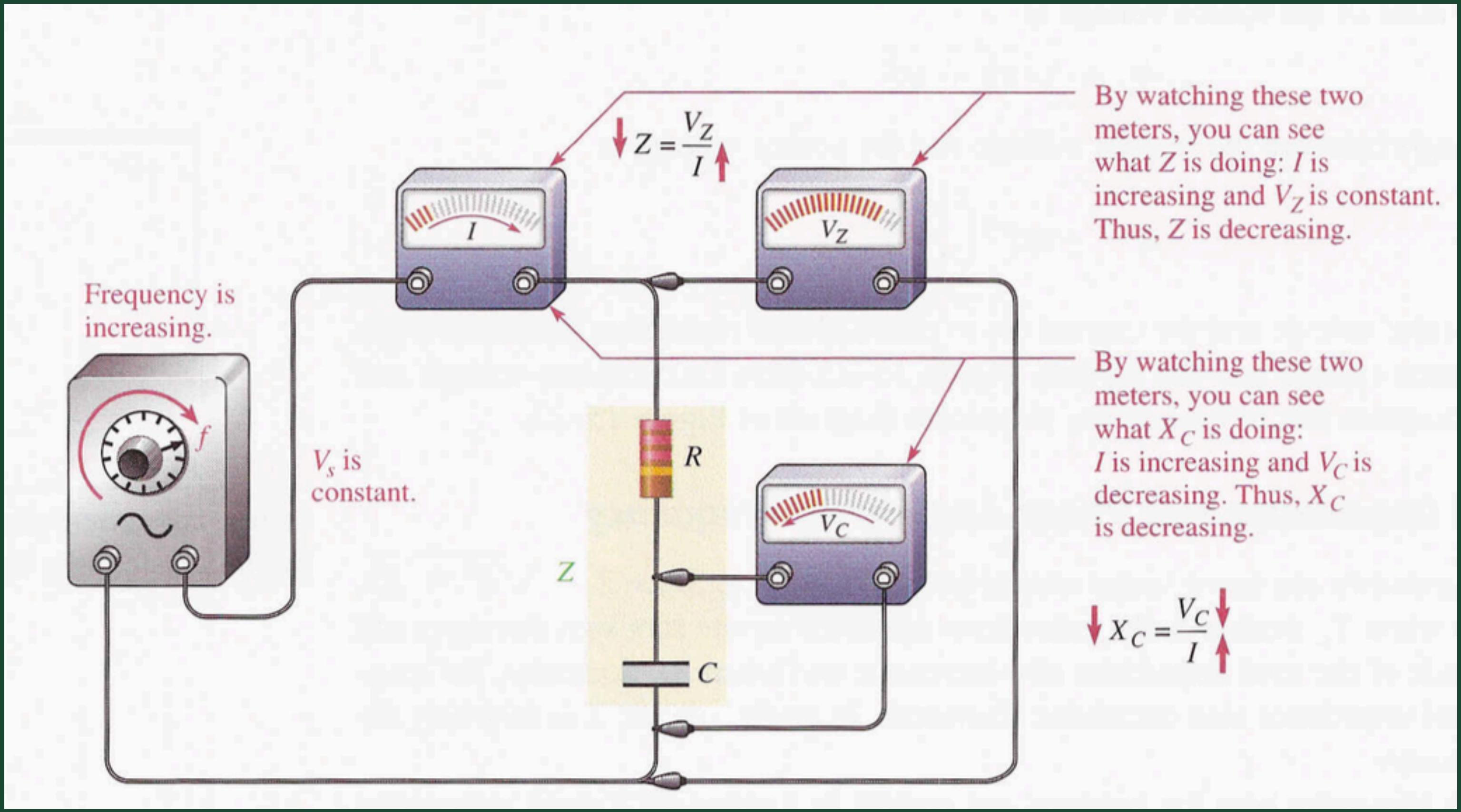
Variation of Impedance & Phase Angle with Frequency



(a) As frequency is increased, Z decreases as X_C decreases, causing I and V_R to increase and V_C to decrease.

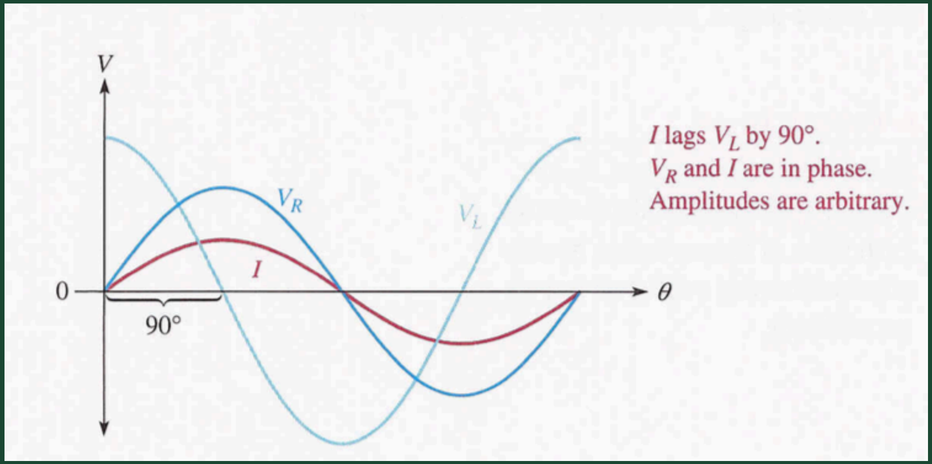
(b) As frequency is decreased, Z increases as X_C increases, causing I and V_R to decrease and V_C to increase.

Variation of Impedance & Phase Angle with Frequency (cont.)



Phase Relationships of RL Series Circuit

- ✓ In a series RL circuit, the current is the same through both the resistor and the inductor.
- ✓ Thus, the resistor voltage is in phase with the current and the inductor voltage leads the current by 90°.
- ✓ Therefore, there is a phase difference of 90° between the resistor voltage V_R and the inductor voltage V_L as shown below:



Phase Relationships of RL Series Circuit (cont.)

- ✓ The sum of the voltage drops must equal to the applied voltage.
- ✓ However, since V_R and V_L are not in phase with each other, they must be added as phasor quantities with V_L leading V_R by 90°.
- ✓ Then V_s is the phasor sum of V_R and V_L:

$$V_s = V_R + jV_L$$

- ✓ In the polar form it can be expressed as:

$$V_s = \sqrt{V_R^2 + V_L^2} \angle \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

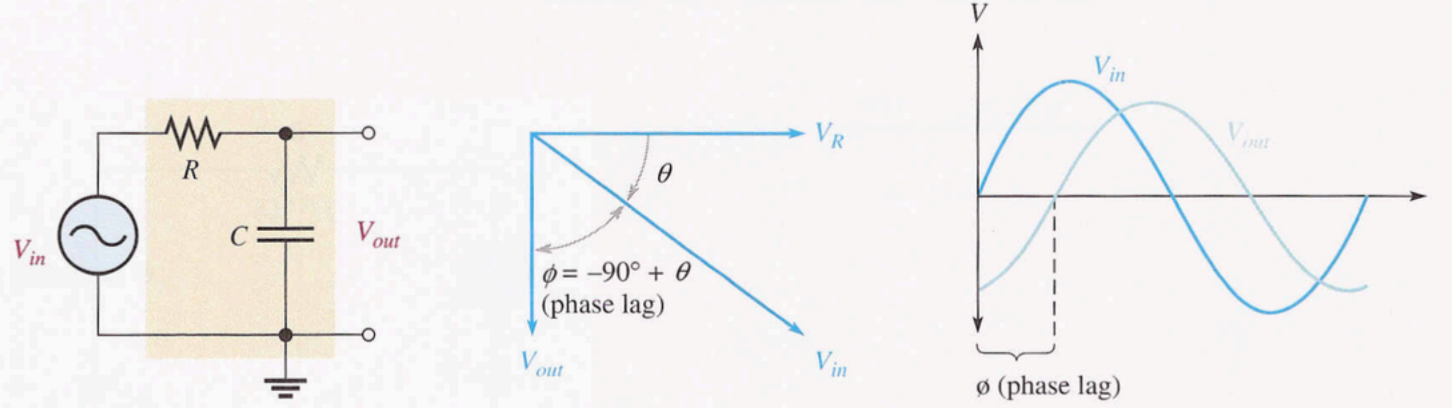
Variation of Impedance & Phase Angle with Frequency

- ✓ As we know inductive reactance varies directly with frequency.
- ✓ When X_L increases, the magnitude of the total impedance also increases and when X_L decreases the magnitude of the total impedance decreases.
- ✓ Thus, Z is directly dependent in frequency.
- ✓ The phase angle θ also varies directly with frequency because $\theta = \tan^{-1}(X_L/R)$. As X_L increases with frequency so does θ and vice versa.

RC with Lag & Lead Current

RC Lag Circuit

✓ An RC lag circuit is a phase shift circuit in which the output voltage lags the input voltage by a specified amount.



(a) A basic *RC* lag circuit

(b) Phasor voltage diagram showing the phase lag between V_{in} and V_{out}

(c) Input and output voltage waveforms

RC Lag Circuit (cont.)

- ✓ <u>Phase Difference Between Input and Output:</u>
  - ✓ θ is the phase angle between I and V_{in}.
  - ✓ The angle between V_{out} and V_{in} is designated ϕ and is developed as follows:
  - ✓ The polar expressions for the input voltage and the current are V_{in}<0° and O<θ respectively.
  - ✓ The output voltage in polar form is:

$$V_{out} = (I\angle\theta)(X_C\angle -90^\circ) = IX_C\angle(-90^\circ + \theta)$$

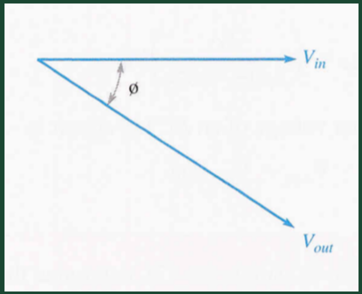
  - ✓ The angle ϕ between the input and output is:

$$\phi = -90 + \tan^{-1}\left(\frac{X_C}{R}\right)$$

  - ✓ This angle can also be expressed as: $\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$

RC Lag Circuit (cont.)

- ✓ This angle is always negative, indicating that the output voltage lags the input voltage as shown below:



- ✓ To evaluate the output voltage in terms of its magnitude, visualize the RC lag circuit as a voltage divider.

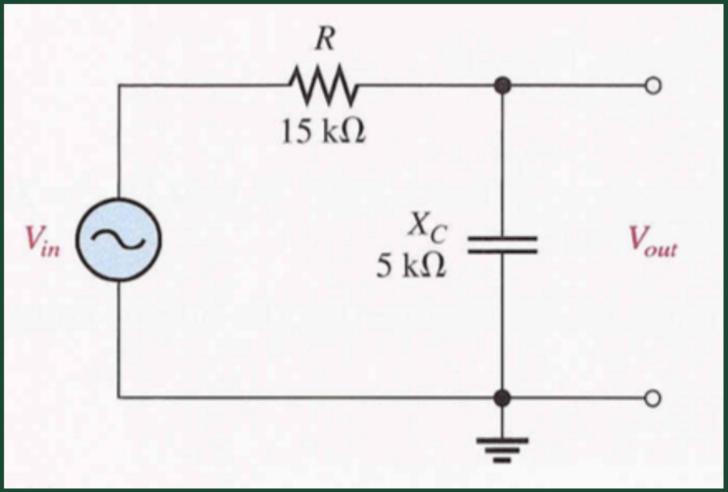
$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}} \right) V_{in}$$

- ✓ The phasor expression for the output voltage of an RC lag circuit is:

$$V_{out} = V_{out} \angle \phi$$

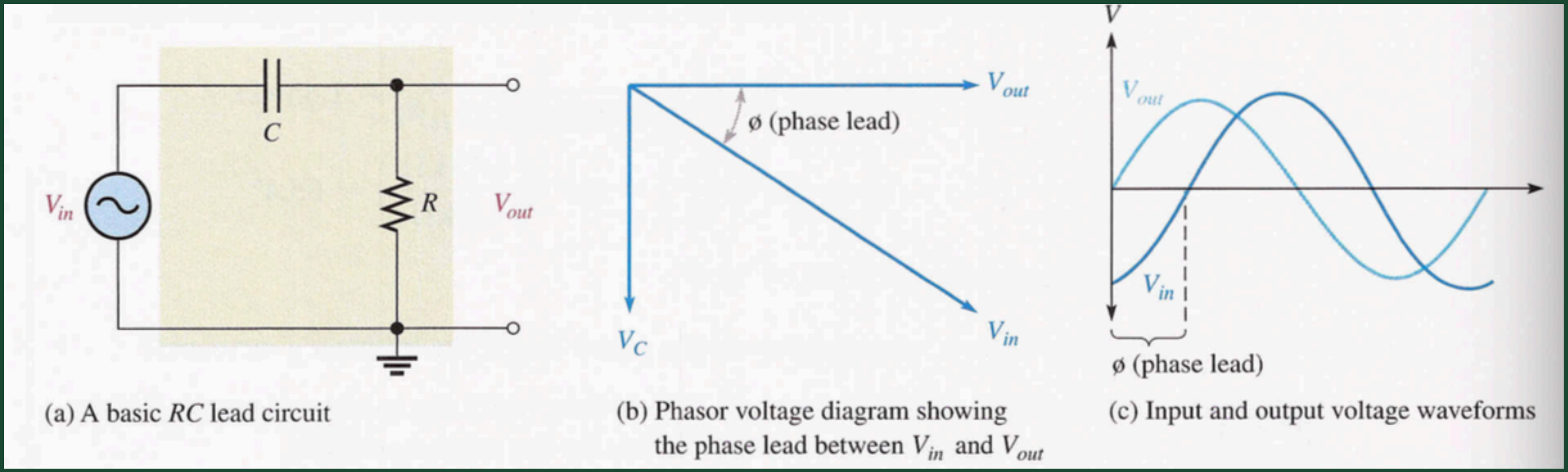
Example #6

✓ Determine the amount of phase lag from input to output in following lag circuit:



RC Lead Circuit

- ✓ An RC lead circuit is a phase shift circuit in which the output voltage leads the input voltage by a specified amount.
- ✓ When the output of a series RC circuit is taken across the resistor rather than across the capacitor it becomes a lead circuit.

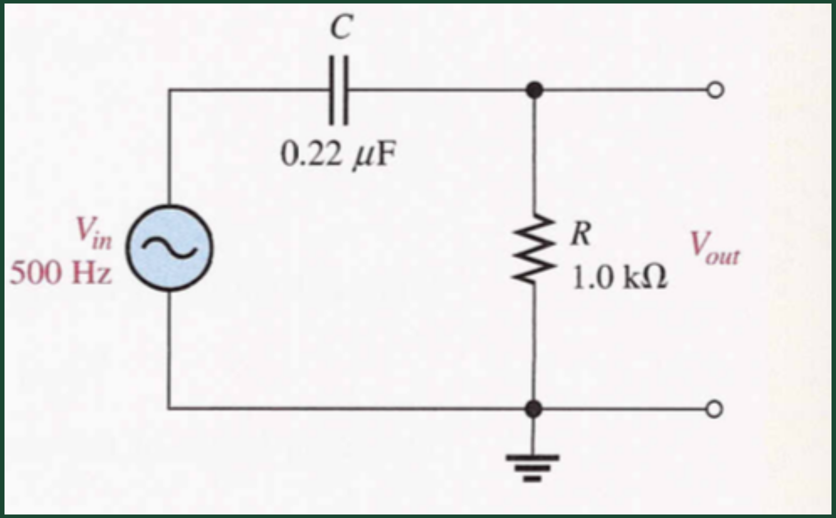


Phase Difference b/w Input & Output

- ✓ In series RC circuit, the current leads the input voltage.
- ✓ Also the resistor voltage is in phase with the current.
- ✓ Since the output voltage is taken across the resistor, the output leads the input.
- ✓ The amount of phase difference b/w the input and output and the magnitude of the output voltage in the lead circuit are dependent on the relative values of the resistance and the capacitive reactance.
- ✓ When the input voltage is assigned a reference angle of 0°, the angle of the output voltage is the same as θ because the resistor voltage and the current are in phase with each other.
- ✓ Therefore, since ϕ=θ, the expression is: $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

Example #7

✓ Calculate the output phase angle for following circuit:



Magnitude of the Output Voltage

- ✓ The voltage divider formula for the RC lead circuit is:

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}} \right) V_{in}$$

- ✓ The expression for the output voltage in phasor form is:

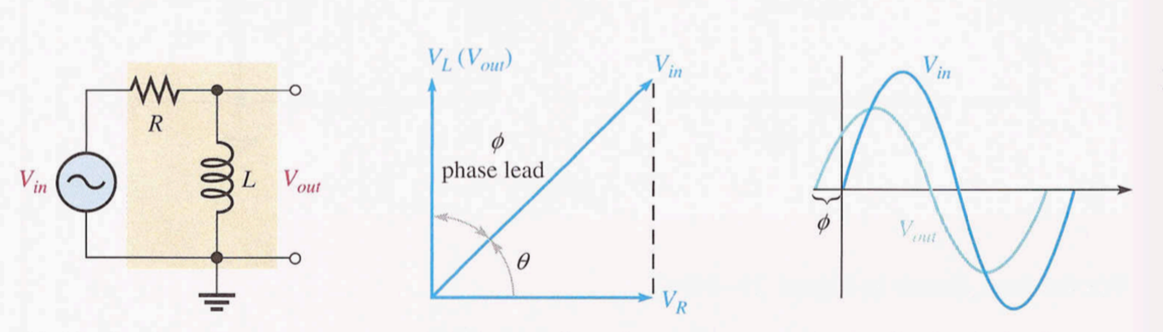
$$V_{out} = V_{out} \angle \phi$$

RL with Lag & Lead Current

RL Lead Circuit

- ✓ An RL lead circuit is a phase shift circuit in which the output voltage leads the input voltage by a specified amount.
- ✓ The source voltage is the input V_{in}.
- ✓ As θ is the angle between the current and the input voltage, it is also the angle between the resistor voltage and the input voltage because V_R and I are in phase.
- ✓ Since V_L leads V_R by 90°, the phase angle between the inductor voltage and the input voltage is the difference between 90° and θ.
- ✓ The inductor voltage is the output; it leads the input and thus creating a basic lead circuit.

RL Lead Circuit (cont.)



(a) A basic RL lead circuit

(b) Phasor voltage diagram showing V_{out} leading V_{in}

(c) Input and output voltage waveforms

- ✓ <u>Phase Difference B/w Input and Output:</u>
- ✓ The polar expressions for the input voltage and the current are V_{in}<0° and I<-θ respectively.
- ✓ The output voltage in polar form is:

$$V_{out} = \left(I\angle-\theta\right)\left(X_L\angle 90°\right) = IX_L\angle\left(90°-\theta\right)$$

RL Lead Circuit (cont.)

- ✓ The angle ϕ between the input and output is:

$$\phi = 90 - \tan^{-1}\left(\frac{X_L}{R}\right) \quad or \quad \phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

- ✓ <u>Magnitude of the Output Voltage:</u>
- ✓ The voltage divider formula is:

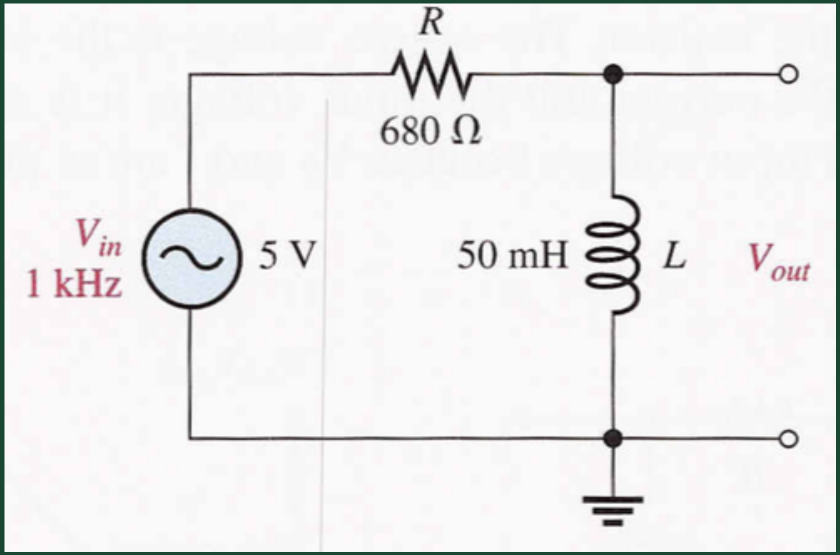
$$V_{out} = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

- ✓ The phasor expression for the output voltage of an RL lead circuit is:

$$V_{out} = V_{out}\angle\phi$$

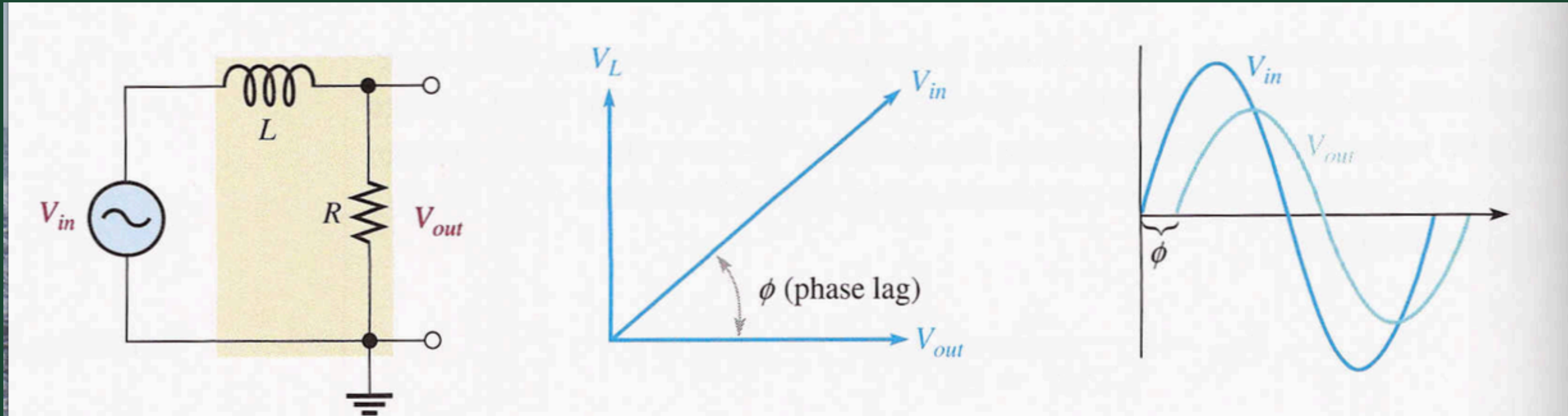
Example #8

- ✓ For the lead circuit shown below, determine the output voltage in phasor form when the input voltage has an rms value of 5V. Draw the input and output voltage waveforms showing their peak values.
- ✓ The inductive X_L is 314Ω .



RL Lag Circuit

- ✓ An RL lag circuit is a phase shift circuit in which the output voltage lags the input voltage by a specified amount.
- ✓ When the output of a series RL circuit is taken across the resistor rather than the inductor, it becomes a lag circuit.



(a) A basic *RL* lag circuit

(b) Phasor voltage diagram showing phase lag between V_{in} and V_{out}

(c) Input and output waveforms

RL Lag Circuit (cont.)

- ✓ Phase Difference b/w Input & Output:
- ✓ Since the output voltage is taken across the resistor, the output lags the input.
- ✓ When the input voltage is assigned a reference angle of 0°, the angle of the voltage with respect to the input voltage equals θ because the resistor voltage and the current are in phase with each other.
- ✓ The expression for the angle is:

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

RL Lag Circuit (cont.)

- ✓ <u>Magnitude of the Output Voltage:</u>
- ✓ The voltage divider formula is:

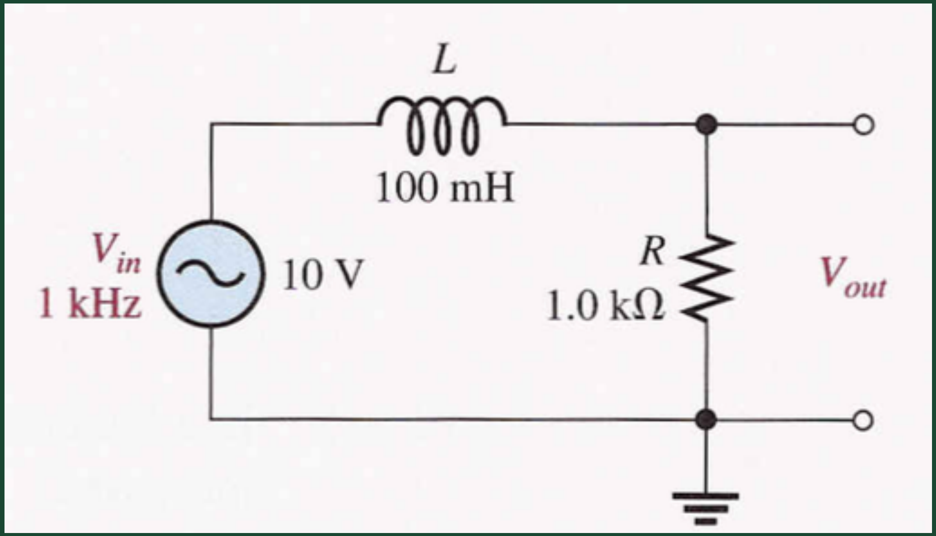
$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_L^2}} \right) V_{in}$$

- ✓ The expression for the output voltage in phasor form is:

$$V_{out} = V_{out} \angle \phi$$

Example #9

- ✓ The input voltage in the figure below, has an rms value of 10V. Determine the phasor expression for the output voltage. Draw the input and output voltage waveforms.
- ✓ The inductive X_L is 628Ω.



Thank You